Control of Laser-Beam Self-Focusing, Filamentation, and Rogue-Wave Formation Using Structured Light Beams

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The visuals of this talk will be posted at boydnlo.ca/presentations

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Ottawa

Rochester

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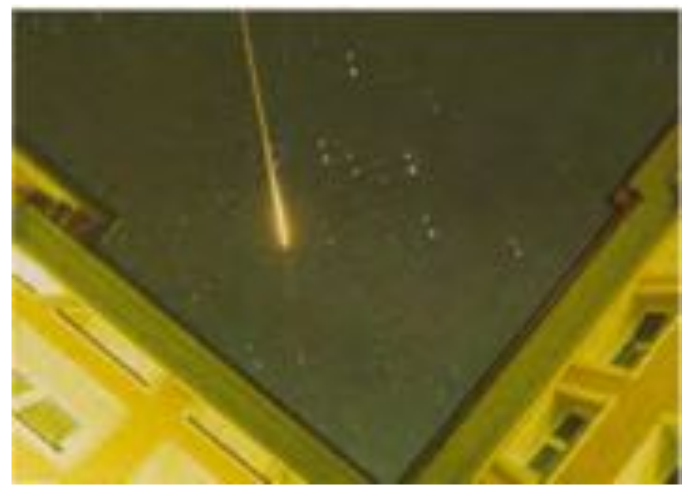
1. Background on self-action effects
2. Control using spatially structured polarization
3. Caustics and rogue waves
4. Control through the spatial spectrum of phase noise
5. Polarization knots

Why Care About Self-Focusing and Filamentation

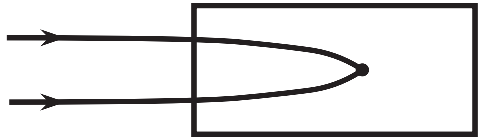
- Optical switching
- Laser modelocking
- Directed energy
  - prevent filamentation
  - controlled self focusing



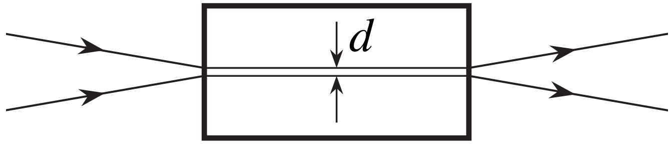
Self-Action Effects in Nonlinear Optics

Self-action effects: light beam modifies its own propagation

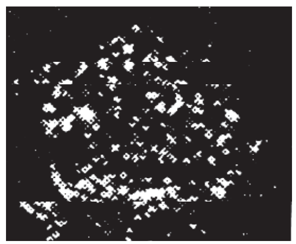
self-focusing



self-trapping



small-scale filamentation
or filamentation
or nonlinear beam breakup



SOVIET PHYSICS JETP VOLUME 15, NUMBER 6 DECEMBER, 1962

EFFECTS OF THE GRADIENT OF A STRONG ELECTROMAGNETIC BEAM ON ELECTRONS AND ATOMS

G. A. ASKAR'YAN

P. N. Lebedev Physics Institute, Academy of Sciences, U.S.S.R.

Submitted to JETP editor December 22, 1961

J. Exptl. Theoret. Phys. (U.S.S.R.) **42**, 1567-1570 (June, 1962)

It is shown that the transverse inhomogeneity of a strong electromagnetic beam can exert a strong effect on the electrons and atoms of a medium. Thus, if the frequency exceeds the natural frequency of the electron oscillations (in a plasma or in atoms), then the electrons or atoms will be forced out of the beam field. At subresonance frequencies, the particles will be pulled in, the force being especially large at resonance. It is noted that this effect can create either a rarefaction or a compression in the beam and at the focus of the radiation, maintain a pressure gradient near an opening from an evacuated vessel to the atmosphere, and create a channel for the passage of charged particles in the medium.

It is shown that the strong thermal ionizing and separating effects of the ray on the medium can be used to set up waveguide propagation conditions and to eliminate divergence of the beam (self-focusing). It is noted that hollow beams can give rise to directional flow and ejection of the plasma along the beam axis for plasma transport and creation of plasma current conductors. The possibilities of accelerating and heating plasma electrons by a modulated beam are indicated.

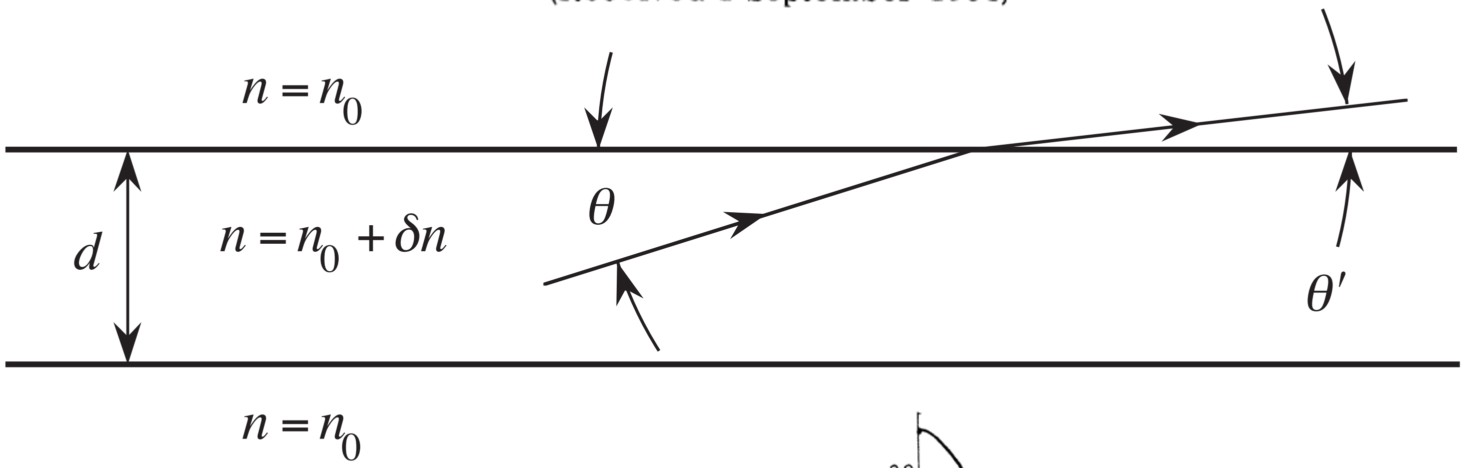
First use of the term "self-focusing".

Prediction of Self Trapping

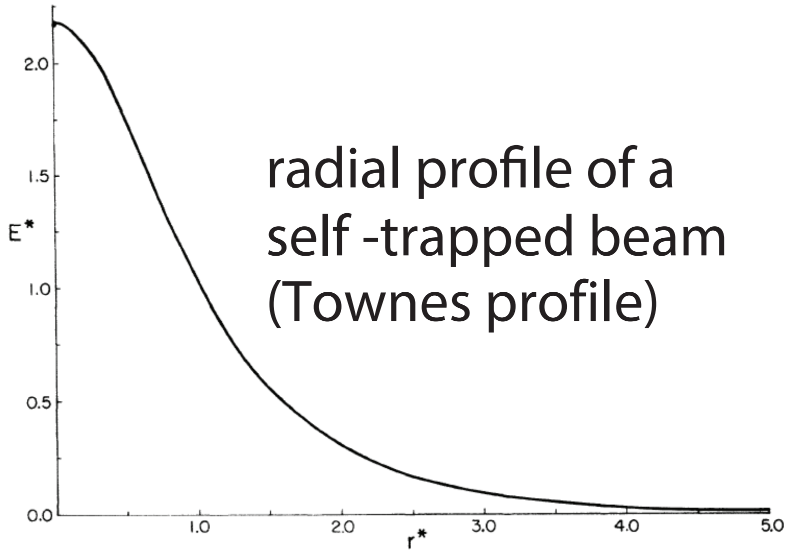
VOLUME 13, NUMBER 15 PHYSICAL REVIEW LETTERS 12 OCTOBER 1964

SELF-TRAPPING OF OPTICAL BEAMS*

R. Y. Chiao, E. Garmire, and C. H. Townes
Massachusetts Institute of Technology, Cambridge, Massachusetts
(Received 1 September 1964)



$$P_{cr} = \frac{\pi (0.61)^2 \lambda_0^2}{8 n_0 n_2}$$

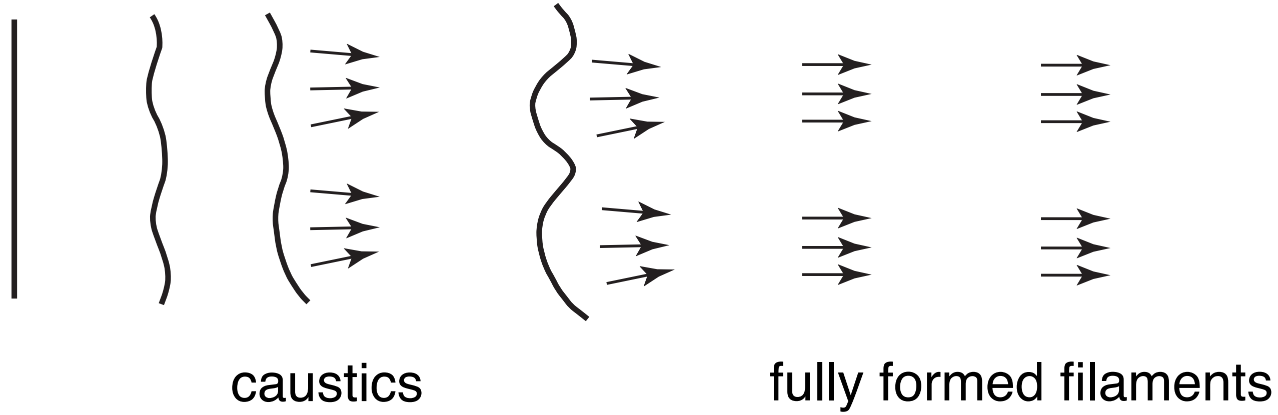


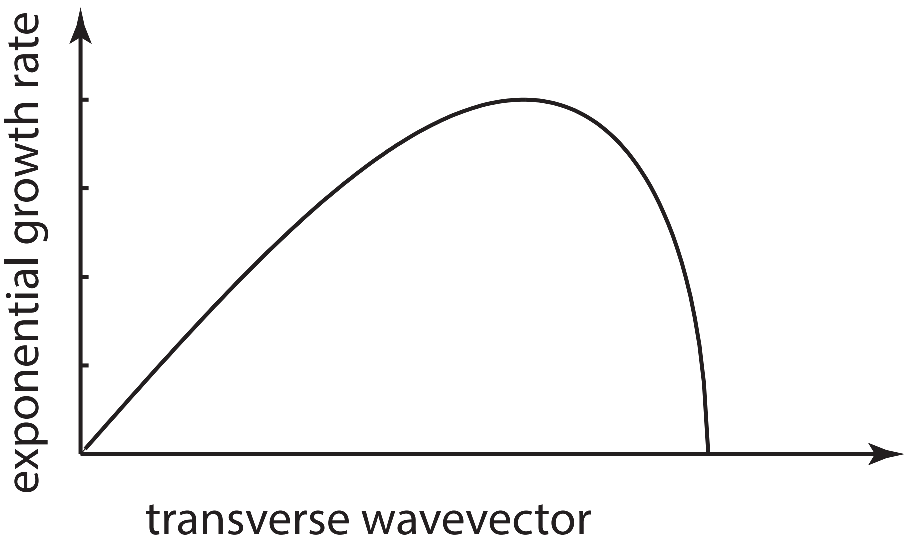
radial profile of a self -trapped beam (Townes profile)

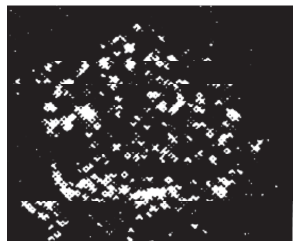
Beam Breakup by Small-Scale Filamentation

Predicted by Bespalov and Talanov (1966)

Exponential growth of wavefront imperfections by four-wave mixing processes







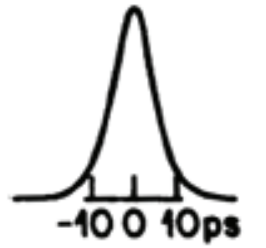
What is the spatial correlation length of noise present on the laser beam?

Optical Solitons

Field distributions that propagate without change of form

Temporal solitons (nonlinearity balances gvd)

$$\frac{\partial \tilde{A}_s}{\partial z} + \frac{1}{2} i k_2 \frac{\partial^2 \tilde{A}_s}{\partial \tau^2} = i\gamma |\tilde{A}_s|^2 \tilde{A}_s.$$

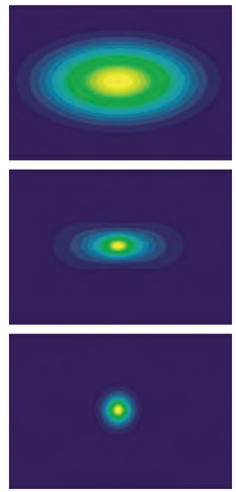


1973: Hasegawa & Tappert

1980: Mollenauer, Stolen, Gordon

Spatial solitons (nonlinearity balances diffraction)

$$2ik_0 \frac{\partial A}{\partial z} + \frac{\partial^2 A}{\partial x^2} = -3\chi^{(3)} \frac{\omega^2}{c^2} |A|^2 A$$



1964: Garmire, Chiao, Townes

1974: Ashkin and Bjorkholm (Na)

1985: Barthelemy, Froehly (CS2)

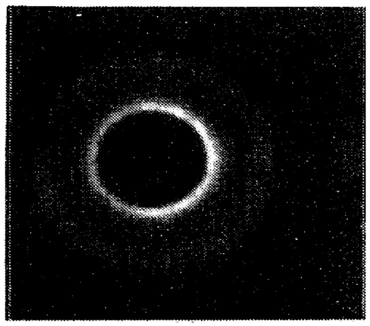
1991: Aitchison et al. (planar glass waveguide

1992: Segev, (photorefractive)

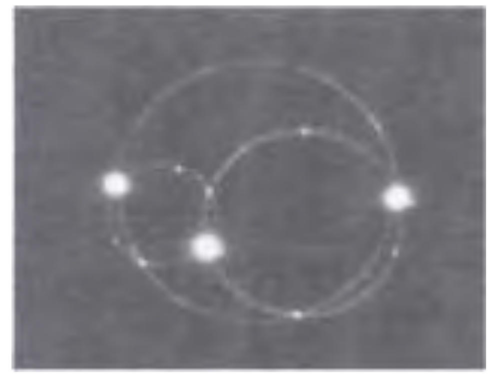
Self-Focusing Can Produce Unusual Beam Patterns

Pattern depends sensitively upon initial conditions

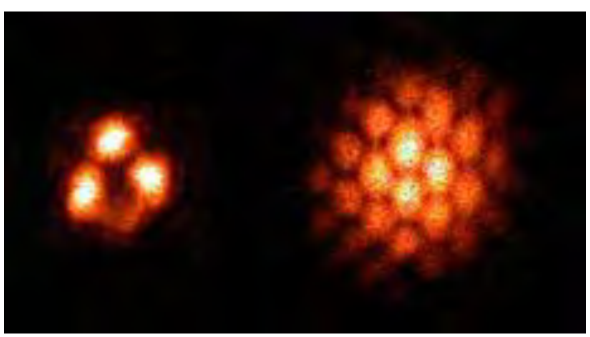
- Conical emission
  Harter et al., PRL 46, 1192 (1981)



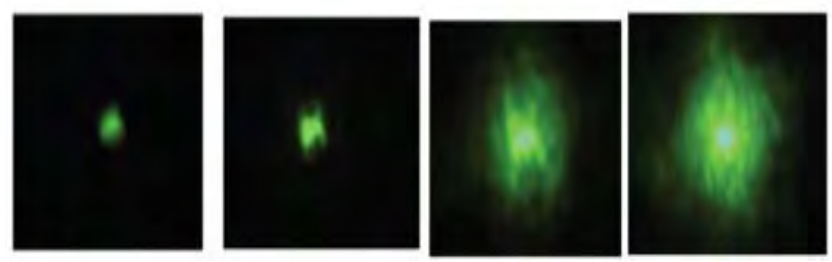
- Multiple ring patterns
  Kauranen et al, Opt. Lett. 16, 943, 1991;



- Honeycomb pattern formation
  Bennink et al., PRL 88, 113901 2002.



- Loss of spatial coherence
  Schweinsberg et al., Phys. Rev. A 84, 053837 (2011).



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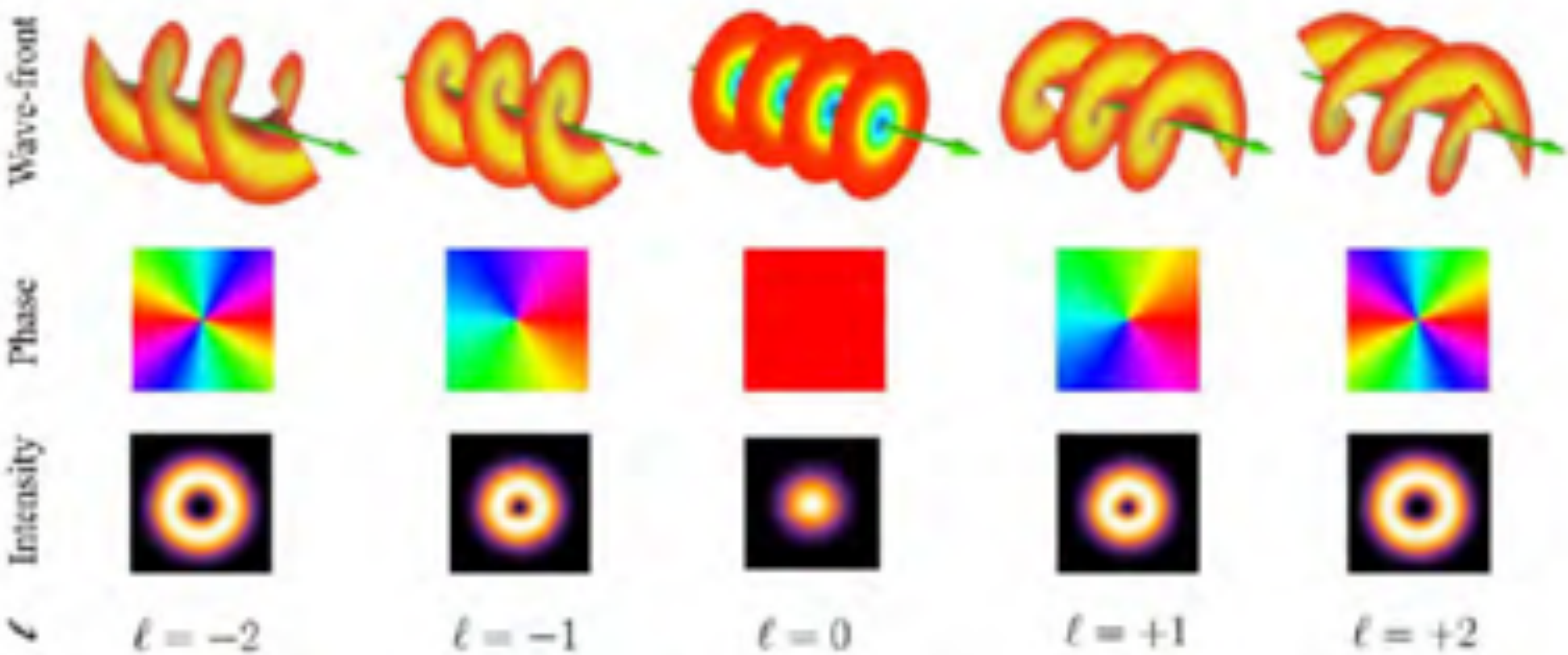
Self-Focusing of Structured Light: OAM States of Light

- Light can carry spin angular momentum by means of its circular polarization.
- Light can also carry orbital angular momentum by mean of a phase winding of the optical wavefront
- A well-known example are the Laguerre-Gauss modes. These modes contain a phase factor of $\exp(i\ell\phi)$ and carry angular momentum of $\ell\hbar$ per photon

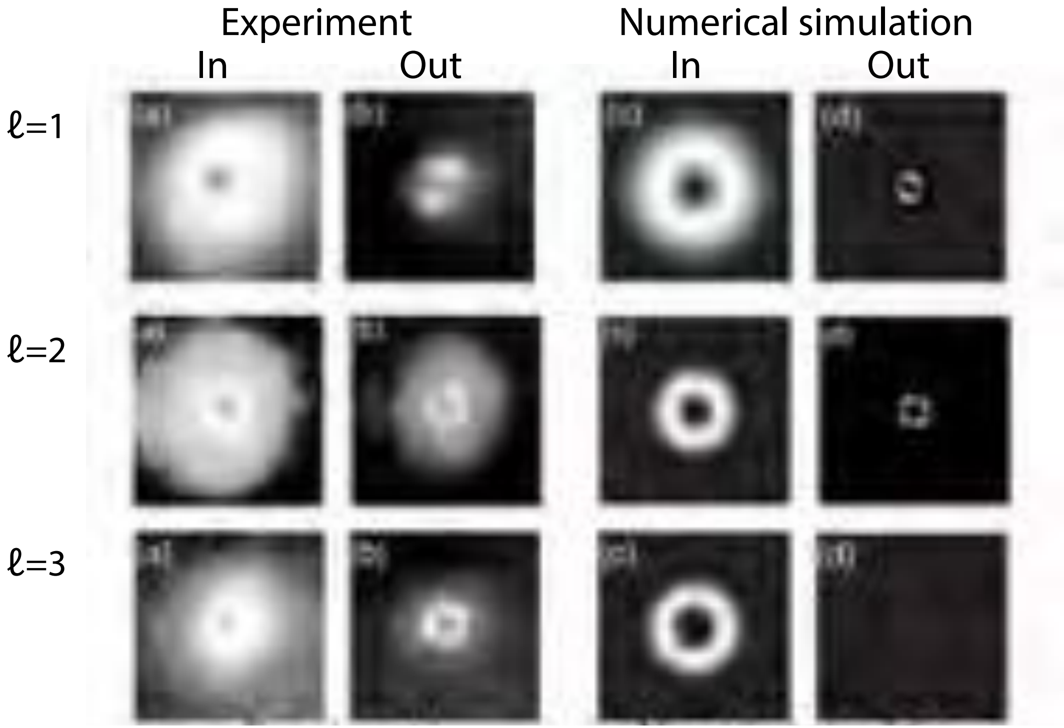


- How is self-focusing modified by the structuring of a light beam?

Breakup of Ring Beams Carrying Orbital Angular Momentum (OAM) in Sodium Vapor

- Firth and Skryabin predicted that ring shaped beams in a saturable Kerr medium are unstable to azimuthal instabilities.
- Beams with OAM of $\ell\hbar$ tend to break into 2ℓ filaments.

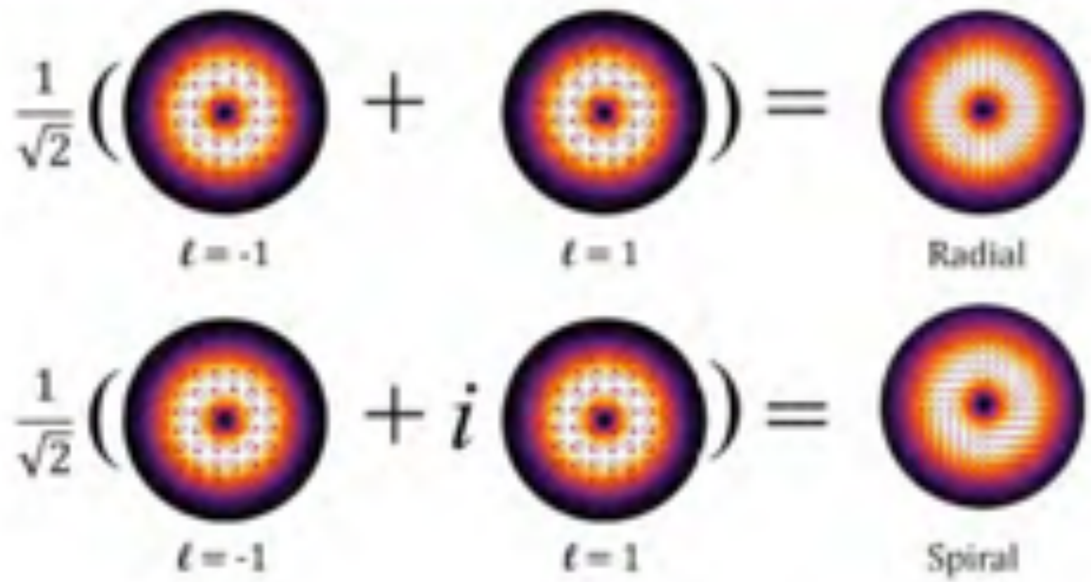
  (But aberrated OAM beams tend to break into $2\ell + 1$ filaments.)



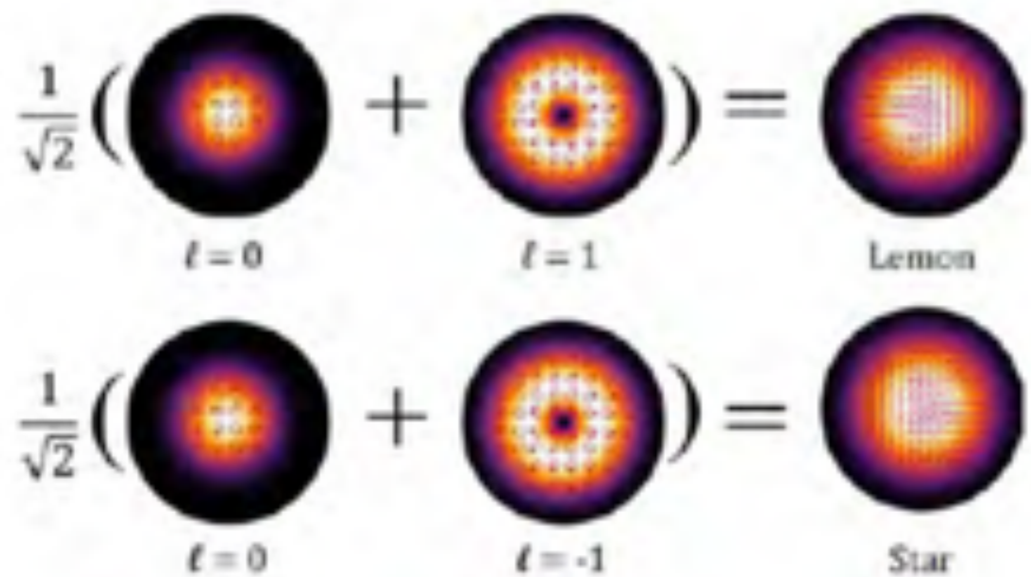
M.S. Bigelow, P. Zerom, and R.W. Boyd, Phys. Rev. Lett 92, 083902 (2004)

Space-Varying Polarized Light Beams

– Vector Vortex Beams



– Poincare Beams

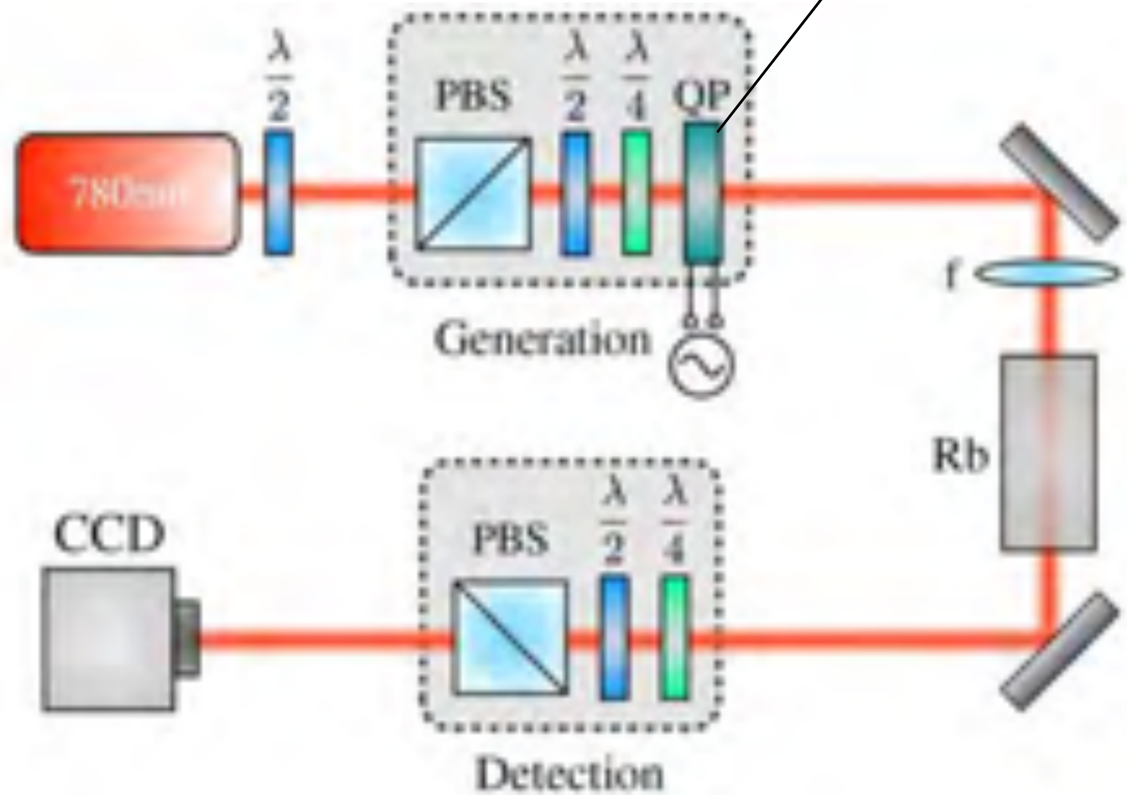


- How do these beams behave under conditions of self-focusing and filamentation?

Bouchard et al, PRL 117, 233903 (2016).

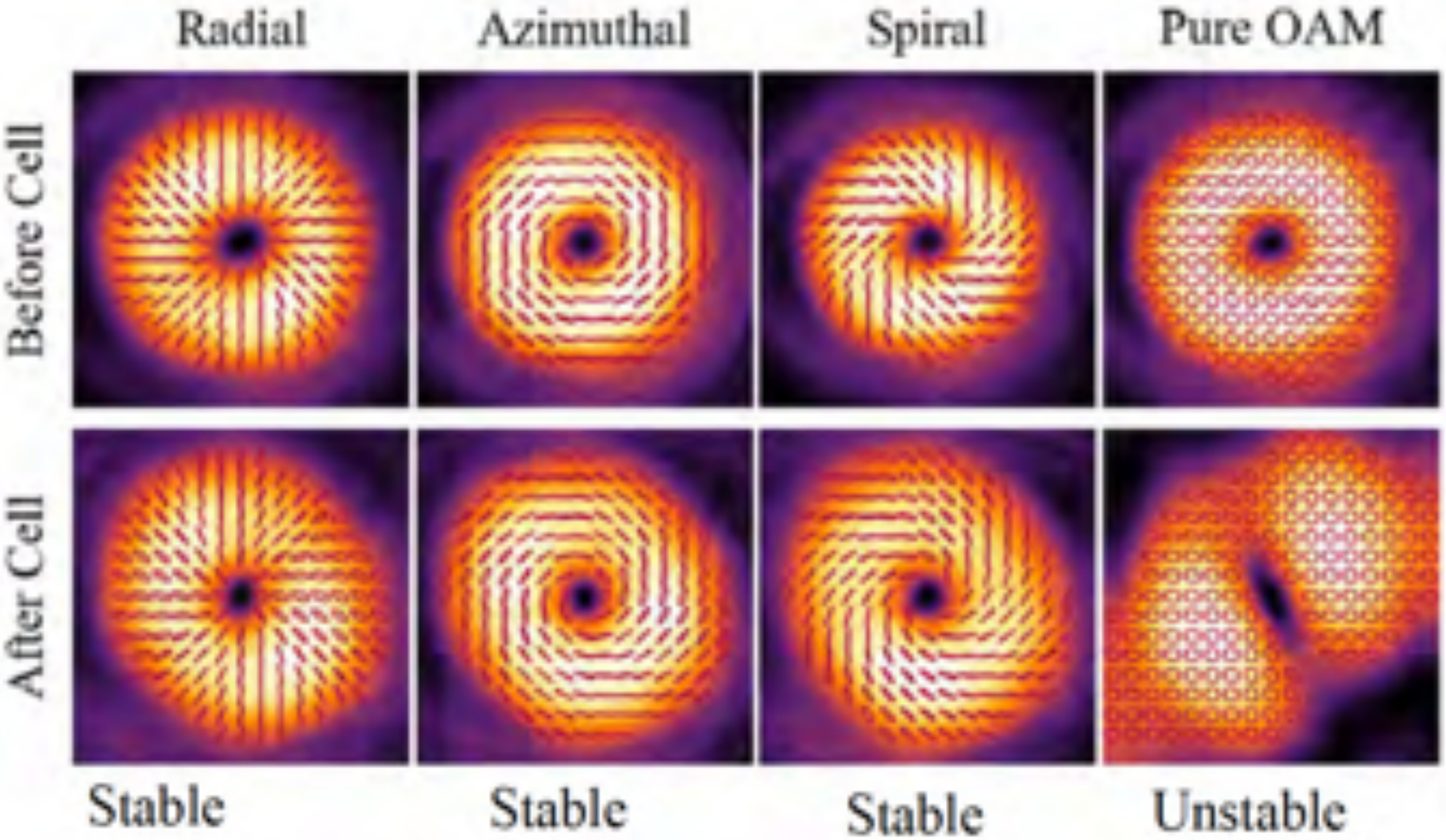
Experimental Setup

Q-plate: SAM to OAM converter



Results – Vector Beams

(Experimental Results)



Intensity and polarization distributions of vector and LG beams before and after propagating through the Rb atomic vapour.

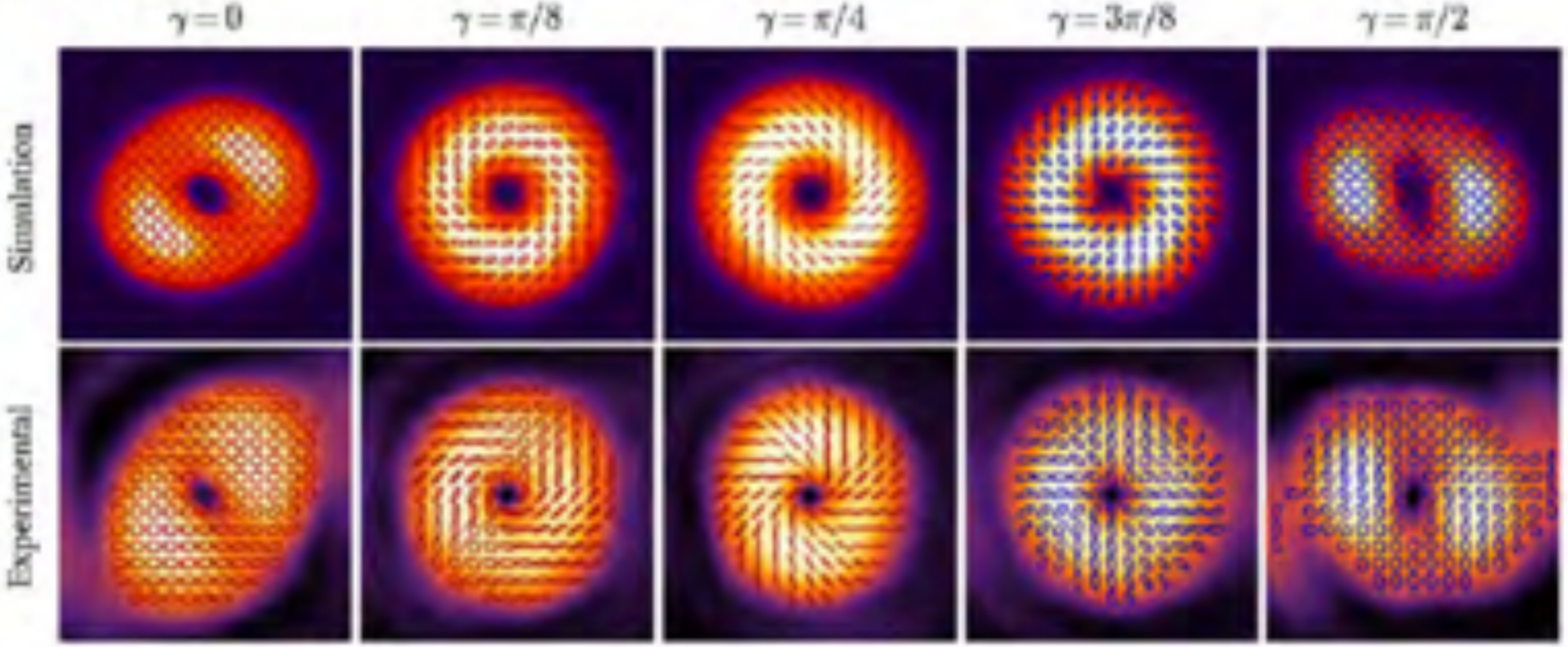
Numerical Modeling of the Experimental Results

- Coupled nonlinear propagation equations

$$\frac{\partial E_L}{\partial \zeta} - \frac{i}{2}\nabla_\perp^2 E_L = i\gamma \frac{|E_L|^2 + \nu |E_R|^2}{1+\sigma\left(|E_L|^2 + \nu |E_R|^2\right)} E_L$$

$$\frac{\partial E_R}{\partial \zeta} - \frac{i}{2}\nabla_\perp^2 E_R = i\gamma \frac{|E_R|^2 + \nu |E_L|^2}{1+\sigma\left(|E_R|^2 + \nu |E_L|^2\right)} E_R$$

- Comparison



Conclusions: stability of vector OAM beams

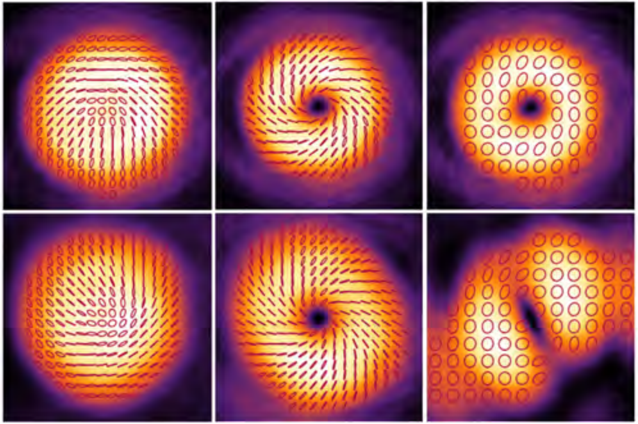
- Pure OAM beam:
  beam breakup
- Vector vortex beams:
  stable propagation
- Poincaré beams:
  stable propagation

Bouchard et al, PRL 117, 233903 (2016).

Poincaré beam | Vector vortex beam | Pure OAM beam

In

Out

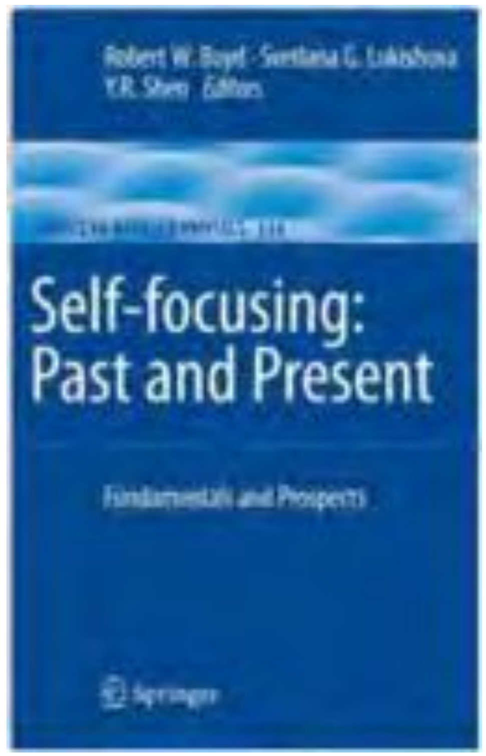


Stable | Stable | Unstable

Summary

- Even more than 50 years after their inceptions, self-focusing and filamentation remain fascinating topics for investigation.

- If you want to learn more:



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Influence of Nonlinearity on the Creation of Rogue Waves

- Study rogue-wave behavior in a well-characterized optical system

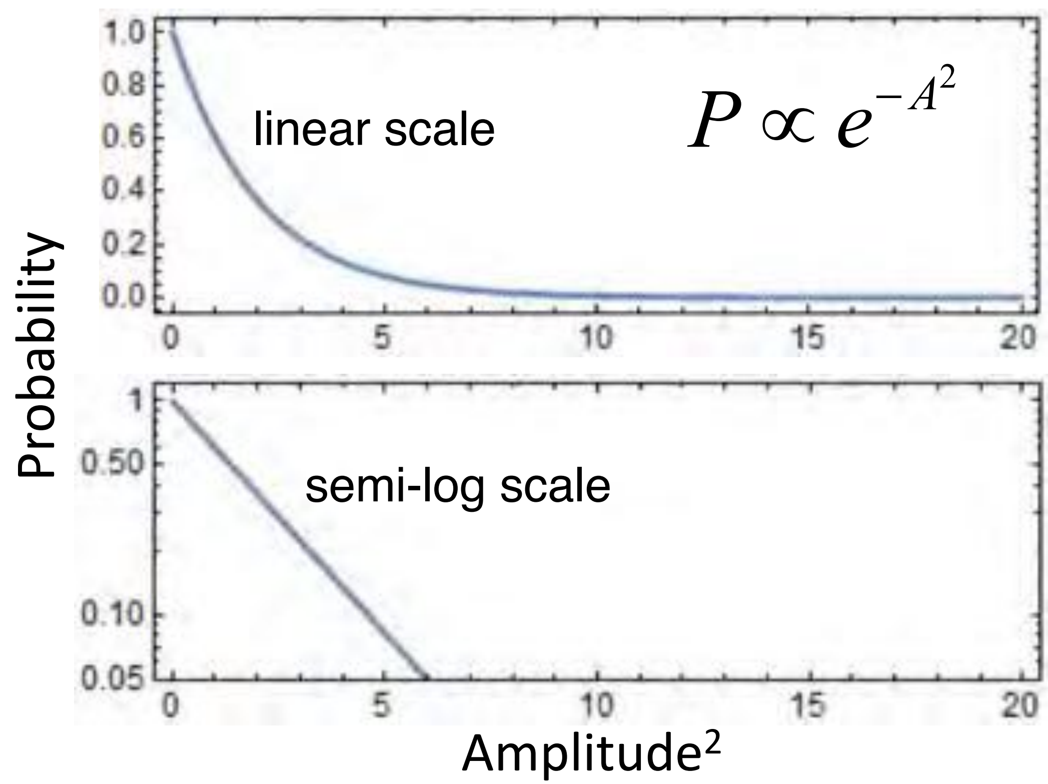
- Is nonlinearity important? Required? Or does it actually inhibit rogue-wave formation?

A. Safari, R. Fickler, M. J. Padgett and R. W. Boyd, Phys. Rev. Lett. 119, 203901 (2017).

Oceanic rogue waves

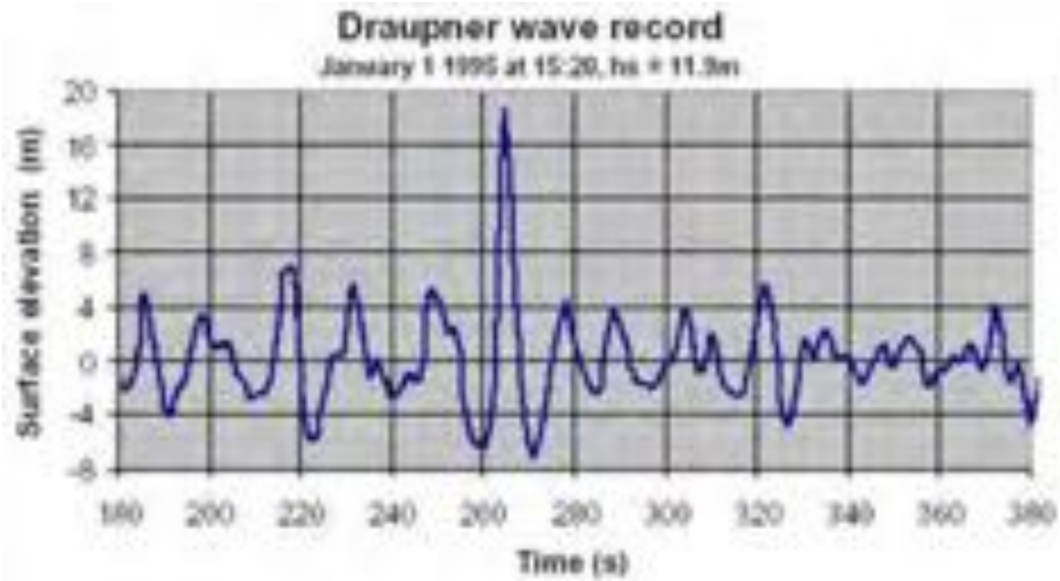
Before 1995

Sailors: we see gigantic waves.
Scientists: it is a fairy tale!
Ocean waves follow
Gaussian distribution.



Oceanic rogue waves

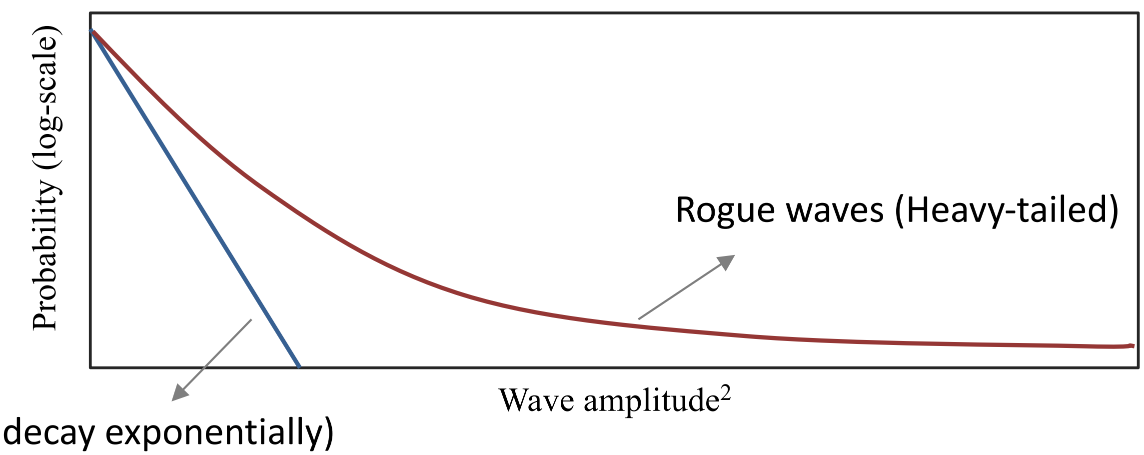
First scientific observation of rogue waves in Draupner oil platform (1995):



Characteristics of rogue waves

- Rogue waves appear from nowhere and disappear without a trace.
- Rogue waves ≠ accidental constructive interference
- They occur much more frequently than expected in ordinary wave statistics.

Probability distribution in rogue systems:



- Not limited to ocean: Observed in many other wave systems including **optics**.

Rogue waves in 1D vs 2D systems

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"Nonlinear Schrödinger equation" explains the wave dynamics in the ocean as well as in optics.

Rogue events studied extensively in 1D systems, such as optical fibers.

$$\frac{\partial A}{\partial x} + \frac{1}{2} i k_2 \frac{\partial^2 A}{\partial t^2} = i\gamma \left| A \right|^2 A$$

D. R. Solli, C. Ropers, P. Koonath & B. Jalali, Nature 450, 1054 (2007).
J.M. Dudley et al, Nat. Photon, 8, 755 (2014)

Water waves are not 1D.

$$2ik \frac{\partial A}{\partial x} + \nabla_\perp^2 A = i\gamma \left| A \right|^2 A$$

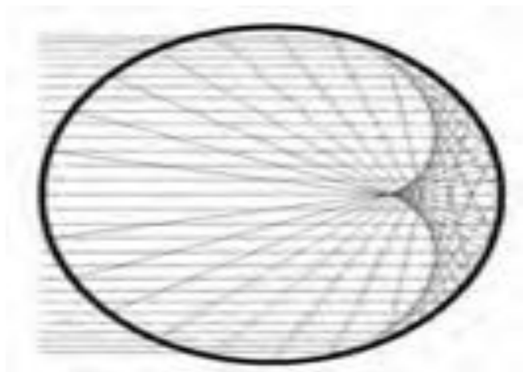
Two focusing effects in 2D systems:

- **Linear**: Spatial (geometrical) focusing
- **Nonlinear**: Self focusing

Optical caustics

Swimming pool

Coffee cup



Ray picture

- Caustics are defined as the envelope of a family of rays
- Singularities in ray optics
- Catastrophe theory is required to remove singularity

Books:
J.F. Nye, *Natural Focusing and Fine Structure of Light.*
Y.A. Kravtsov, *Caustics, Catastrophes and Wave Fields.*
O.N. Stavroudis, *The Optics of Rays, Wavefronts, and Caustics*.

How does one quantify caustic properties?

– There are two common metrics

– Scintillation index →

$$\beta^2 = \frac{\langle I^2 \rangle - \langle I \rangle^2}{\langle I \rangle^2}$$

Defines "sharpness" of beam.
= 1 for speckle pattern.

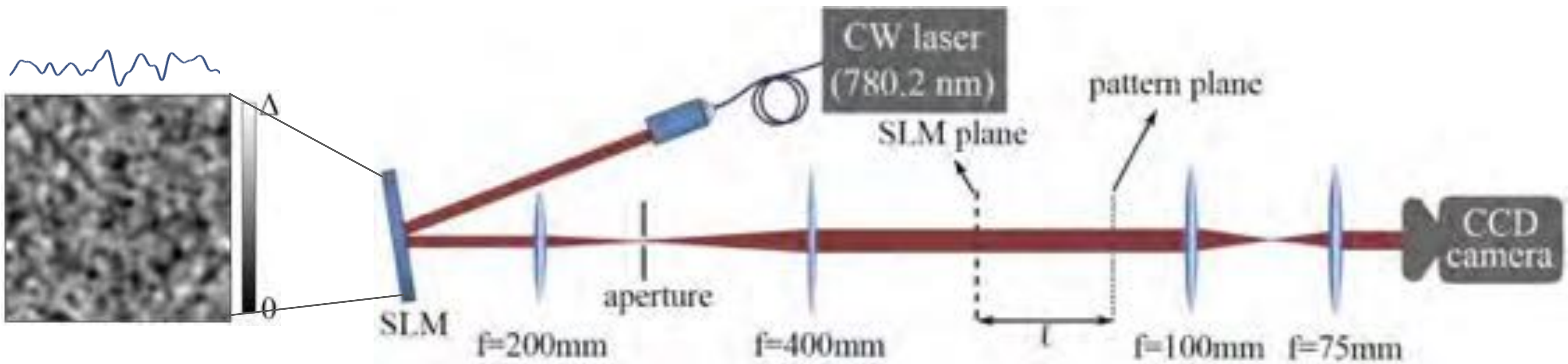
– Statistical distribution of pixel intensities

$$p(I) = A \exp(-BI^C)$$

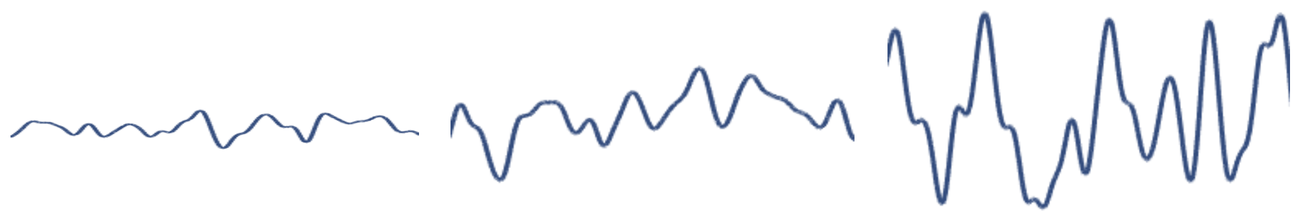
= 1 for speckle pattern
< 1 for caustics (long-tailed distribution)

Distribution [X] (see cover slide)

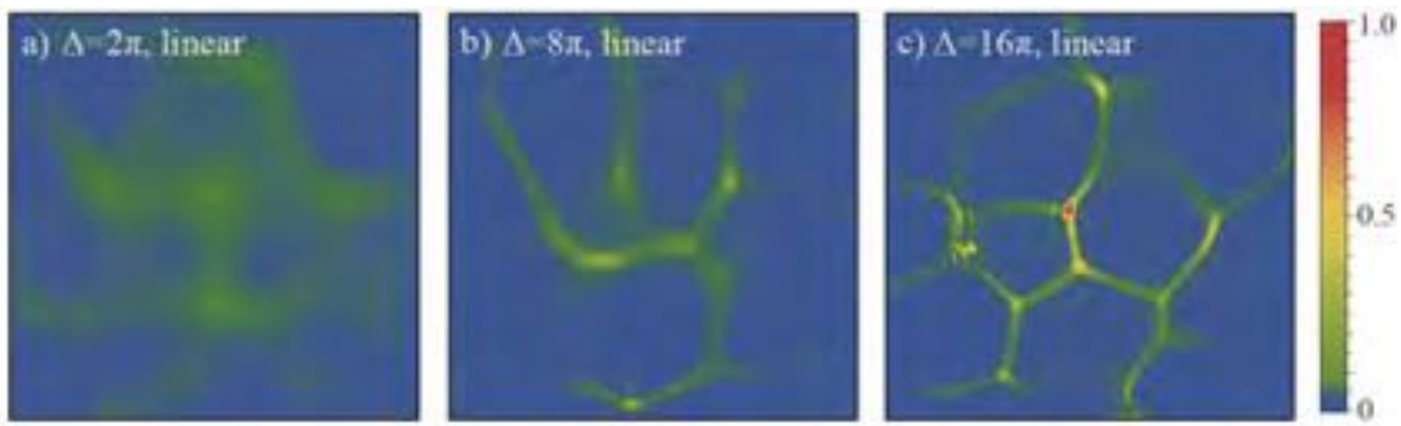
Generation of optical caustics



Phase variations:



Corresponding intensity variations:



A sharp caustic is formed only if the phase variations are large

Statistics of caustics

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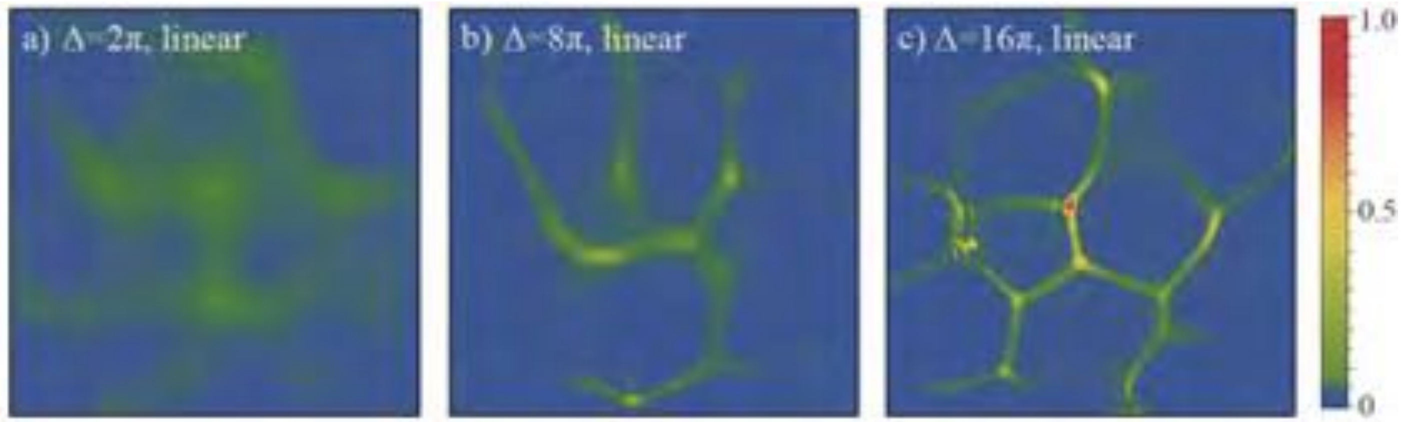
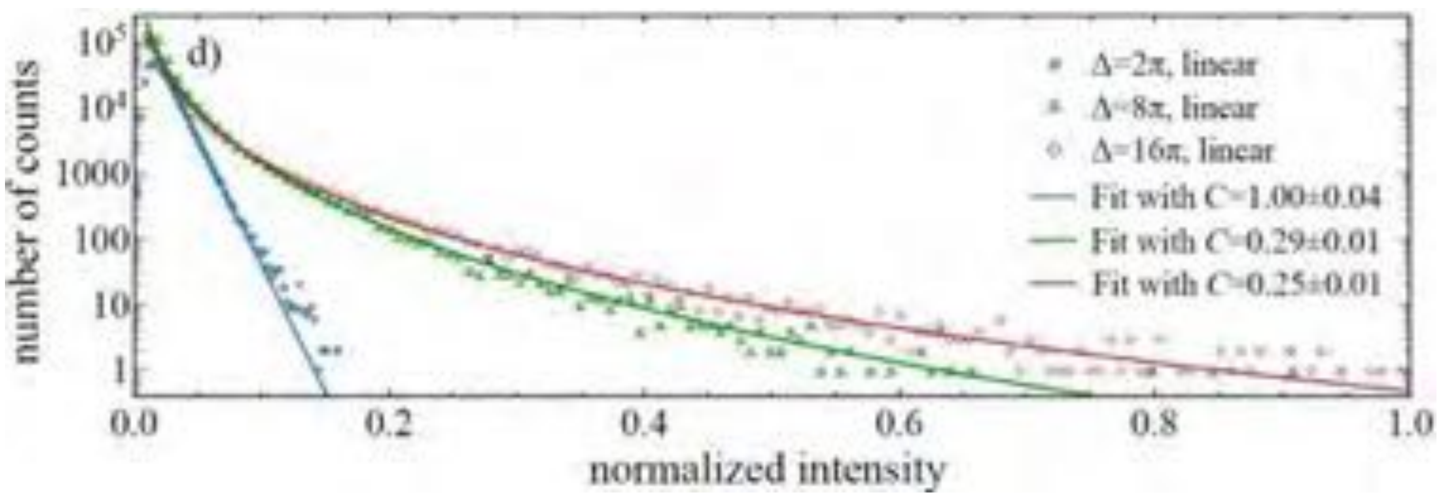
Caustics exhibit long-tailed probability distribution

1000 different patterns for each Δ



Intensity distributions with fit to $A\exp\left(-B\,I^C\right)$



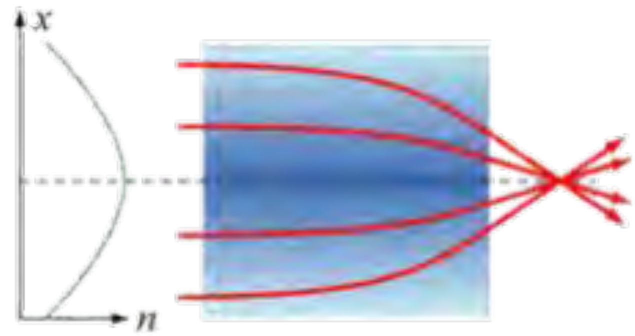
A. Mathis, L. Froehly, S. Toenger, F. Dias, G. Genty & J. Dudley. *Scientific Reports* **5**, 1 (2015).

Nonlinear focusing

Self focusing:

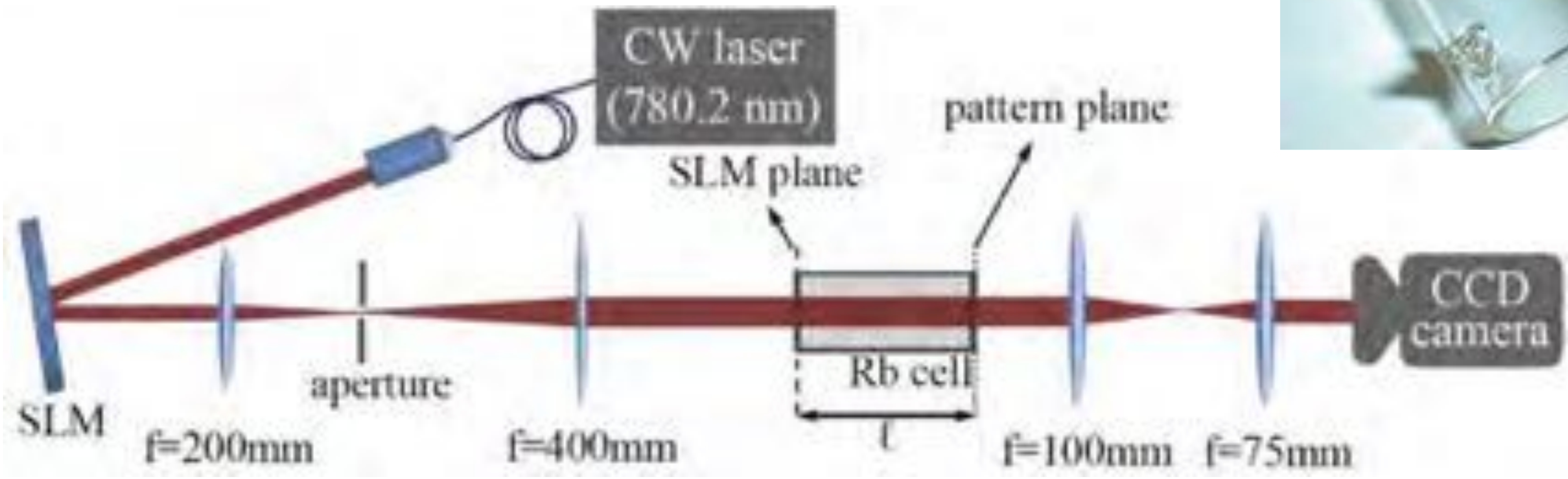
Refractive index depends on intensity:

$$n = n_0 + n_2 I$$



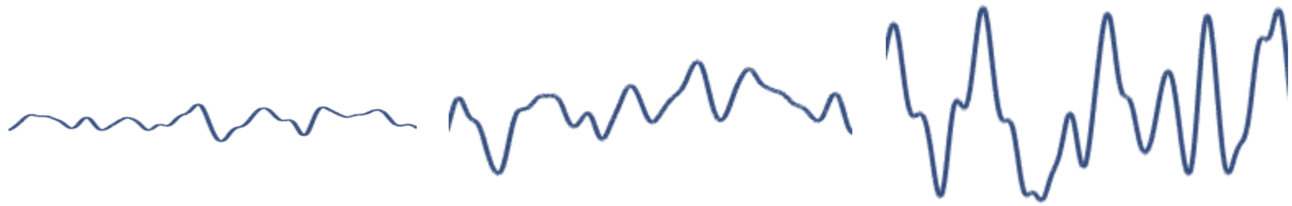
Rubidium cell

Rubidium vapors show large nonlinear effects

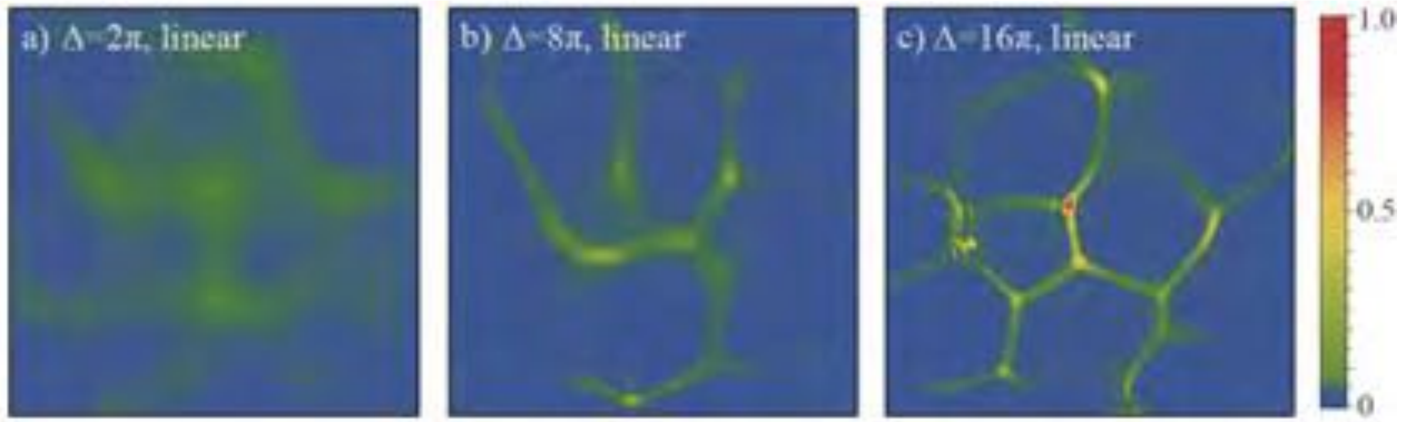


Effect of nonlinearity on caustics

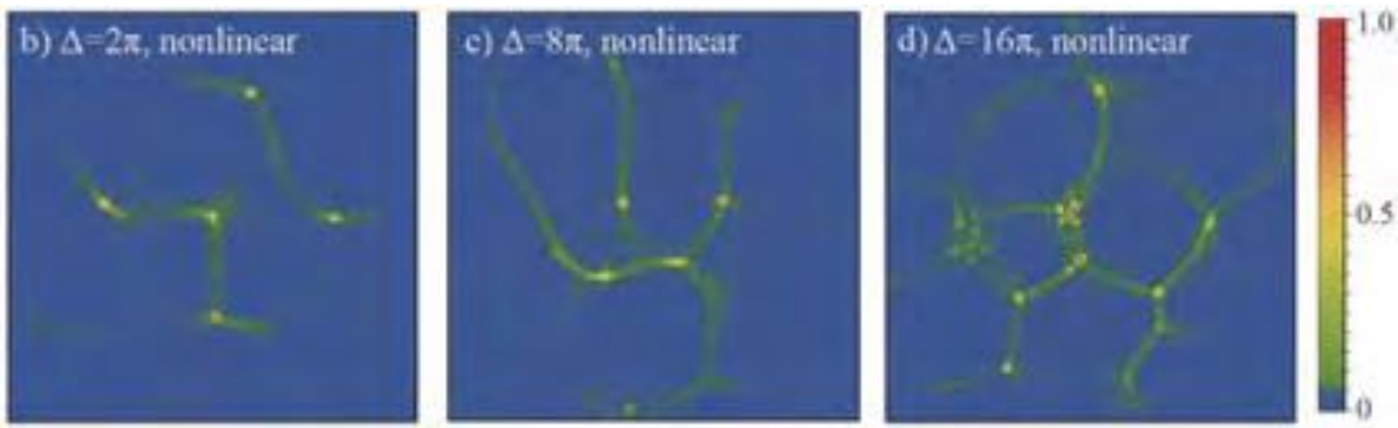
Phase variations:



After linear propagation:



After nonlinear propagation:



Statistics of caustics

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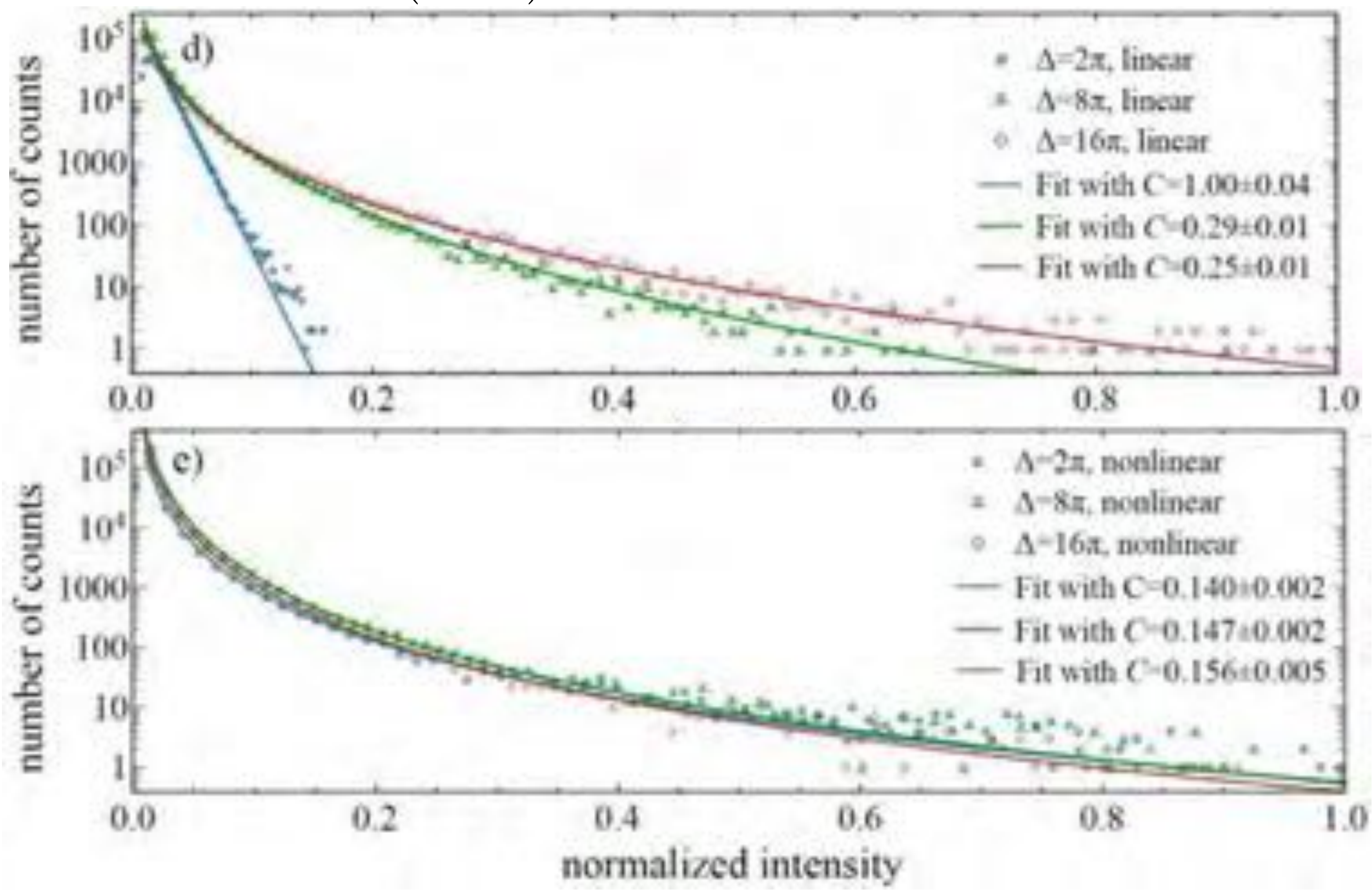
Intensity distributions with fit to $A\exp\left(-BI^{C}\right)$

Linear propagation:

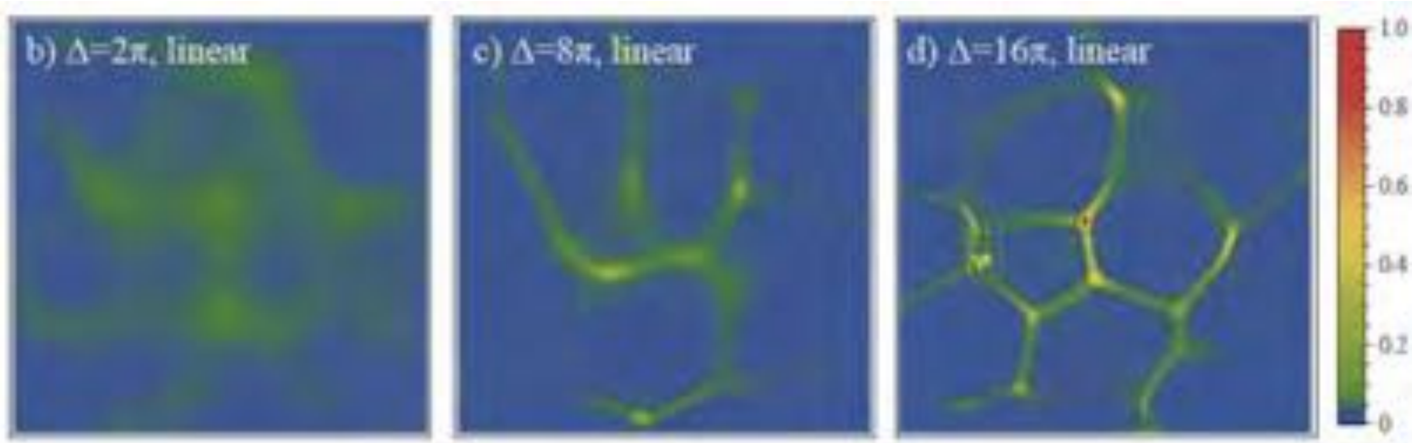
After nonlinear propagation:



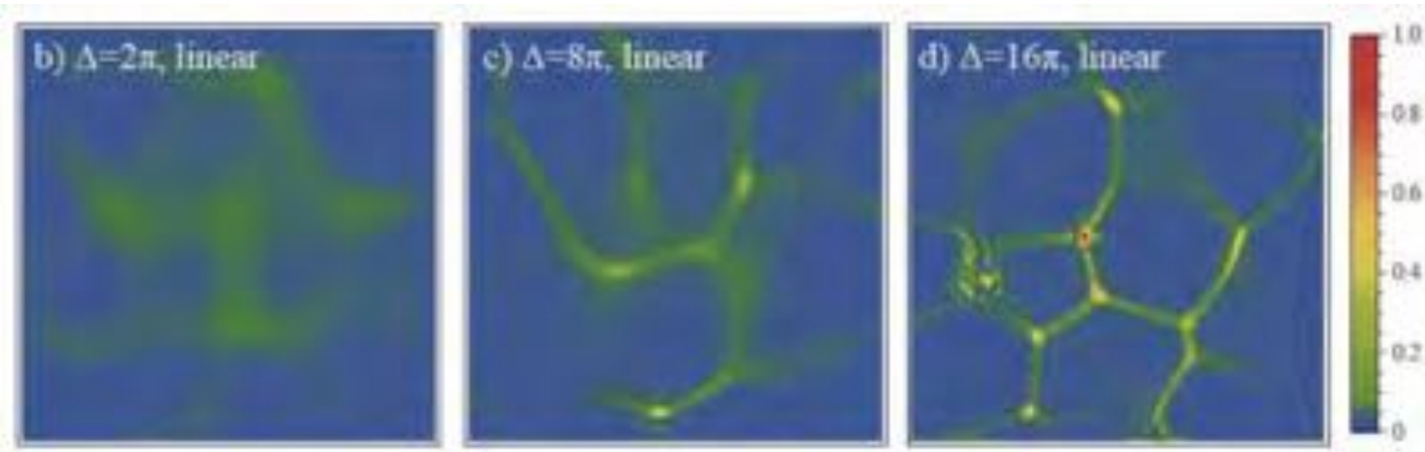
Simulation – Linear propagation

Linear propagation was simulated by FFT beam propagation

Experiment:



Simulation:



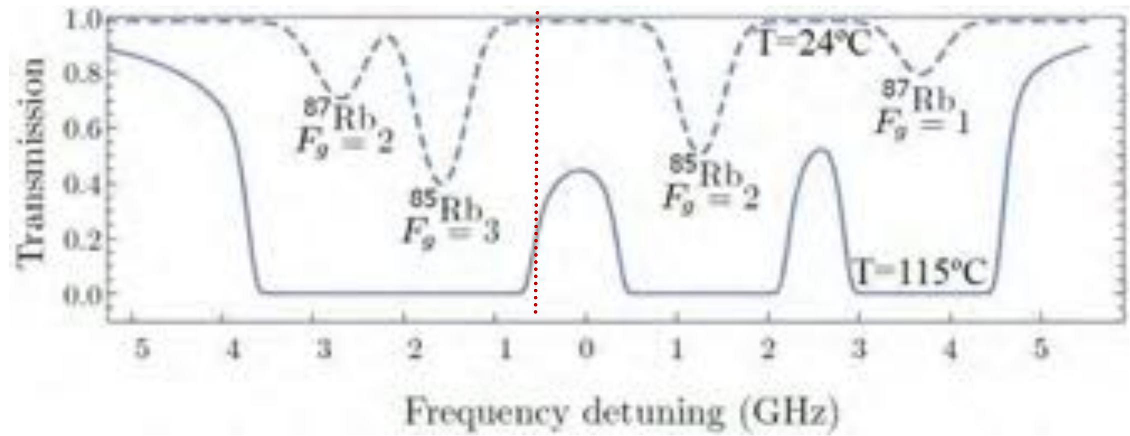
Nonlinear simulation – Rb model

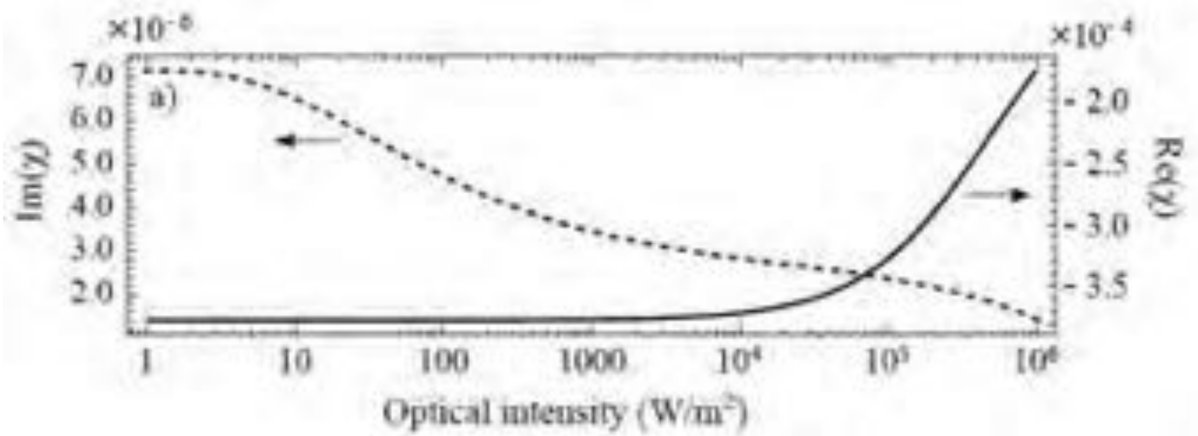
NLSE: $$\frac{\partial \mathcal{E}}{\partial z} - \frac{i}{2k}\nabla_\perp^2 \mathcal{E} = \frac{ik}{2\epsilon_0} P$$

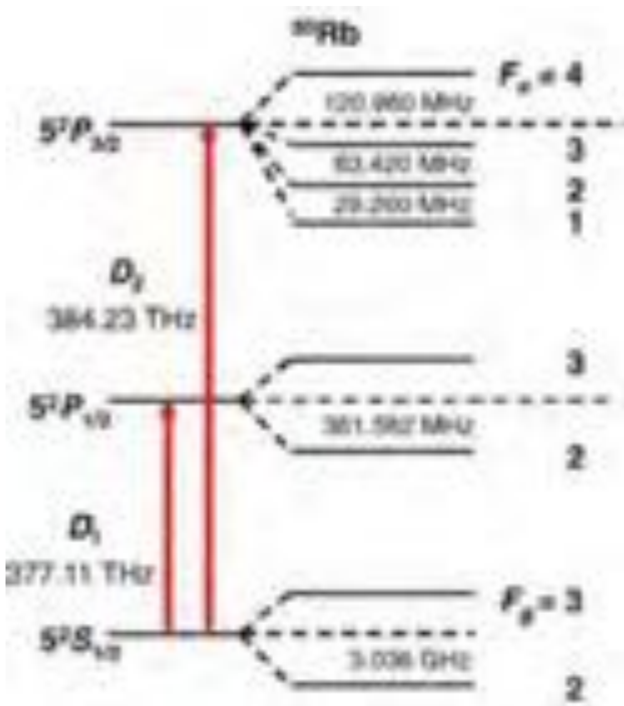
Atomic polarization: $P = \epsilon_0 \chi \mathcal{E}$

Our Rb model includes:

- All hyperfine transitions
- Doppler broadening
- Power broadening
- Collisional broadening
- Optical pumping





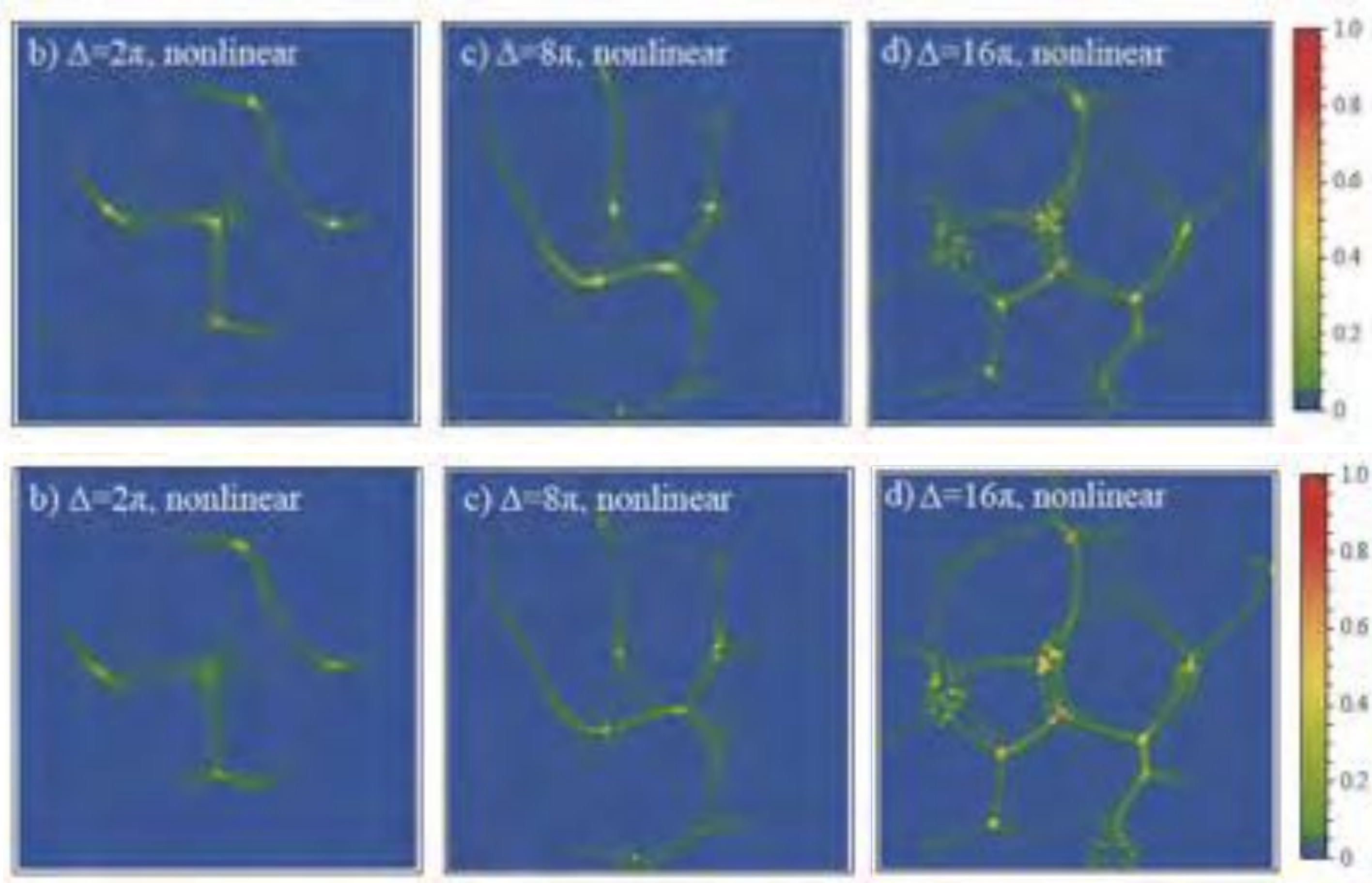


Simulation – Nonlinear propagation

Nonlinear propagation is simulated by a split-step Fourier method.

Experiment:

Simulation:



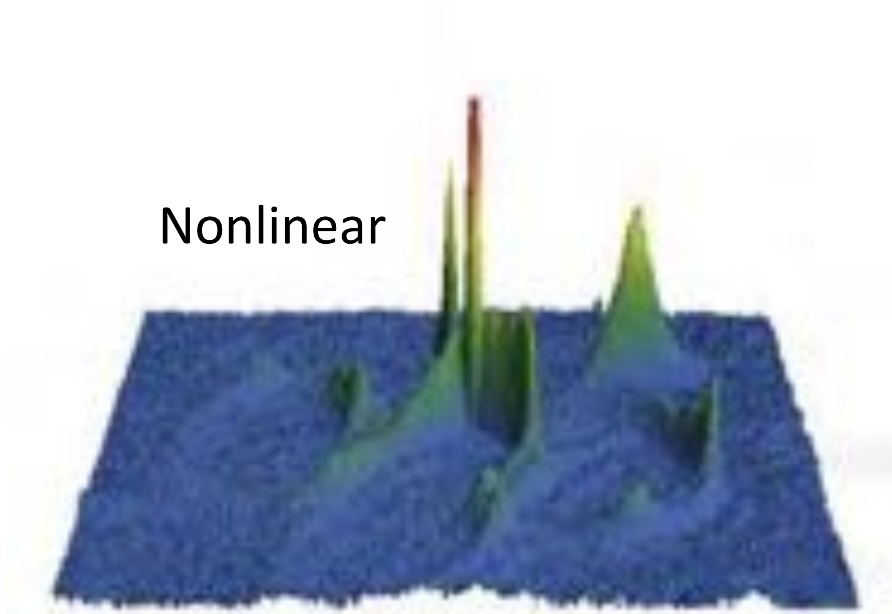
A. Safari, R. Fickler, M. Padgett, and R. Boyd, Physical Review Letters 119, 203901 (2017)

Conclusions

- Caustics are rogue waves!
- Generation of caustics by linear propagation requires large phase fluctuations
- Nonlinear effects can enhance the generation of caustics.

Linear

Nonlinear



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1. Background on self-action effects
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Controlling nonlinear rogue wave formation through the granularity of phase noise

Saumya Choudhary, A. Nicholas Black, Aku Antikainen, and Robert W. Boyd

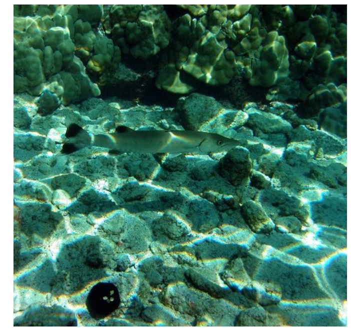
What are caustics?

- Caustics are the envelope of light rays that are intensely concentrated in a small volume.

- Phase fluctuations with a spatial correlation length (L_{corr}) larger than the wavelength of the light convert to intensity variations after propagation.

- Have long-tailed intensity statistics[1, 2] ➡ $$p\left(\frac{I}{\langle I\rangle}\right) = \frac{1}{N}\exp\left[-A\left(\frac{I}{\langle I\rangle}\right)^{B}\right]; 0 < B < 1$$

- Longer tail ⇒ Larger probability of forming `rogue' waves



Caustics formed by a wavy water surface.
https://en.wikipedia.org/wiki/Caustic_(optics)

[1] Pierangeli, D., Di Mei, F., Conti, C., Agranat, A.J. and DelRe, E., 2015. *Physical review letters*, *115*(9), p.093901.
[2] Safari, A., Fickler, R., Padgett, M.J. and Boyd, R.W., 2017. *Physical Review Letters*, *119*(20), p.203901.

Motivation

- A self-focusing nonlinearity enhances the likelihood of formation of rogue waves for small phase instabilities present on the beam[2].
- The effect that the granularity (or the spectral bandwidth) of phase instability present on the beam has on caustic and rogue wave formation has not yet been studied.
- Understanding this effect could yield insight into how the strength of turbulence would affect nonlinear caustic formation on a beam propagating through the atmosphere.

[2] Safari, A., Fickler, R., Padgett, M.J. and Boyd, R.W., 2017. *Physical Review Letters*, *119*(20), p.203901.

Some background on nonlinear caustics (recall our caustics paper)

PRL 119, 203901 (2017) PHYSICAL REVIEW LETTERS week ending 17 NOVEMBER 2017

Generation of Caustics and Rogue Waves from Nonlinear Instability

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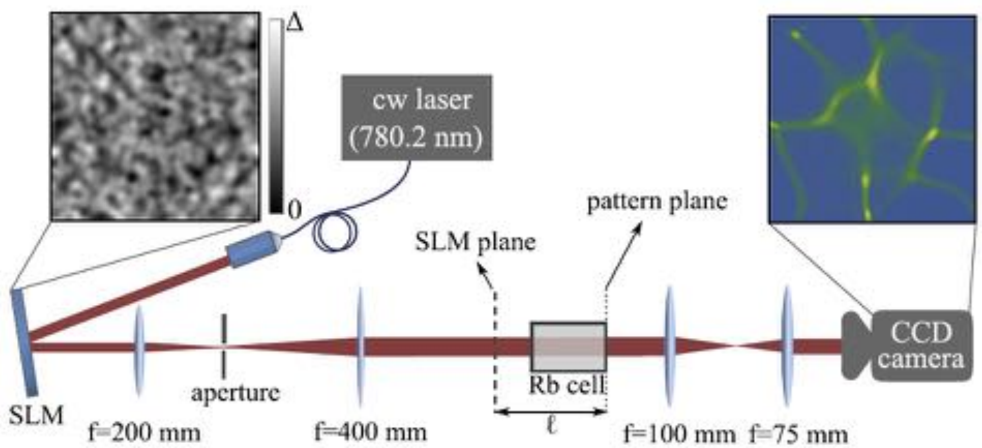
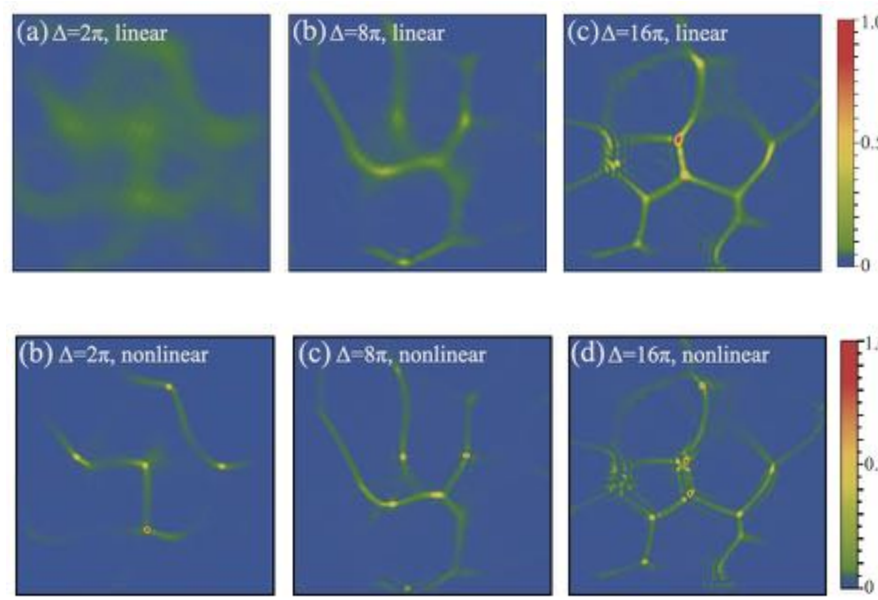
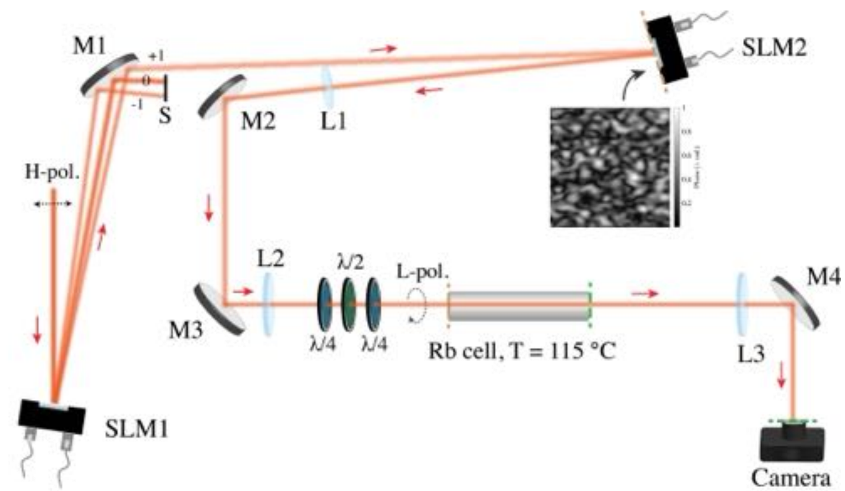


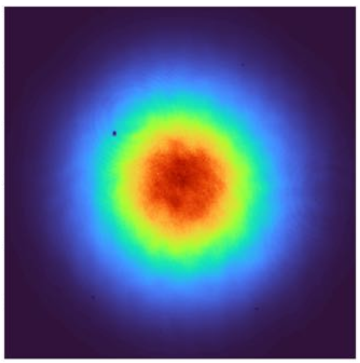


Experimental setup

- SLM1 has a phase-only grating for generating a clean Gaussian beam in its 1st diffractive order.
- SLM2 adds a random phase mask on the waist of the beam with a maximum amplitude of π radians, and a Gaussian angular spectrum of width $\propto 1/L_{\text{corr}}$. The smaller the value of L_{corr}, the more granular is the phase noise added to the beam.
- SLM2 is then imaged on the entrance of a heated rubidium cell.
- The output facet of the cell is imaged on a camera, which records images of the beam after propagation through the cell.
- We vary the beam power and L_{corr} of the phase mask.
- Intensity histograms are obtained from the beam images recorded for 500 different phase masks for each dataset.

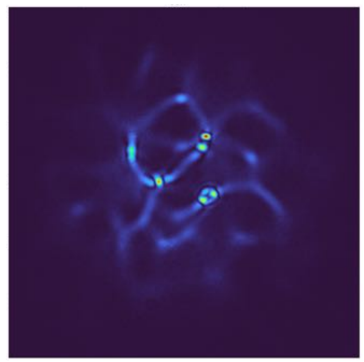


Intensity statistics

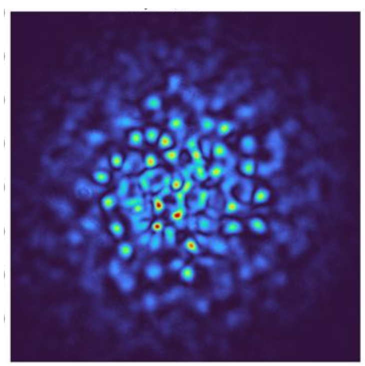


Input beam

Rubidium cell

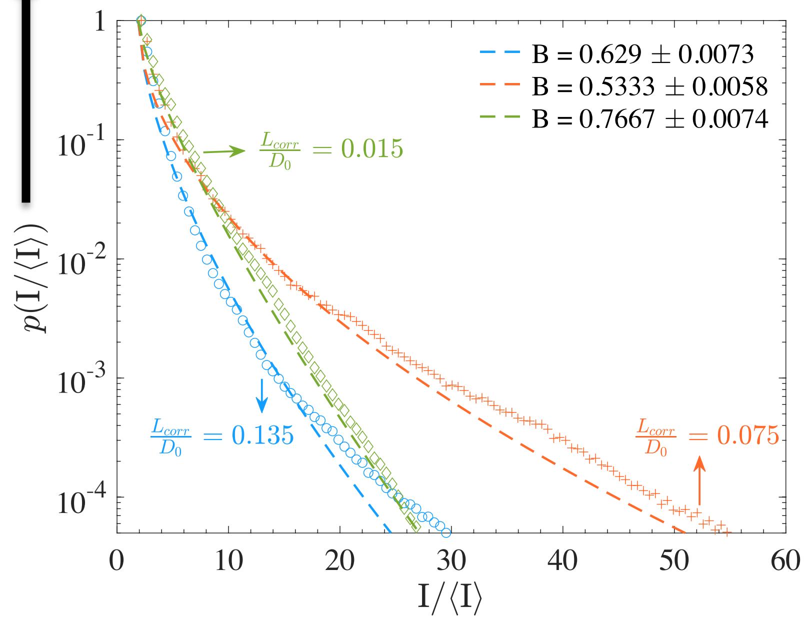


$P_{in} = 90\ mW, L_{corr} = 250\ \mu m$



$P_{in} = 90\ mW, L_{corr} = 50\ \mu m$

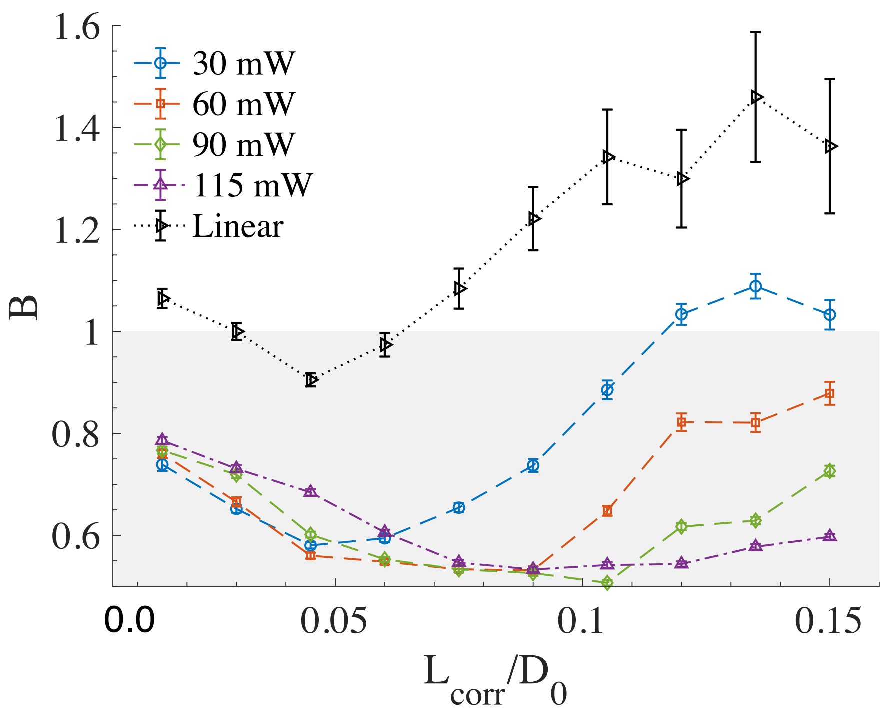
$$p\left(\frac{I}{\langle I\rangle}\right) = \frac{1}{N} exp\left[-A\left(\frac{I}{\langle I\rangle}\right)^{B}\right]$$



Measured intensity histograms (markers) for beam powers of 90 mW, and various L_{corr} normalized to the beam diameter $\boldsymbol{D_0}$. The dashed lines are the maximum likelihood estimation (MLE) fits of the stretched exponential function to the respective intensity statistics (shown by same color markers). The intensity exponent 'B' obtained from the fit is stated in the legend.

More Intensity statistics

$$p\left(\frac{I}{\langle I\rangle}\right) = \frac{1}{N} exp\left[- A \left(\frac{I}{\langle I\rangle}\right)^{B}\right]$$



The intensity exponent 'B' of the measured intensity histograms for various input beam powers, and normalized correlation length L_{corr} w.r.t beam diameter D_0. B < 1 (the gray shaded region) implies long-tailed intensity statistics.

- Smaller value of 'B' ⇒ longer-tailed intensity statistics ⇒ larger probability of formation of 'rogue' waves where the intensity could be larger than 10x the average.
- Nonlinearity enhances the likelihood for rogue waves generally.
- This enhancement is quite large for $\frac{L_{\text{corr}}}{D_0} \geq 0.8$.
- For smaller L_{corr} (that is, when the phase structure has a smaller grain size), this nonlinear enhancement is reduced.
- For $\frac{L_{\text{corr}}}{D_0} \leq 0.06$, the likelihood of rogue wave formation is less sensitive to beam power and appears to saturate.

Numerical Modeling

- The 2+1– D nonlinear Schrödinger equation was used to model beam propagation within the rubidium cell.

$$\frac{\partial E}{\partial z} - \frac{i}{2k}\nabla_{\perp}^{2}E = \frac{i\,k}{2}\,\chi(E)E$$

Here 'E' is the field envelope, and $\chi(E)$ is the total susceptibility of rubidium[3].

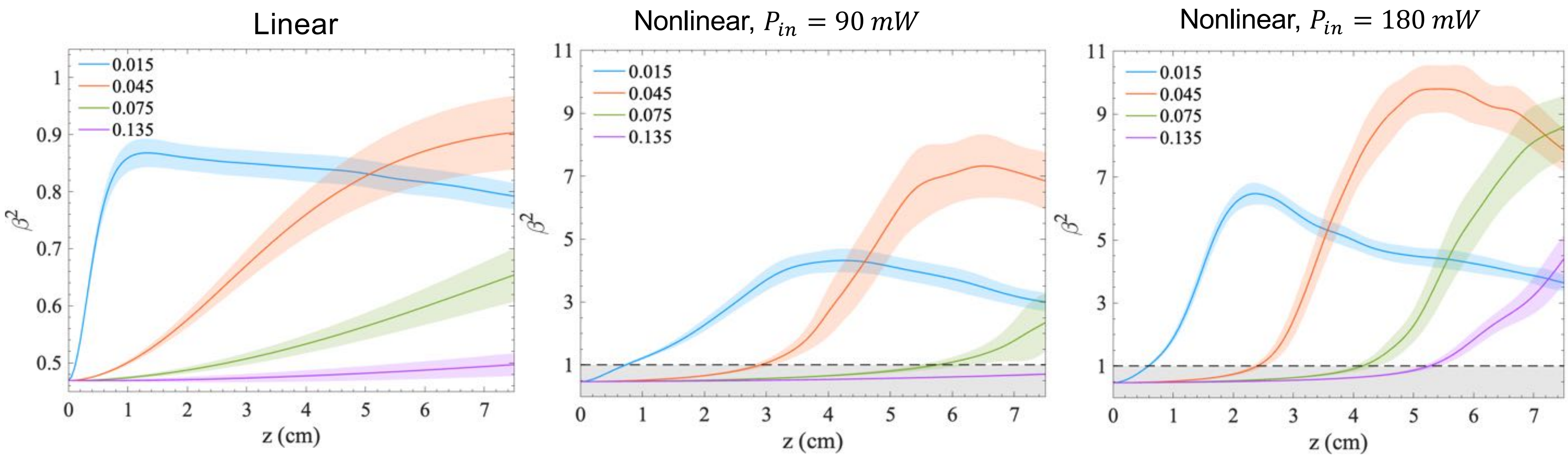
- We use the split-step Fourier method for propagation[4].

- We calculate the scintillation index β^2 along the length of the cell. β^2 quantifies the degree of sharpness of features in the caustic patterns.[2]

$$\beta^2 = \frac{\langle I^2\rangle - \langle I\rangle^2}{\langle I\rangle^2};\ \mathrm{I = Intensity\ of\ the\ beam}$$

[2] Safari, A., Fickler, R., Padgett, M.J. and Boyd, R.W., 2017. *Physical Review Letters*, *119*(20), p.203901.
[3] Boyd, R.W., 2020. *Nonlinear optics*. Academic press.
[4] Agrawal, G.P., 2000. *Nonlinear fiber optics*. Springer, Berlin, Heidelberg.

Scintillation index vs. propagation



- The legends indicate the value of $\frac{L_{\text{corr}}}{D_0}$ in each panel.
- $\beta^2 > 1$ indicates long-tailed intensity statistics (above the gray region in middle and right panels).
- Linear propagation does not reach the threshold scintillation index for a speckle pattern ($\beta^2 = 1$)
- Larger β^2 implies sharper and more intense caustics.
- The maxima of β^2 shifts to smaller z with increasing nonlinearity
- For the most granular noise ($\frac{L_{corr}}{D_0}$ = 0.015), β^2 at the output facet of the cell remains almost the same even when the beam power is doubled.

Conclusions and future work

- We have explored how the granularity/bandwidth of phase instability influences the likelihood of caustic and rogue wave formation in the presence of a saturable self-focusing nonlinearity.
- The nonlinearity significantly enhances the probability of rogue wave formation for relatively narrowband phase instabilities even for modest phase amplitudes (~ π radians).
- For very granular/broadband phase instability, the probability of rogue wave formation is not very sensitive to the nonlinearity.
- We can also interpret this effect in terms of the spatial coherence of the input field after the addition of the phase instability. A more granular phase instability reduces the spatial coherence of the field, and hence counteracts the self-focusing effect of the nonlinearity. ⇒ *We are currently working on this analysis.*

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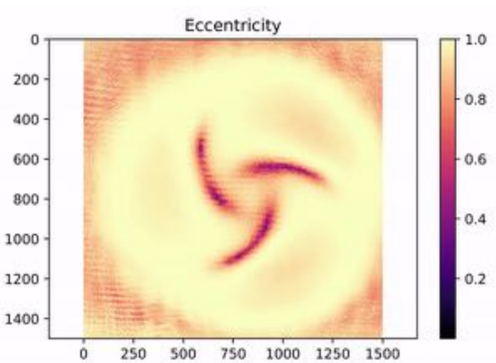
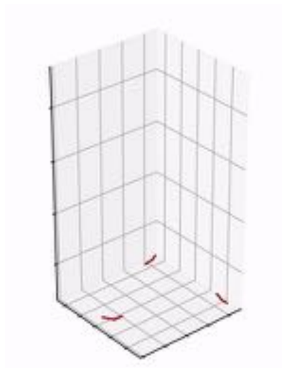
Polarization Knots

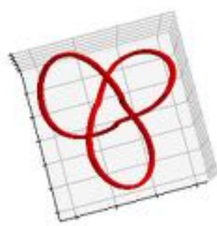
–Motivation :

In polarization knotted beams, the trajectories of the points of purely circular polarization on the cross-section form 3D knotted structures under propagation.
The behavior of these beams is known for propagation through linear media. , it would be of interest to determine how these polarization-structured beams behave under nonlinear effects.

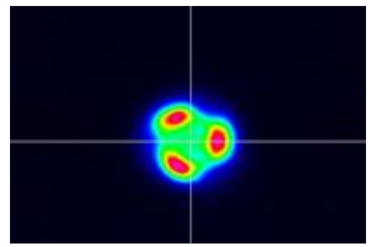


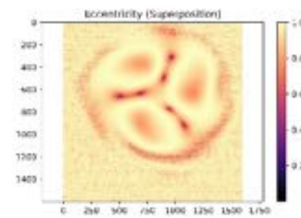


–Video showing linear propagation of a trefoil knot. Dark spots indicate circular polarization (eccentricity of polarization ellipse is zero) and how they twist and form a knot with propagation.

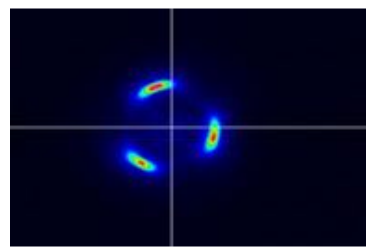


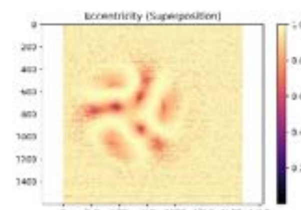
–Top view





–Input beam profile and plot of polarization ellipse eccentricity





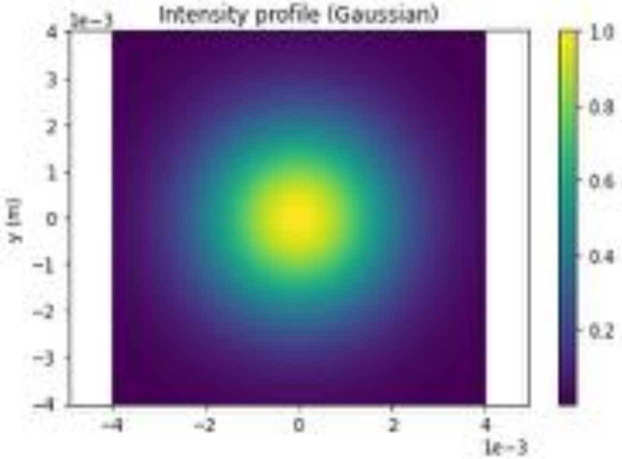
–Beam profile (120 mW power CW) and plot of polarization ellipse eccentricity after nonlinear propagation through the Rb cell (115.1°C).

–Under nonlinear propagation, the polarization structure appears to be preserved.

THEORY – TREFOIL KNOT

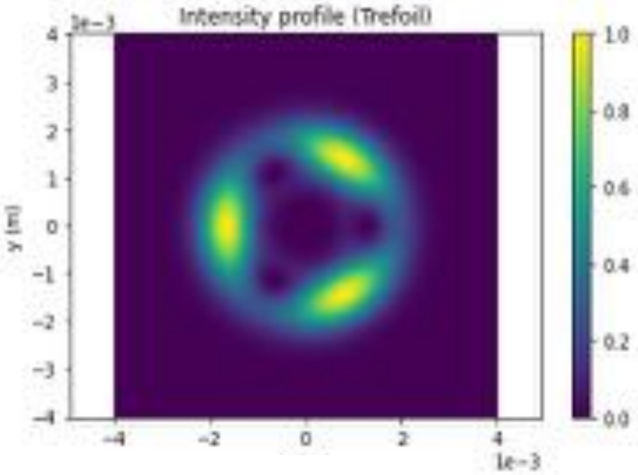
Position: beam waist (z = 0)

LG_{00}

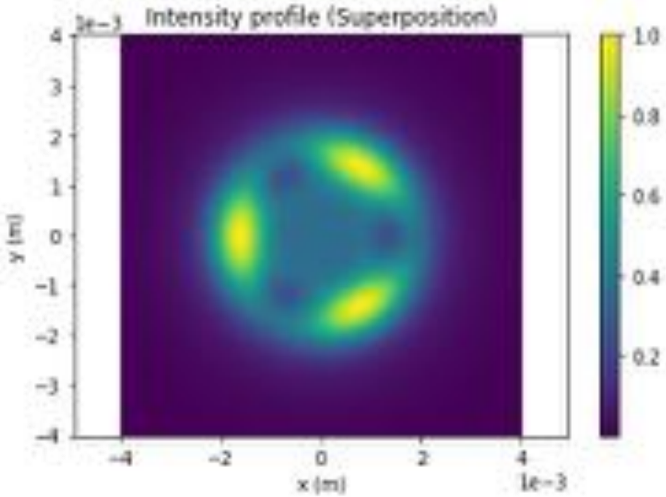


Left circular polarization

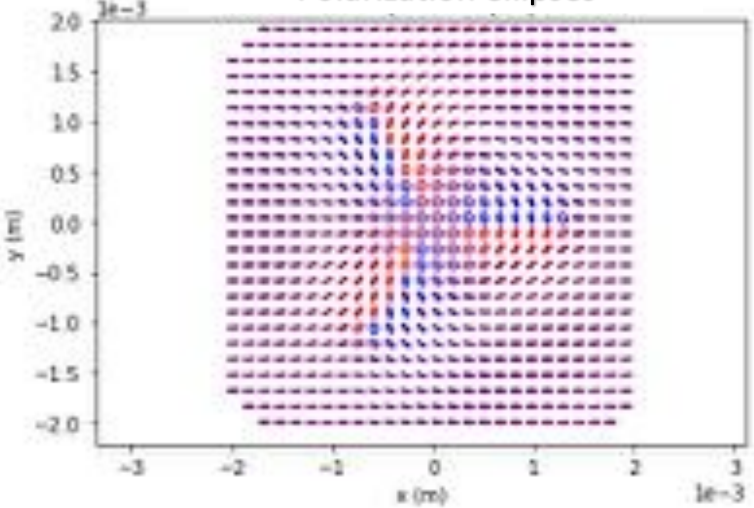
OAM ($1.71LG_{00}-5.66LG_{01}+6.38LG_{02}-2.3LG_{03}-4.36LG_{30}$)



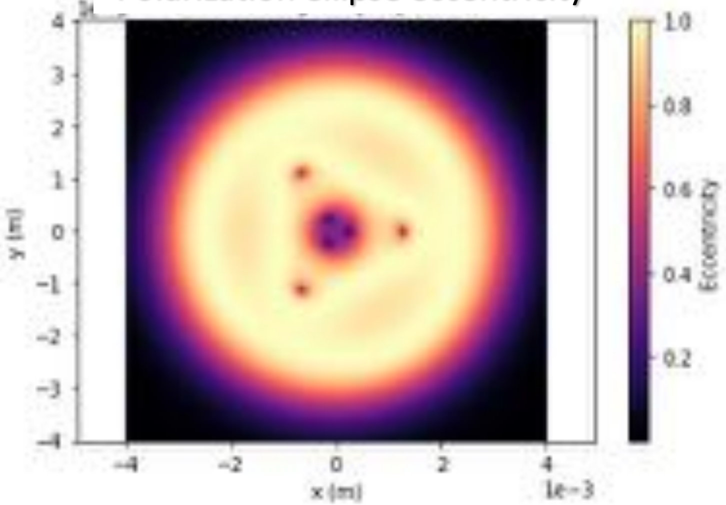
Right circular polarization



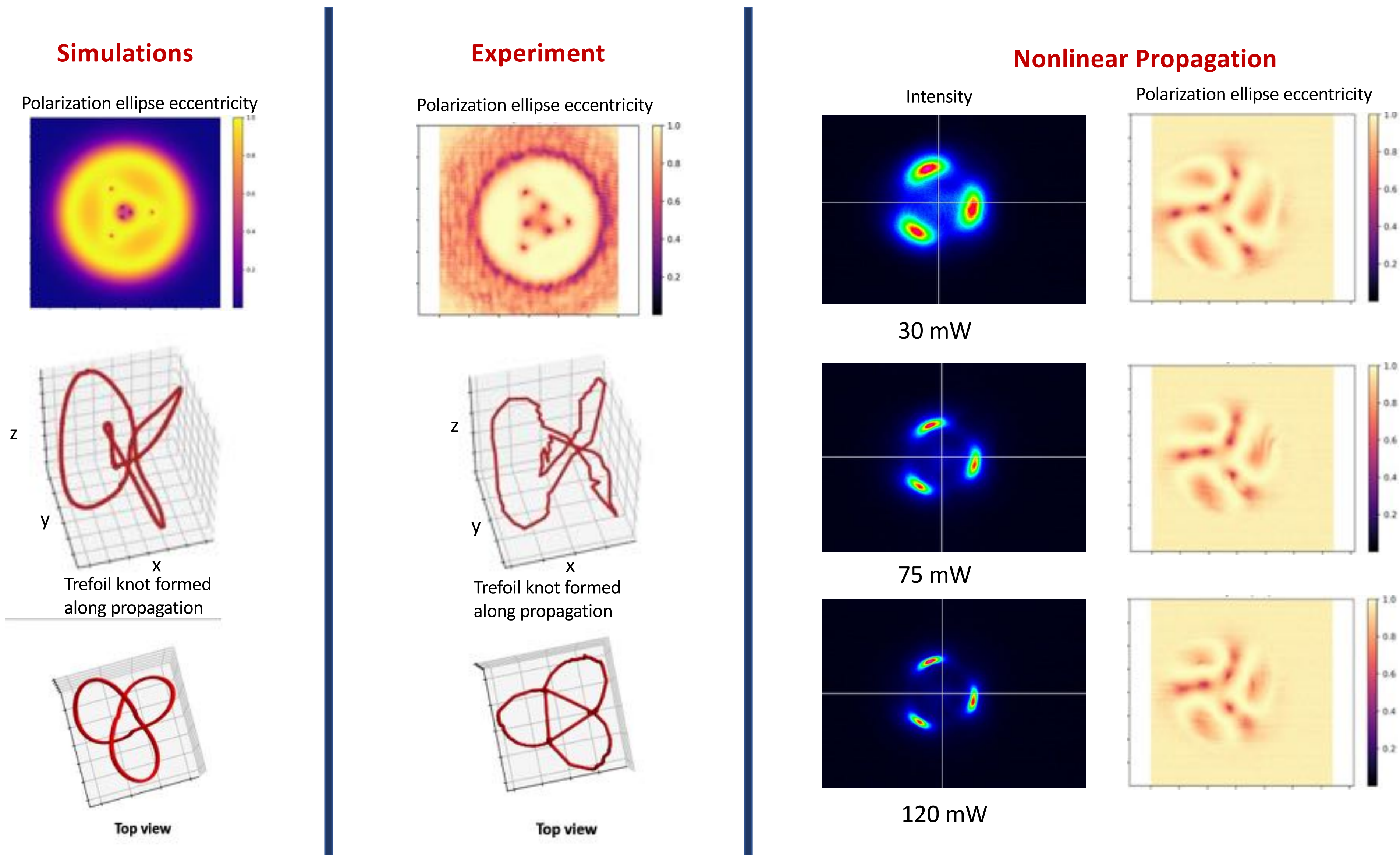
Polarization ellipses



Polarization ellipse eccentricity



We speculate that the number of singular points (six in this case) is conserved even under nonlinear propagation and through atmospheric turbulence.



Simulations
Polarization ellipse eccentricity
z
y
x
Trefoil knot formed along propagation
Top view
Experiment
Polarization ellipse eccentricity
z
y
x
Trefoil knot formed along propagation
Top view
Nonlinear Propagation
Intensity
Polarization ellipse eccentricity
30 mW
75 mW
120 mW

Special Thanks To My Students and Postdocs!

Ottawa Group

Rochester Group