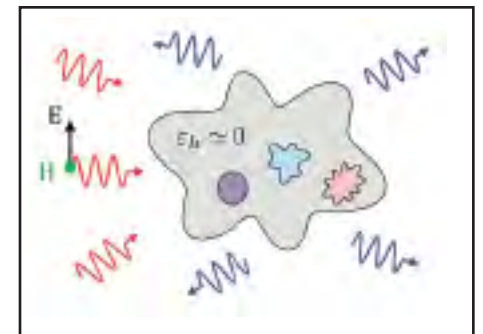
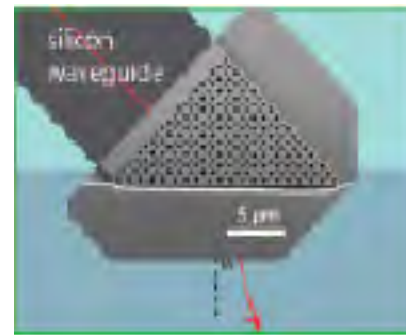
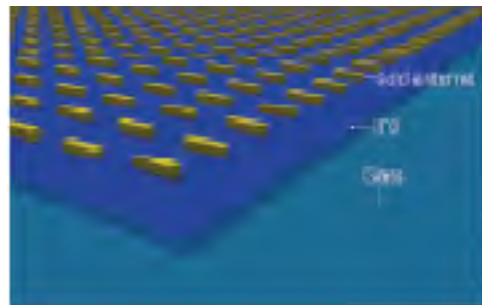
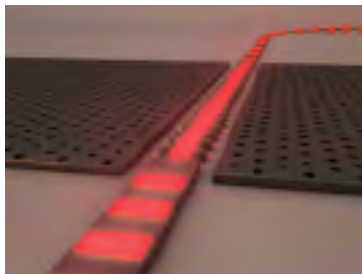




Nonlinear Optics of Time-Varying Media (especially epsilon-near-zero media)

Robert W. Boyd

Department of Physics and
Max-Planck Centre for Extreme and Quantum Photonics
University of Ottawa
Institute of Optics and Department of Physics and Astronomy
University of Rochester



The visuals of this talk are posted at boydnlo.ca/presentations

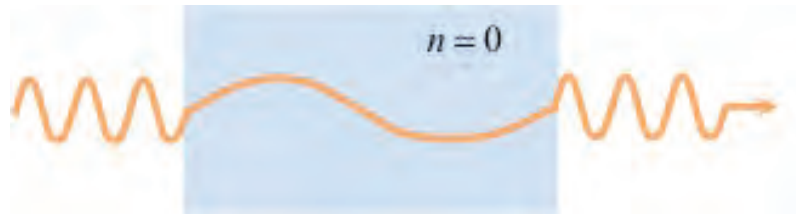
Presented at the virtual Workshop on Time-Varying Media, Imperial College, London, May 26, 2021.

Physics of Near-Zero-Index (NZI) and Epsilon-Near-Zero (ENZ) Materials

- The wavelength of light is given by

$$\lambda = \lambda_{\text{vac}}/n$$

and is significantly lengthened in a NZI material. The wavelength approaches infinity as n approaches zero.



- The phase velocity of light is given by

$$v = c/n$$

and also approaches infinity as n approaches zero.

- For n approaching zero, the field oscillates in time but not in space; oscillations are in phase everywhere

Brown, Proc. IEE 100, 5 (1953).

Ziolkowski, Phys. Rev. E 70, 046608 (2004).

Silveirinha and Engheta, Phys. Rev. Lett. 97, 157403 (2006).

Physics of Epsilon-Near-Zero (ENZ) Materials

- Radiative processes are modified in ENZ materials

Einstein A coefficient (spontaneous emission lifetime = $1/A$)

$$A = n A_{\text{vac}}$$

We can control (inhibit!) spontaneous emission!

Einstein B coefficient

Stimulated emission rate = B times EM field energy density

$$B = B_{\text{vac}} / (n n_g)$$

Optical gain is very large!

Einstein, *Physikalische Zeitschrift* 18, 121 (1917).

Milonni, *Journal of Modern Optics* 42, 1991 (1995).

Equations are shown for nonmagnetic ($\mu = 1$) materials

- Implications:
 - If we can inhibit spontaneous emission, we can build thresholdless lasers.
 - Expect superradiance effects to be pronounced in ENZ materials.

Nonlinear Optical Properties of Epsilon-Near-Zero Materials

Giant Nonlinear Response of ENZ Metastructures

- Nonlinear Optics is important for a variety of reasons:

Photonic Devices

All-optical switching, buffers and routers based on slow light

Used to create quantum states of light for

Quantum Computing/Communications/Imaging

Fundamental understanding of light-matter interactions

Not “just” Lorentz oscillator formalism

Understand rogue waves

Control filamentation process

- However, the nonlinear response is usually much weaker than the linear response
- Means to enhance the nonlinear response
 - Resonance interactions (atomic vapors)
 - Plasmonic systems
 - Electromagnetically induced transparency (EIT)
 - Metamaterials (composite materials)
- Our approach: Use epsilon-near-zero (ENZ) materials and metamaterials

Implications of ENZ Behavior for Nonlinear Optics

Here is the intuition for why the ENZ condition is of interest in NLO

Recall the standard relation between n_2 and $\chi^{(3)}$

$$n_2 = \frac{3\chi^{(3)}}{4\epsilon_0 c n_0 \operatorname{Re}(n_0)}$$

Note that under ENZ conditions the denominator becomes very small, leading to a very large value of n_2

Footnote:

Standard notation for perturbative NLO

$$\mathbf{P} = \chi^{(1)}\mathbf{E} + \chi^{(2)}\mathbf{E}^2 + \chi^{(3)}\mathbf{E}^3 + \dots$$

\mathbf{P} is the induced dipole moment per unit volume and \mathbf{E} is the field amplitude.

Also, the refractive index changes according to

$$n = n_0 + n_2 I + n_4 I^2 + \dots$$

How to Choose an Epsilon-Near-Zero Material

- Electrical conductors

All conductors display ENZ behavior at their (reduced) plasma frequency

Recall the Drude formula

$$\epsilon(\omega) = \epsilon_{\infty} - \frac{\omega_p^2}{\omega(\omega + i\gamma)}$$

Note that $\text{Re } \epsilon = 0$ for $\omega = \omega_p / \sqrt{\epsilon_{\infty}} \equiv \omega_0$.

ENZ wavelength restricted to a limited range in the visible.

- Electrical insulators (dielectrics)

Dielectrics can show ENZ behavior at their (optical) phonon resonance.

ENZ wavelength restricted to a limited range in the mid-IR.

- Metamaterials

Can design the material so that the ENZ or EMNZ wavelengths are at any desired value.

- Challenge (for any material system). For low loss, we want $\text{Im } \epsilon$ as small as possible at the wavelength where $\text{Re } \epsilon = 0$.

Nonlinear Optics of Indium Tin Oxide (ITO)

- We recently reported that, at its ENZ wavelength, ITO possesses a nonlinear coefficient n_2 that is 100 times larger than those of previously reported materials [1].
- ITO is a degenerate semiconductor (so highly doped as to be metal-like).
- ITO has a large density of free electrons, and a bulk plasma frequency corresponding to a wavelength of approximately 1.24 μm .
- Dielectric properties of ITO are well described by the Drude formula.

$$\epsilon(\omega) = \epsilon_{\infty} - \frac{\omega_p^2}{\omega(\omega + i\gamma)}$$

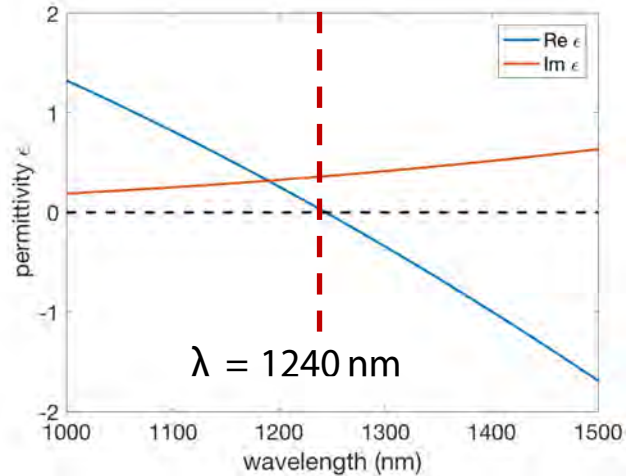
- Note that aluminum-doped zinc oxide (AZO), another transparent conducting oxide, also has strong nonlinear response at its ENZ wavelength [2].

1. Alam, De Leon and Boyd, Science 352, 795–797 (2016)

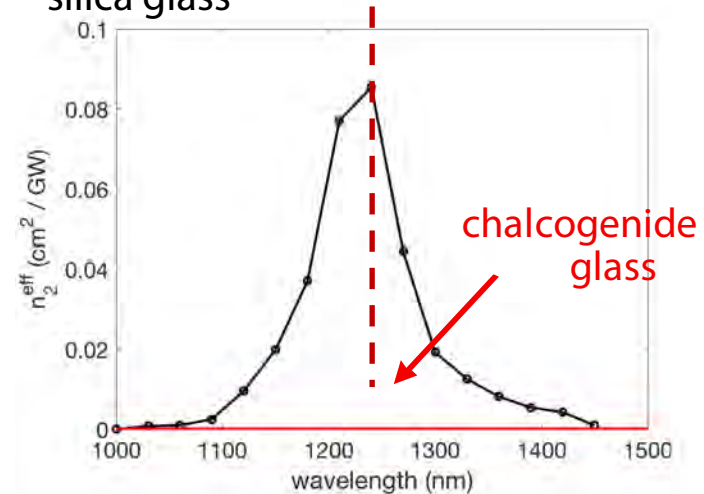
2. Caspani, Shalaev, Boltasseva, Faccio et al., Phys. Rev. Lett. 116, 233901 (2016).

Huge, Fast NLO Response of Indium Tin Oxide at its ENZ Wavelength

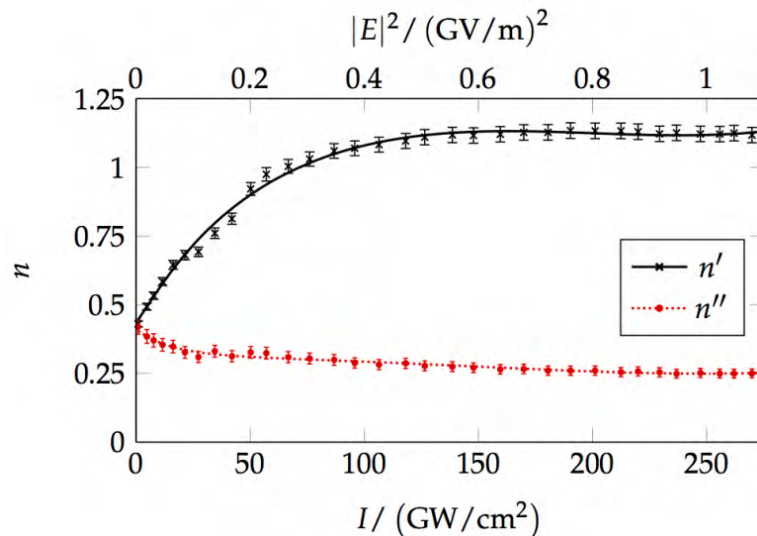
- ellipsometry



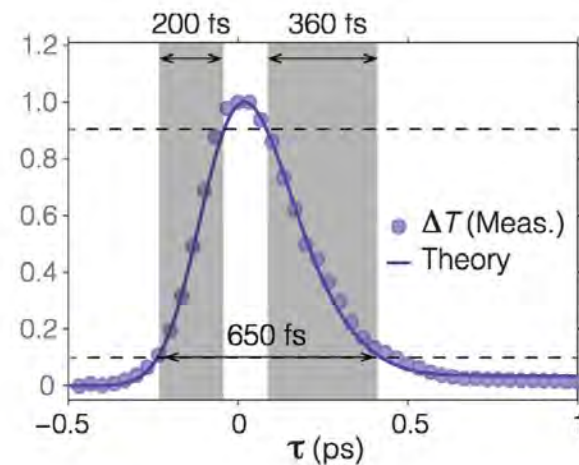
- n_2 can be 3.4×10^5 times larger than that of silica glass



- overall change in refractive index of 0.8



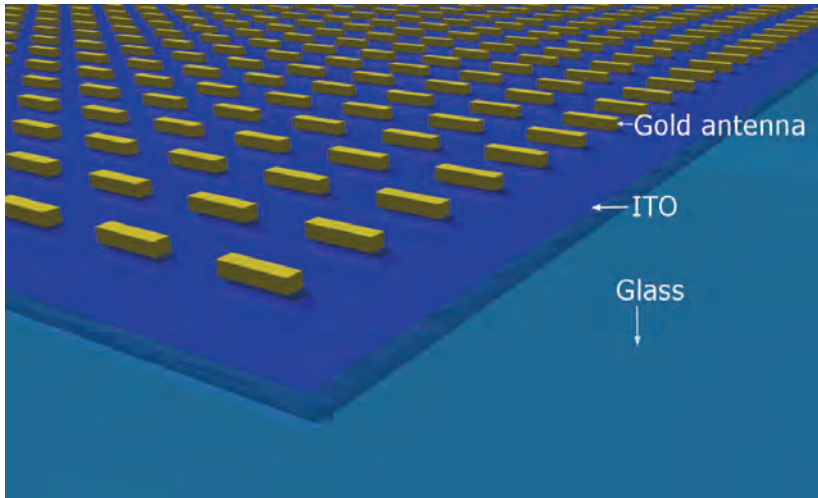
- sub picosecond response time



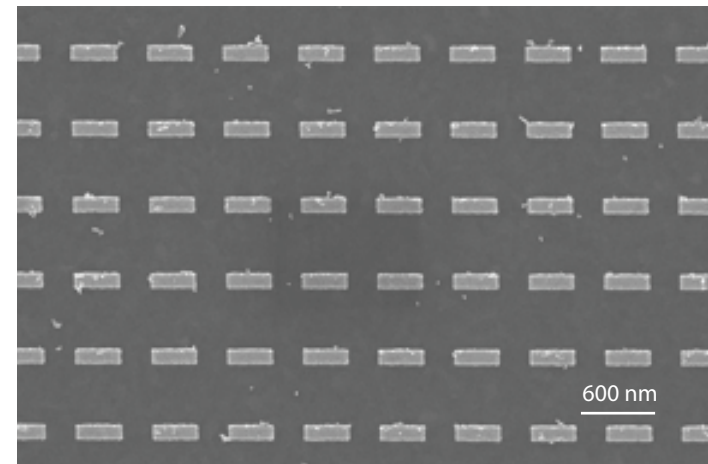
An ENZ Metasurface

- We functionalize ITO by creating a photonic metasurface
- We obtain an even larger NLO response by placing a gold antenna array on top of ITO.
 - Lightning rod effect: antennas concentrate the field within the ITO
 - Coupled resonators: ENZ resonance and nano-antennas

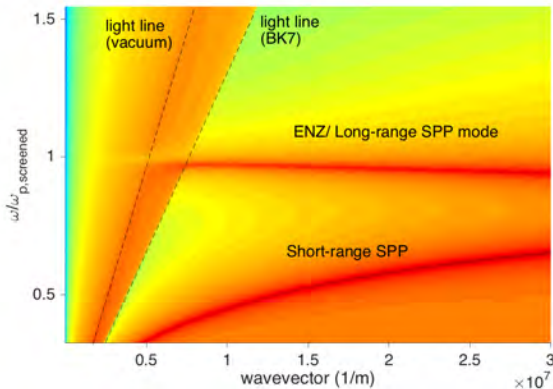
Concept:



SEM:



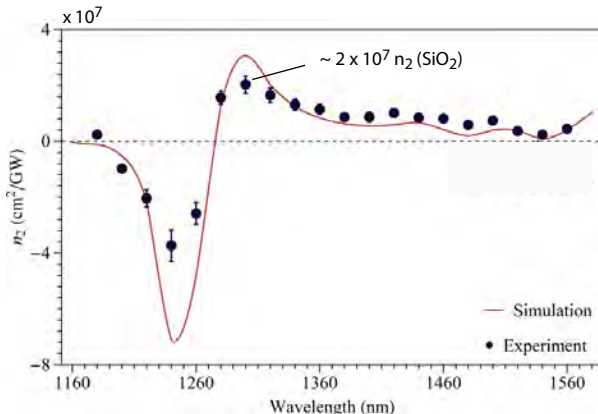
A thin ENZ medium supports a bulk plasma mode.



A thin layer of ITO supports two modes

- the bulk plasma mode, also called the ENZ or long-range SPP mode
- the short range surface plasmon polariton (SPP) mode

NLO response of the coupled antenna-ENZ system



The structure exhibits an extremely large n_2 value over a broad spectral range. The on-resonance n_2 value is **seven orders of magnitude** larger than that of silica glass.

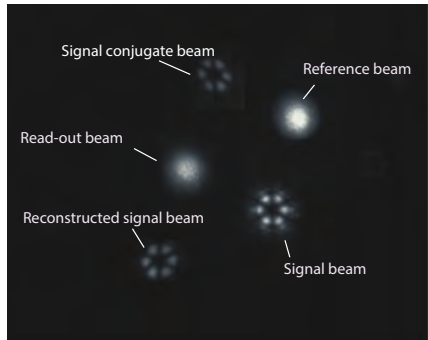
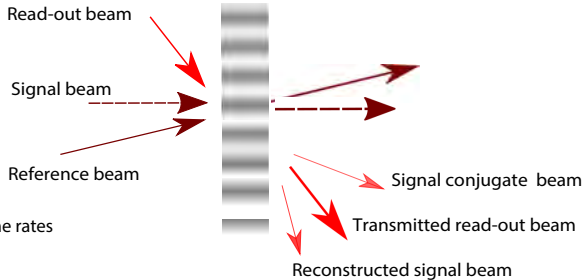
Applications of Time-Varying Epsilon-Near-Zero Materials

Real-Time Holography with THz Refresh Rates

- Goal: Real-time holography with video or much faster refresh rates.
- The ultrafast response of ITO permits THz refresh rates
- Important applications involve image processing and signal processing
- Current real-time holographic materials cannot even support video frame rates



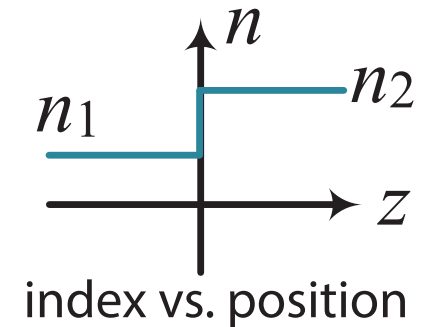
- Demonstration of image processing (edge enhancement)



Adiabatic Wavelength Conversion through Time Refraction

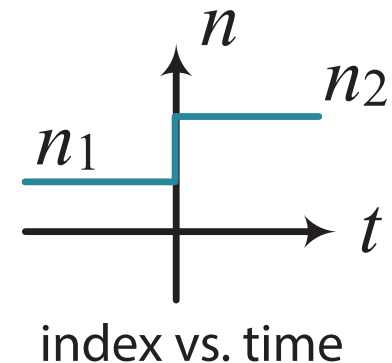
- Space refraction (e.g., Snell's law)

$$\frac{c}{f} = n \cdot \lambda \longrightarrow n_1 \lambda_1 = n_2 \lambda_2$$



- Time refraction (analog of Snell's law)

$$\frac{c}{f} = n \cdot \lambda \longrightarrow n_1 f_1 = n_2 f_2$$



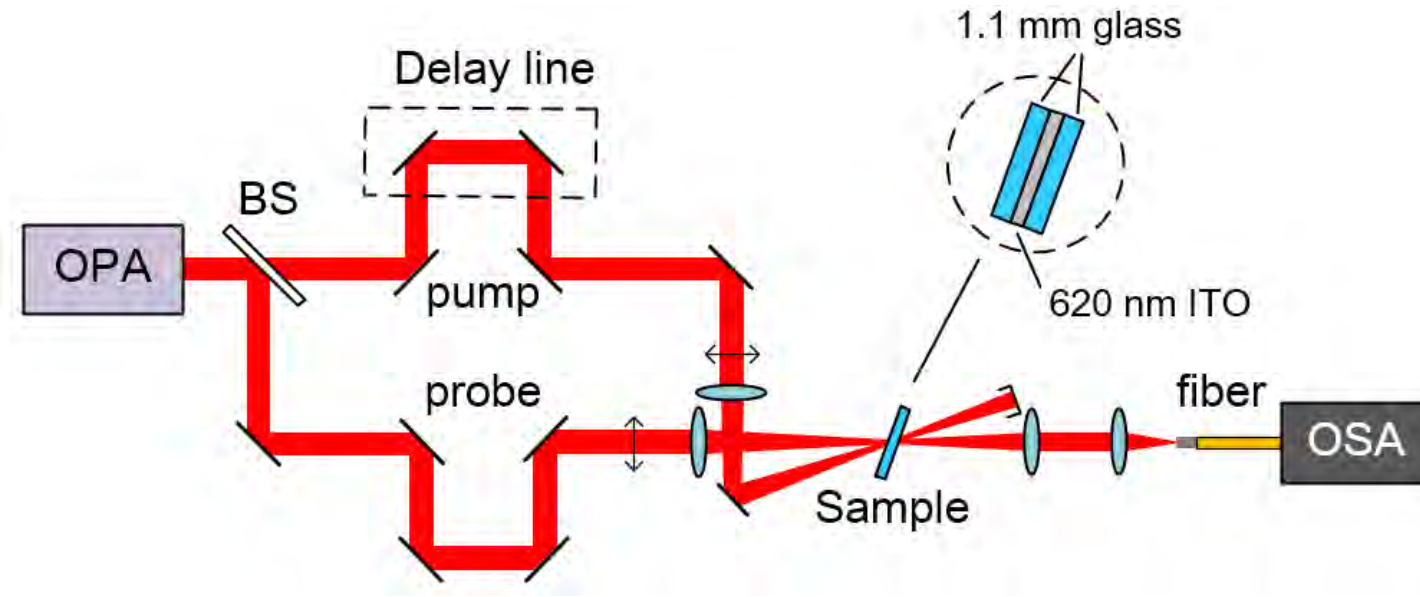
Photon frequency (energy) is changed because of the temporal change in index, but the wavelength (inverse of momentum) is conserved in the absence of any spatial asymmetry

- Time refraction is an alternative way of understanding frequency broadening and shifting by self-phase modulation:

$$\delta\omega(t) = \frac{d}{dt}\phi_{\text{NL}} = \frac{d}{dt}[n_2 I(t)\omega/c]$$

Laboratory Study of Wavelength Conversion by Time Refraction

- Pump beam creates a time-varying refractive index in ITO sample
- Frequency of probe beam is thereby modified.



OPA = optical parametric amplifier

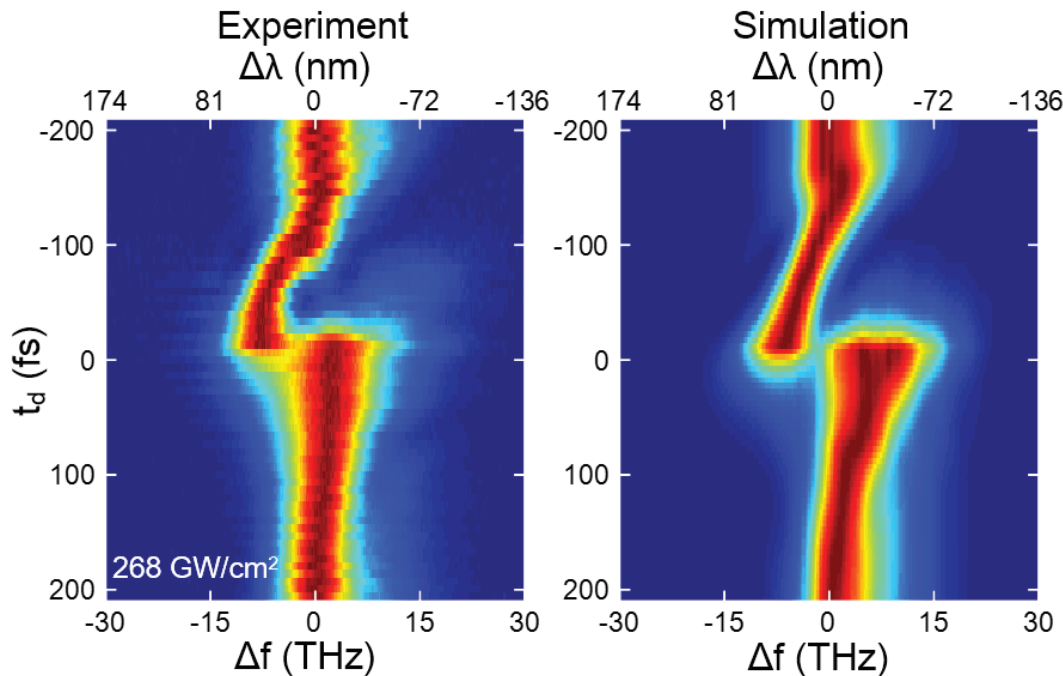
wavelength = 1240 nm

pulse duration = 120 fs

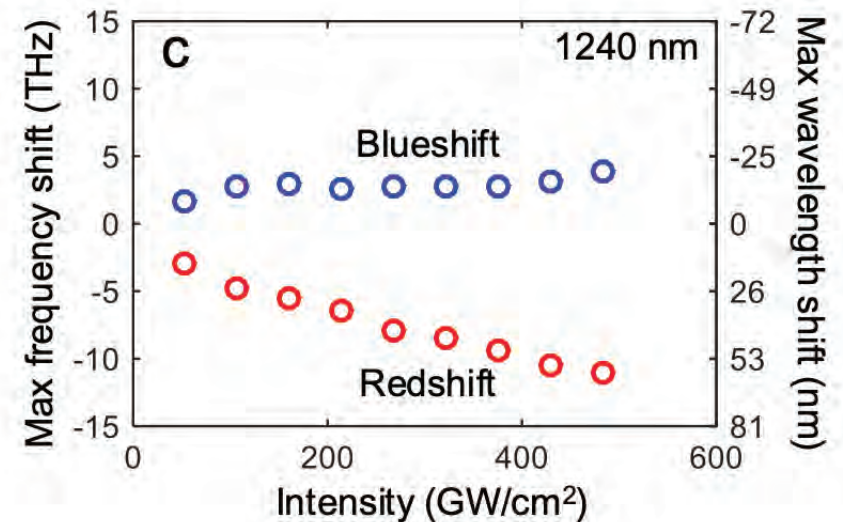
OSA = optical spectrum analyzer

Results: Adiabatic Wavelength Conversion by Time Refraction

Experimental results at 1240 nm



Probe phase and amplitude are measured by frequency-resolved optical gating (FROG)



The wavelength shift can be controlled by the pump intensity and the sign of the time delay.

- The observed effect is 100 times larger with almost 100 times smaller propagation distance than previous reports of AWC.
- Application: wavelength-division multiplexing for telecom

More Work on Adiabatic Wavelength Conversion



pubs.acs.org/journal/apchd5

Letter

Photon Acceleration Using a Time-Varying Epsilon-near-Zero Metasurface

Cong Liu,* M. Zahirul Alam, Kai Pang, Karapet Manukyan, Orad Reshef, Yiyu Zhou, Saumya Choudhary, Joel Patrow, Anuj Pennathurs, Hao Song, Zhe Zhao, Runzhou Zhang, Fatemeh Alishahi, Ahmad Fallahpour, Yinwen Cao, Ahmed Almainan, Jahan M. Dawlaty, Moshe Tur, Robert W. Boyd, and Alan E. Willner



Cite This: <https://dx.doi.org/10.1021/acsphotonics.0c01929>



Read Online

Tunable Doppler Shift using a Time-Varying Epsilon-Near-Zero Thin Film near 1550 nm

CONG LIU^{1,2,*}, M. ZAHIRUL ALAM^{3,4}, KAI PANG¹, KARAPET MANUKYAN¹, JOSHUA R. HENDRICKSON⁵, EVAN M. SMITH^{5,6}, YIYU ZHOU⁴, ORAD RESHEF³, HAO SONG¹, RUNZHOU ZHANG¹, HAOQIAN SONG¹, FATEMEH ALISHAHI¹, AHMAD FALLAHPOUR¹, AHMED ALMAIMAN⁷, ROBERT W. BOYD^{3,4}, MOSHE TUR⁸, AND ALAN E. WILLNER¹

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²Dept. of Physics and Astronomy, University of Southern California, Los Angeles, CA 90089, USA

³Department of Physics, University of Ottawa, Ottawa, ON, Canada

⁴The Institute of Optics, University of Rochester, Rochester, New York 14627, USA

⁵Air Force Research Laboratory, Sensors Directorate, Wright-Patterson AFB, Dayton, Ohio 45433, USA ⁶King Saud University, Riyadh, Saudi Arabia

⁶KBRWyle Laboratories, Beavercreek, OH 45431, USA

⁷King Saud University, Riyadh, Saudi Arabia

⁸School of Electrical Engineering, Tel Aviv University, Ramat Aviv 69978, Israel

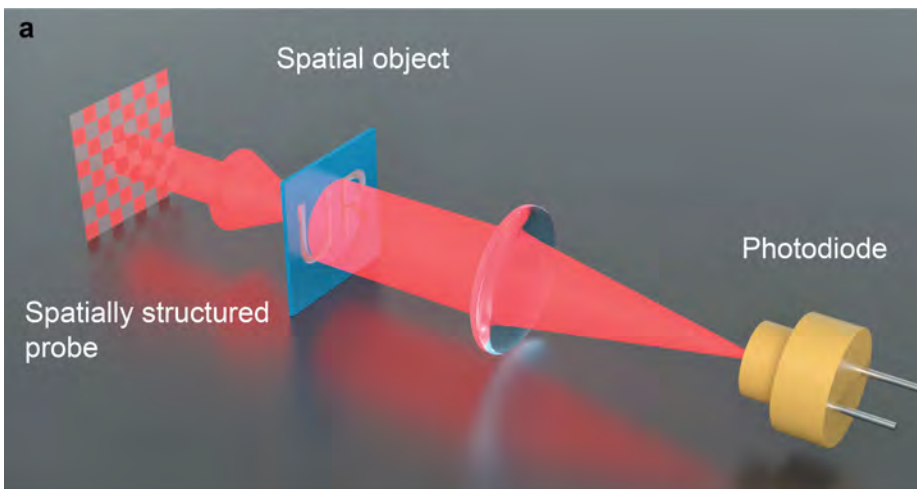
*Corresponding author: liucong@usc.edu

Ultrafast Pulse Measurements by Time-Domain Single-Pixel Imaging

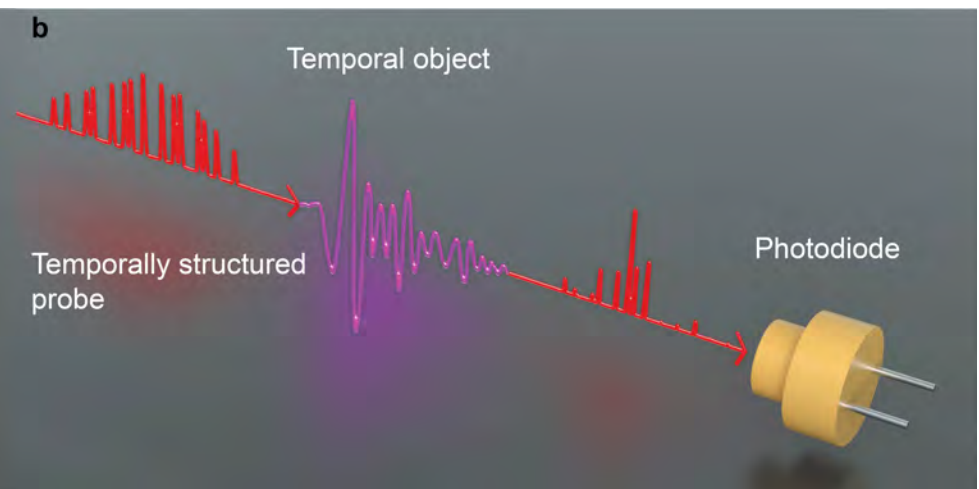
Single-pixel imaging is now a standard method for image acquisition under certain circumstances.

Can we apply the same concepts to ultrafast pulse measurements?

Space-domain

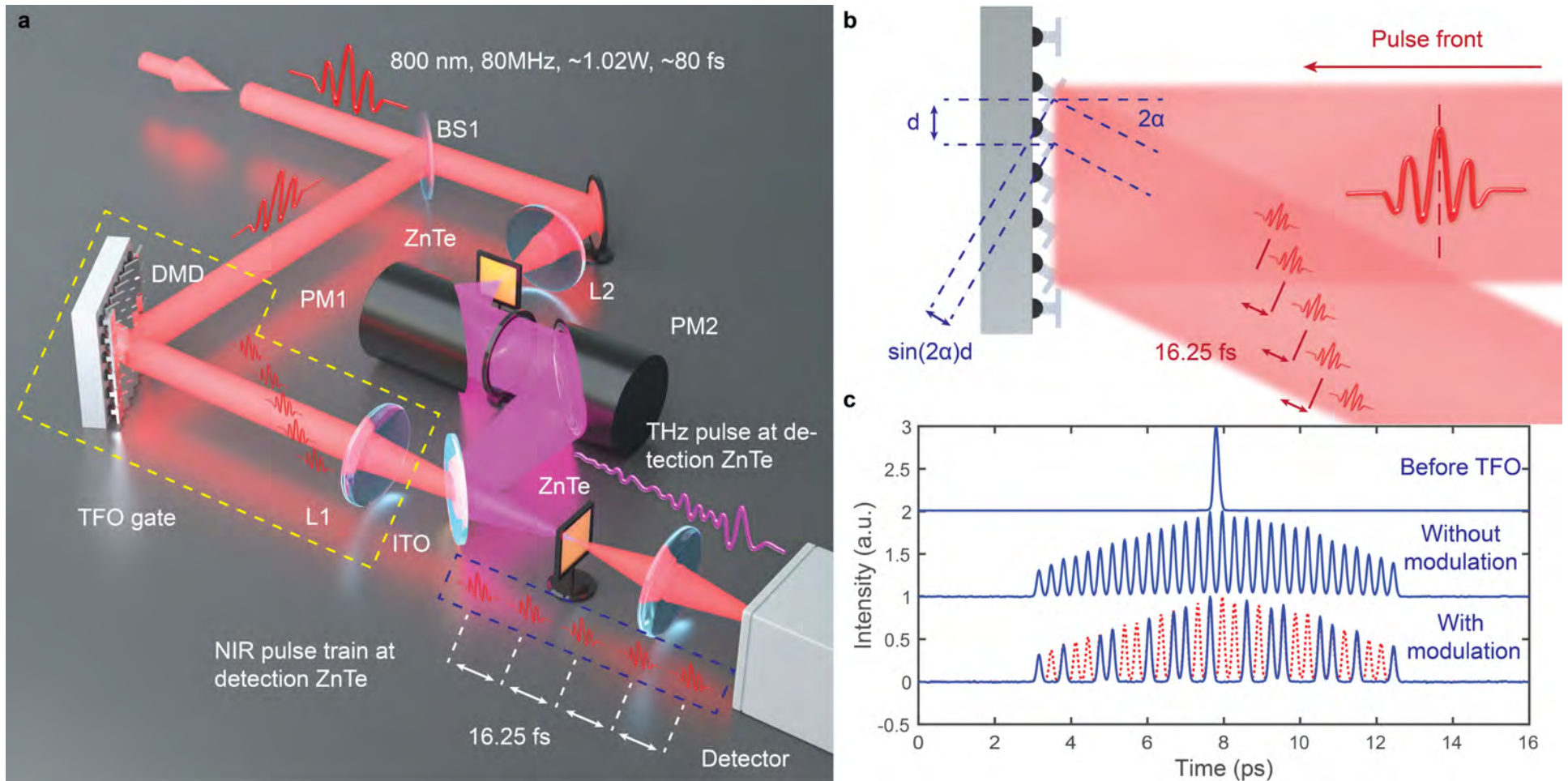


Time-domain



Ultrafast Pulse Measurements by Time-Domain Single-Pixel Imaging

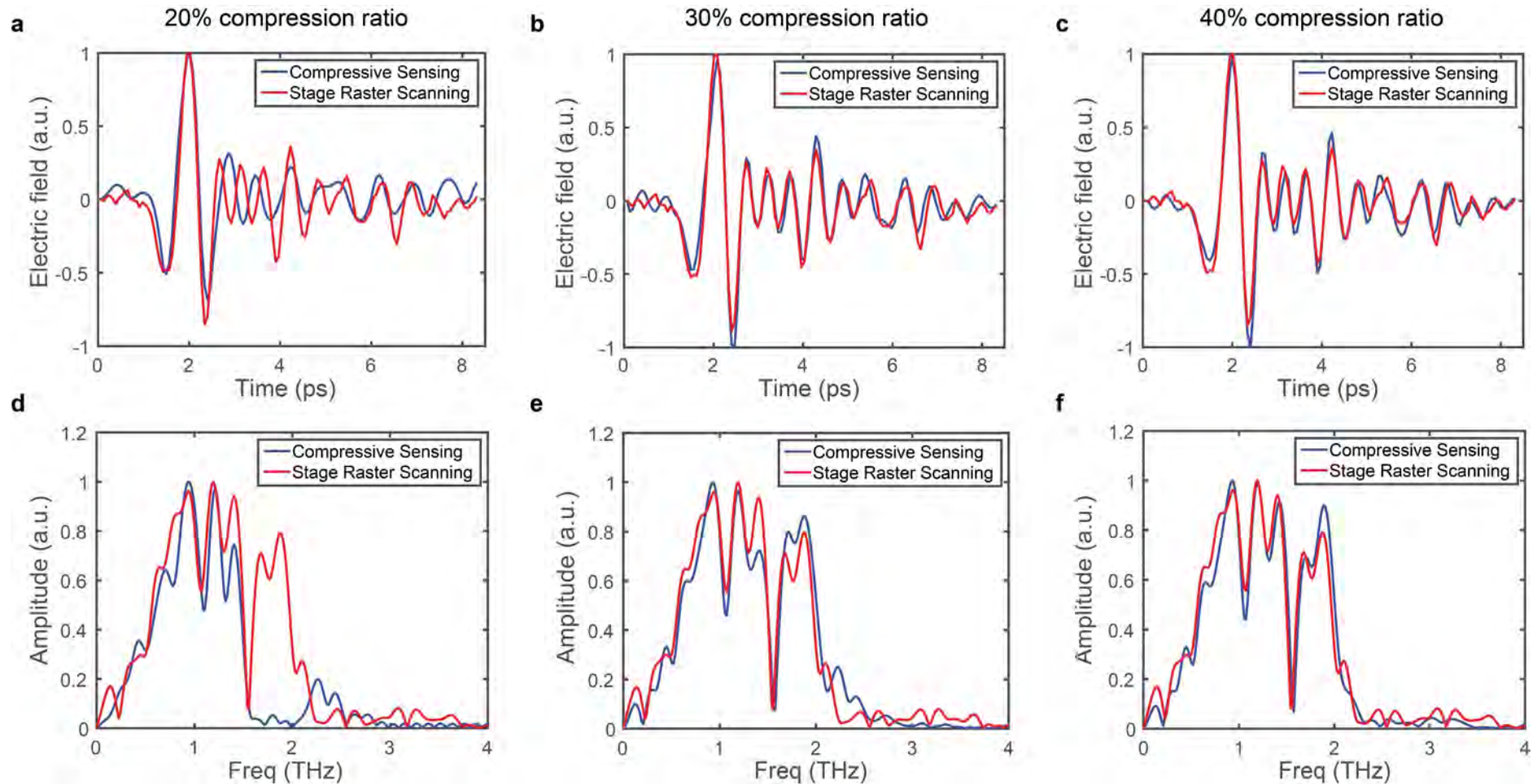
Our laboratory implementation



Ultrafast Pulse Measurements by Time-Domain Single-Pixel Imaging

Demonstration of recovery of a THz pulse with sub-ps features

We use compressive sampling to speed up pulse measurements

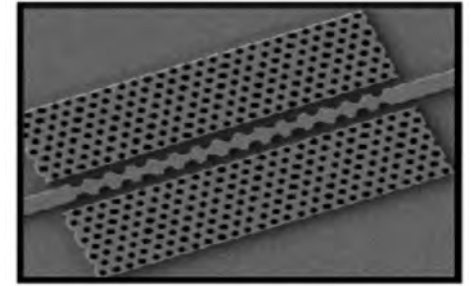


Relaxed Phase-Matching Requirements in ENZ Media

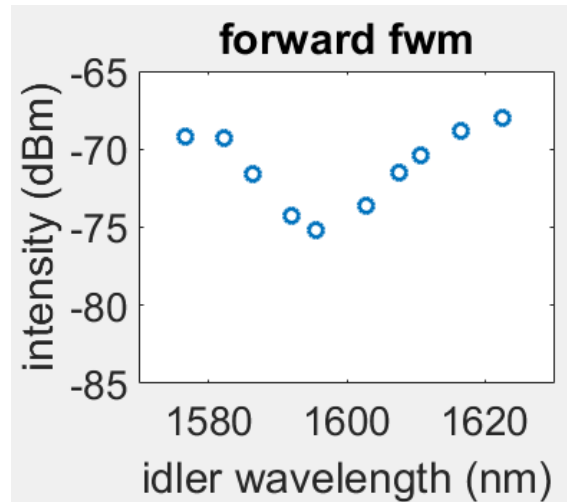
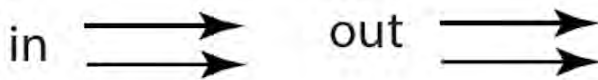
- We study four-wave mixing in a zero-index waveguide

$$2\omega_p = \omega_s + \omega_i$$

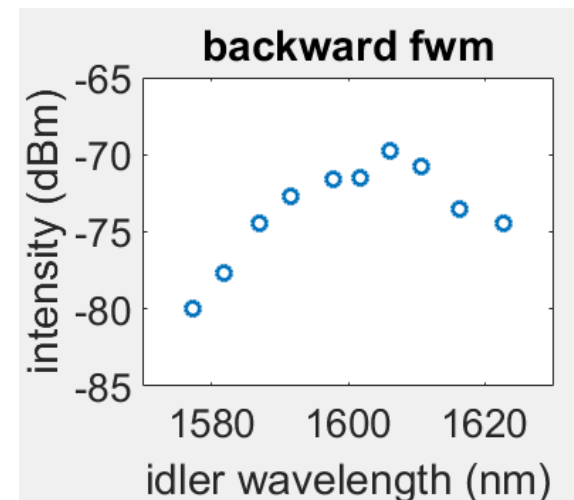
- We find that an idler field is generated in both the forward and backward directions!
- Recall that we need $\Delta k = 0$, but when $n = 0$, $k = n \omega / c$ vanishes for each of the interacting waves and thus so does Δk .



“forward” process



“backward” process



- Significance: Nonlinear optical processes that were previously believed to be too weak to be useful can be excited through use of ENZ materials.



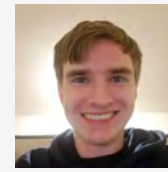
Boris
Braverman



Xialin
Liu



Alexander
Skerjanc



Nicholas
Sullivan



Robert W.
Boyd

Acousto-Optic Modulators for Arbitrary Spatial Mode Control

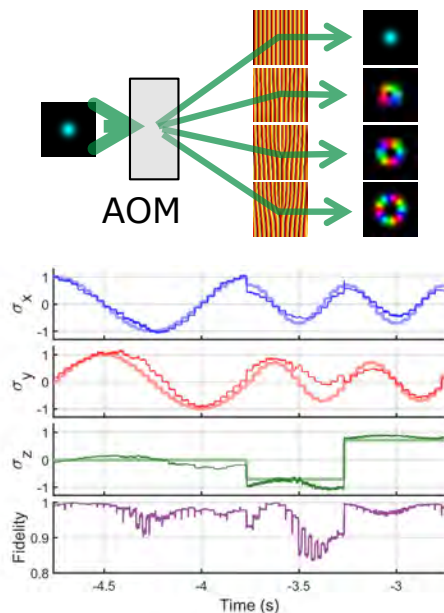
B. Braverman, X. Liu, A. Skerjanc, N. Sullivan, and R.W. Boyd

July 7, 2020

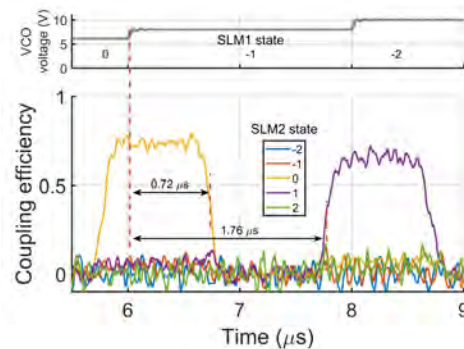
boydnlo.ca

Acousto-Optic Modulators for Arbitrary Spatial Mode Control

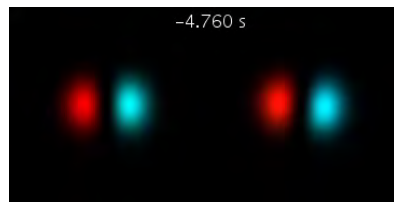
An AOM can be used to **quickly select** one of several static holograms on an SLM



>1 kHz state tomography in 2-D Hilbert space, 97% fidelity

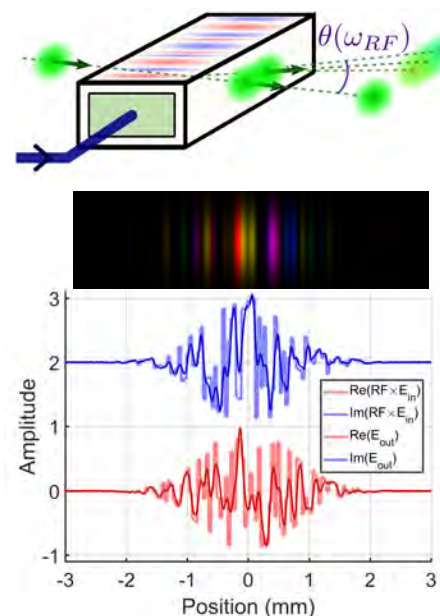


500 kHz switching among any 6 spatial modes

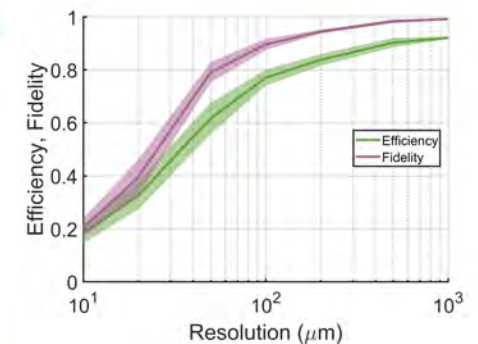


Predicted and reconstructed spatial modes

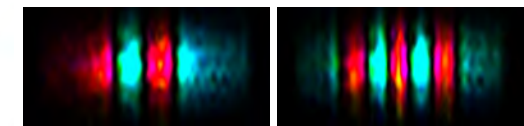
Using an appropriately modulated RF input, the **AOM itself** can generate spatial modes for pulsed light



Highly complex modes can be produced



High fidelity, efficiency at 0.1 mm resolution with RF optimization



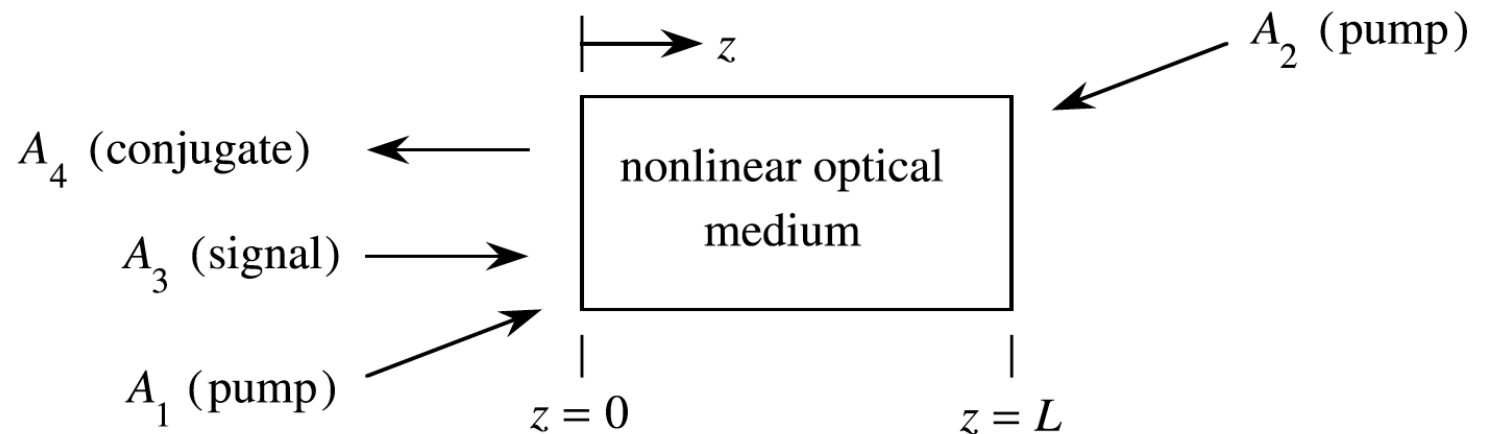
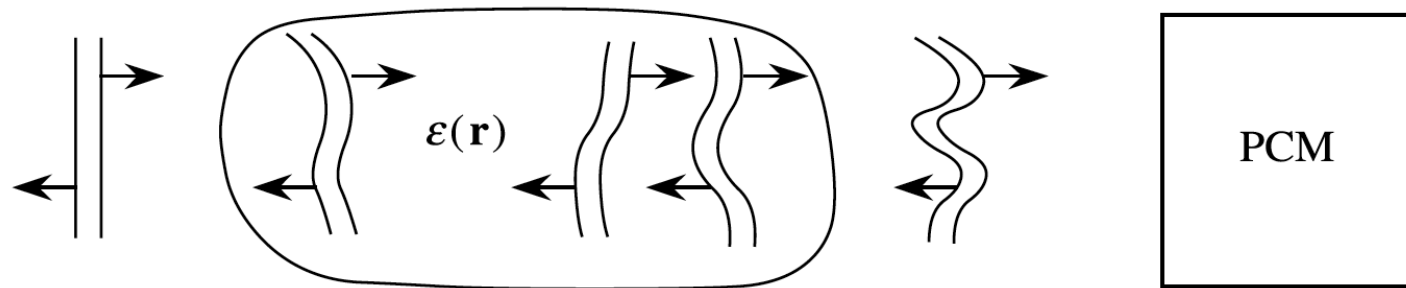
Experimentally generated HG3 and HG6 modes

High-fidelity spatial mode transmission through a 1-km-long multimode fiber via vectorial time reversal

Yiyu Zhou^{1✉}, Boris Braverman², Alexander Fyffe³, Runzhou Zhang⁴, Jiapeng Zhao¹, Alan E. Willner⁴, Zhimin Shi³ & Robert W. Boyd^{1,2}

Phase conjugation (generation of time reversed wavefronts) has historically been implemented through use of a nonlinear optical interaction, and was inefficient.

We instead are implementing a digital form of phase conjugation.

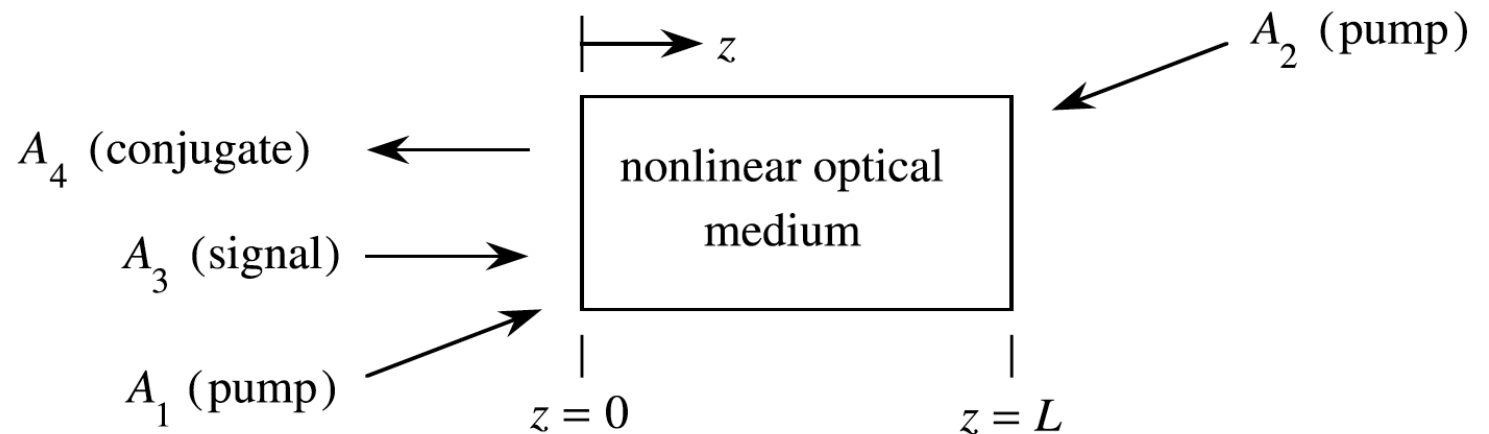
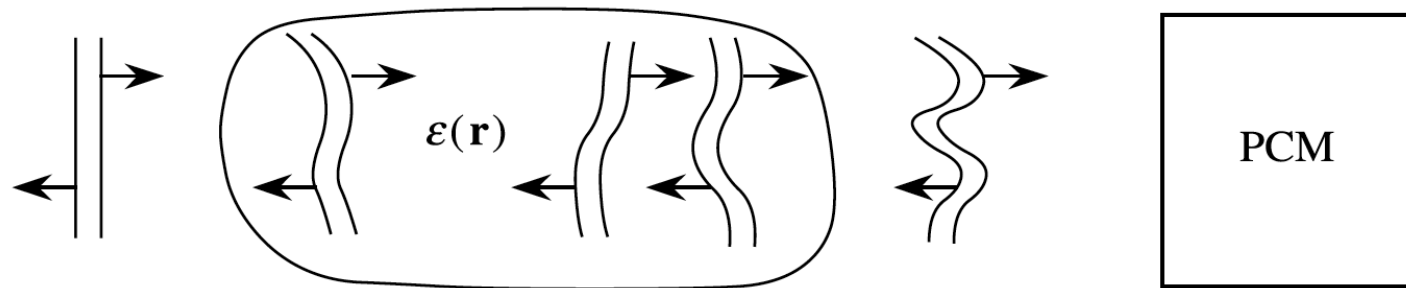


High-fidelity spatial mode transmission through a 1-km-long multimode fiber via vectorial time reversal

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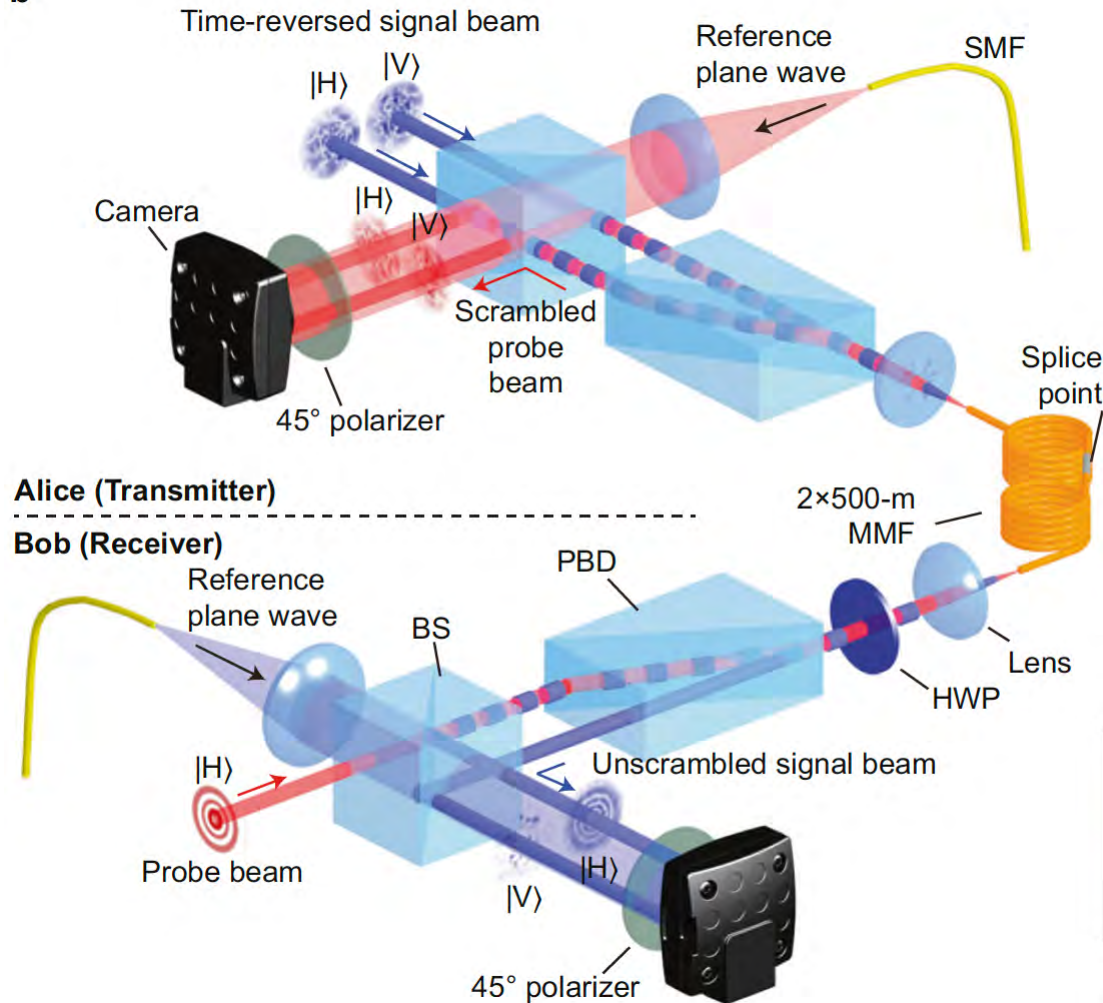
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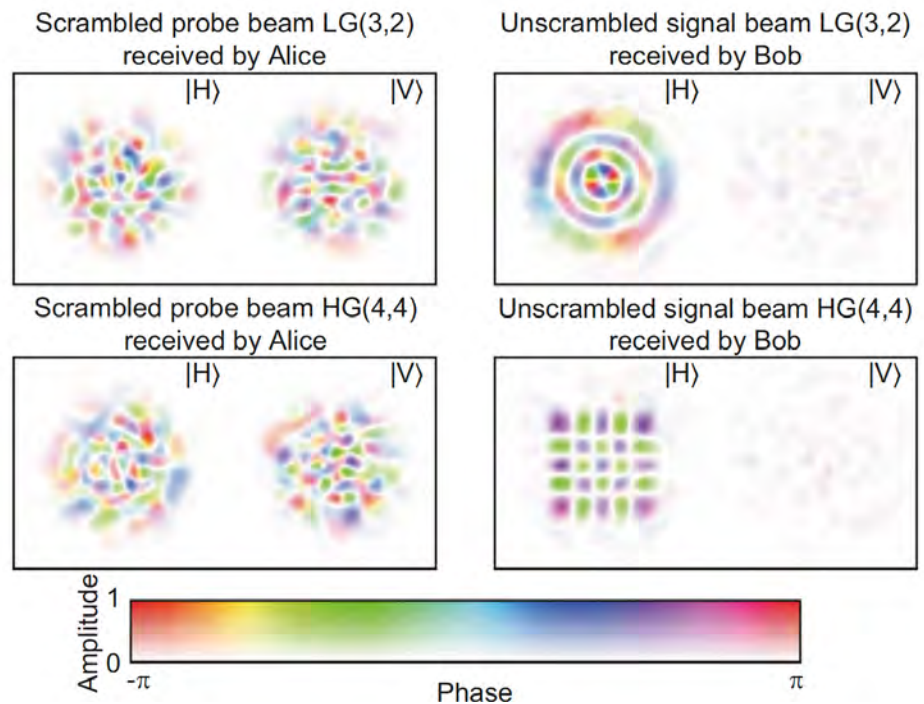
Implementation of Digital Phase Conjugation

b Lab setup



Bob sends a probe beam to Alice which becomes distorted. Alice measures the wavefront distortion and send to Bob a wave that is the phase conjugate of the wave she received.

Results:



Summary: Physics and Applications of ENZ Materials

- Extremely interesting physical processes occur in ENZ materials
- ENZ materials, metamaterials, and metastructures display extremely large NLO response
- The huge, ultrafast NLO response of ENZ materials lend themselves to many important applications

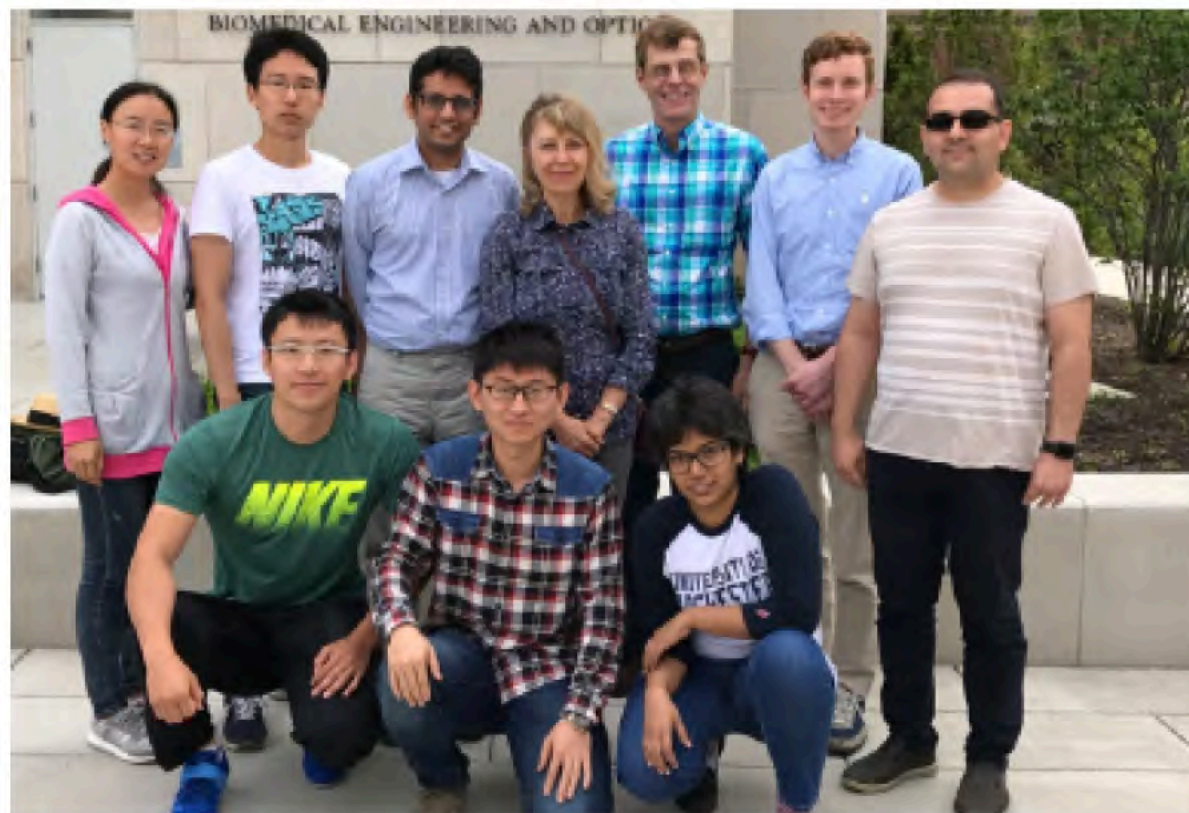
The visuals of this talk are posted at boydnlo.ca/presentations

Special Thanks To My Students and Postdocs!

Ottawa Group



Rochester Group



Summary: Physics and Applications of ENZ Materials

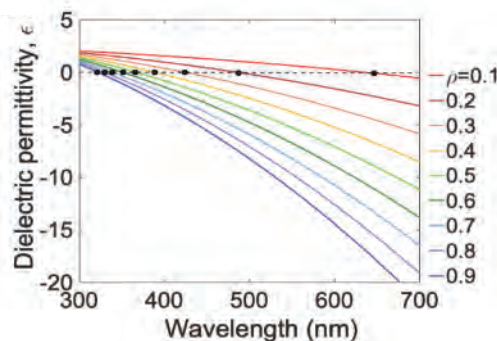
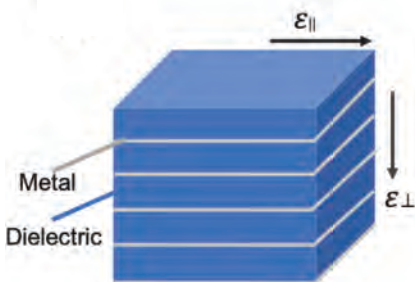
- Extremely interesting physical processes occur in ENZ materials
- ENZ materials, metamaterials, and metastructures display extremely large NLO response
- The huge, ultrafast NLO response of ENZ materials lend themselves to many important applications that require ultrafast time-varying media.

The visuals of this talk are posted at boydnlo.ca/presentations

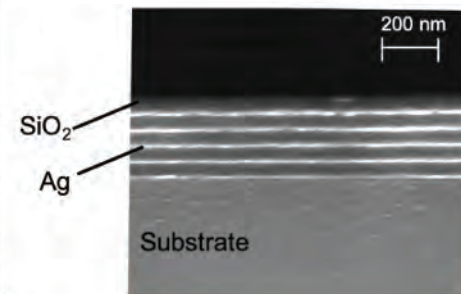
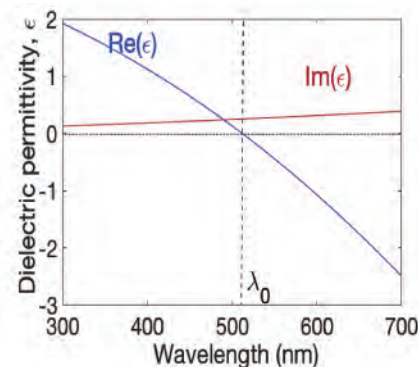
Nonlinear Optical Properties of a Layered Metamaterial in its ENZ Region

Do layered metamaterials also show enhanced NLO response at ENZ wavelength?

- By controlling the metallic fill fraction ρ , we can set the ENZ wavelength to be anywhere from 300 to 700 nm. We use $\rho = 0.2$, which corresponds to 500 nm. We deposit five layer pairs

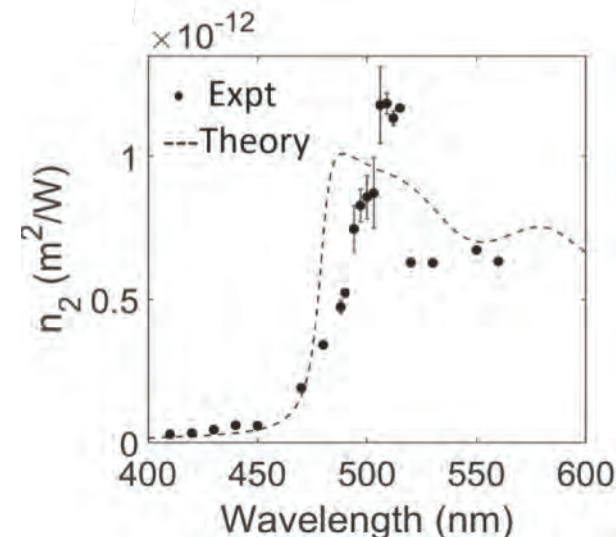
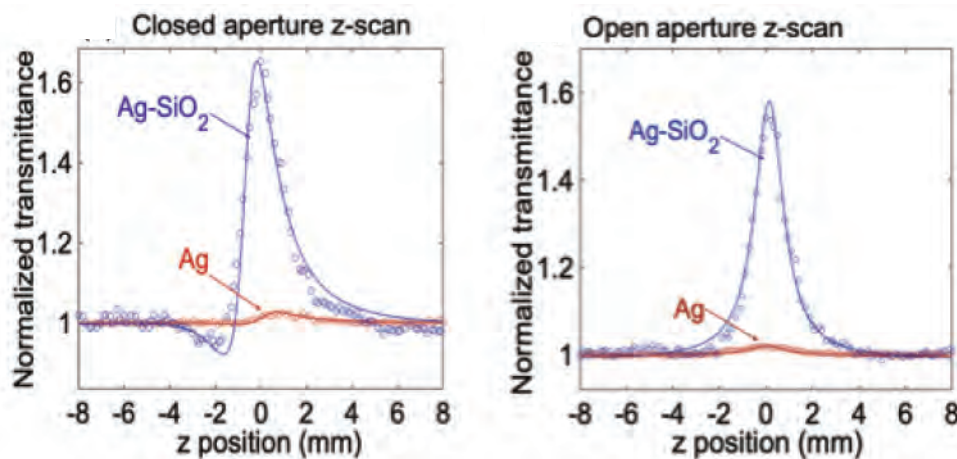


- Note that the real part of epsilon vanishes at 508 nm, close to the design wavelength. The SEM shows our structure. Ag thickness = 16 nm; silica thickness = 65 nm



- We perform Z-scan measurements on the sample. Note the enhanced response of the composite as compared to a single layer of silver.

- Note the pronounced peak in the value of n_2 around the ENZ wavelength. We find a good but not perfect agreement with a simple effective medium theory.



Why Does ENZ Lead to Large NLO Response?

1. From form of n_2
$$n_2 = \frac{3\chi^{(3)}}{4\epsilon_0 c n_0 \text{Re}(n_0)}$$

2. From simple math: $n = n_b + \Delta n$ and $\epsilon = \epsilon_b + \Delta\epsilon$

$$n = \sqrt{\epsilon_b + \Delta\epsilon} \approx \sqrt{\epsilon_b} \left(1 + \frac{\Delta\epsilon}{2\epsilon_b}\right) = n_b + \frac{\Delta\epsilon}{2n_b} \text{ and thus } \Delta n = \frac{\Delta\epsilon}{2n_b}$$

3. Note behavior of wave equation for $\epsilon = 0$

$$\nabla \times \nabla \times \mathbf{E} + \frac{\epsilon\mu}{c^2} \frac{\partial^2}{\partial t^2} \mathbf{E} = -\mu \frac{\partial^2 \mathbf{P}^{\text{NL}}}{\partial t^2}$$

4. From Maxwell's equations, it is easy to show that the nonlinear response scales as

$$\left. \frac{dH_x}{dz'} \right|_{\text{nl}} \frac{1}{|H_x|} \propto \sqrt{\frac{\mu_r}{\epsilon_r}}$$

5. Detailed numerical integration confirms this behavior.

Dependence of Second-Harmonic Generation on the Linear Dielectric Permittivity

- We solve the standard equations for second-harmonic generation

$$\frac{dA_1}{dz} = i \frac{\eta_1 \omega_1 \chi^{(2)}}{c} A_2(z) A_1^*(z) e^{-i\Delta k z},$$

$$\frac{dA_2}{dz} = i \frac{\eta_2 \omega_2 \chi^{(2)}}{2c} A_1^2(z) e^{i\Delta k z},$$

- We take $\Delta k = 0$ and plot the solution for various values of the permittivity ϵ .
- We find that the growth rate increases dramatically as the permittivity is decreased.

