How Light Behaves When The Refractive Index Vanishes

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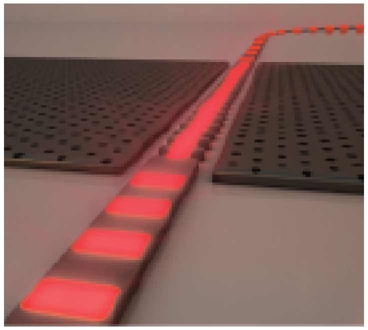
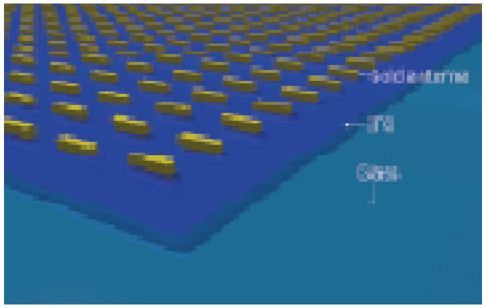
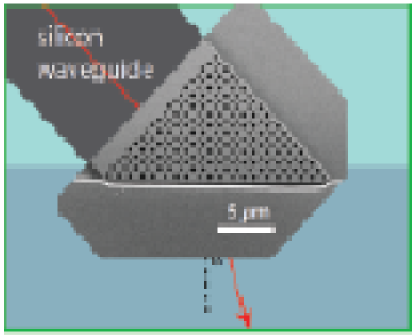
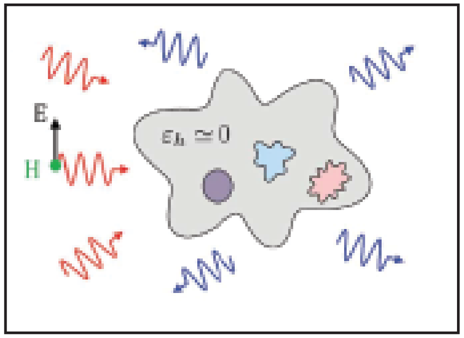








The visuals of this talk are posted at BoydNLO.ca/presentations

Presented at CLEO, San Jose, CA, USA, May 13, 2020.

Physics and Applications of Epsilon-Near-Zero Materials

- **Physics of ENZ Materials**
- Huge NLO Response of ITO and ITO Metastructures
- Materials for ENZ
- Applications of ENZ Materials

Giant Nonlinear Response of ENZ Metastructures

- Nonlinear Optics is important for a variety of reasons:
  - Photonic Devices
    - All-optical switching, buffers and routers based on slow light
  - Used to create quantum states of light for
    - Quantum Computing/Communications/Imaging
  - Fundamental understanding of light-matter interactions
    - Not "just" Lorentz oscillator formalism
    - Understand rogue waves
    - Control filamentation process

- However, the nonlinear response is usually much weaker than the linear response

- Means to enhance the nonlinear response
  - Resonance interactions (atomic vapors)
  - Plasmonic systems
  - Electromagnetically induced transparency (EIT)
  - Metamaterials (composite materials)

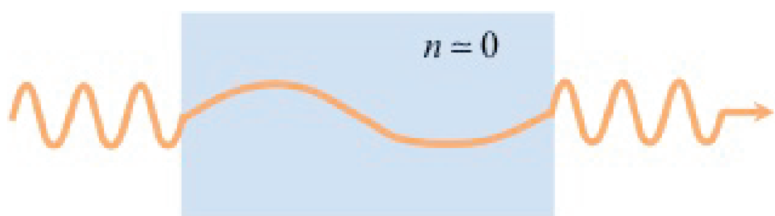
- Our approach: Use epsilon-near-zero (ENZ) materials and metamaterials

Physics of Near-Zero-Index (NZI) and Epsilon-Near-Zero (ENZ) Materials

- The wavelength of light is given by

$$\lambda = \lambda_{\mathrm{vac}}/n$$

and is significantly lenthened in a NZI material. The wavelength approaches infinity as *n* approaches zero.



- The phase velocity of light is given by

$$v = c/n$$

and also approaches infinity as n approaches zero.

- For n approaching zero, the field oscillates in time but not in space; oscillations are in phase everywhere

Brown, Proc. IEE 100, 5 (1953).
Ziolkowski, Phys. Rev. E 70, 046608 (2004).
Silveirinha and Engheta, Phys. Rev. Lett. 97, 157403 (2006).

Some Details from Electromagnetic Theory

- The linear response of any material to electromagnetic radiation can be described by

  - The dielectric permittivity (dielectric constant) ε define through the relation

$$\mathbf{D} = \epsilon_0 \epsilon \mathbf{E}$$

where **D**, known as the dielectric displacement, and **E**, known as the electric field, are the two fieldsthat describe the material response to an electric field.

  - The magnetic permeability μ define through the relation

$$\mathbf{B} = \mu_0 \mu \mathbf{H}$$

where **B,** known as the magnetic field, and **H**, known as the magnetic intensity, are the two fields that describe the magnet response of a material to an applied field.

- It is straightforward to shown from the equations of electromagnetism that

$$n = \sqrt{\epsilon \mu}$$

- Thus, *n*=0 when either ε =0 or μ=0 (or both ε and μ equal zero).

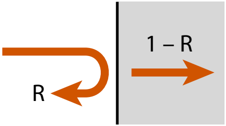
- Terminology:
  ENZ: epsilon near zero
  MNZ: mu near zero
  EMNZ: epsilon and mu near zero

Surface Reflection

- There is a problem getting light into a zero-index material.
- There is always reflection from the boundary between two materials
- The impedance and surface reflectivity are given by

$$Z = \sqrt{\mu/\epsilon} \qquad\qquad R = \left|\frac{Z-1}{Z+1}\right|^2$$

- Thus the reflectivity will be 100% if $\epsilon = 0$ unless $\mu = 0$ as well.



- This is one reason for the interest in developing EMNZ materials (epsilon and mu near zero materials.

Physics of Epsilon-Near-Zero (ENZ) Materials

- Radiative processes are modified in ENZ materials

Einstein A coefficient (spontaneous emission lifetime = $1/A$)

$A = n\, A_{\text{vac}}$

We can control (inhibit!) spontaneous emission!

Einstein B coefficient

Stimulated emission rate = B times EM field energy density

$B = B_{\text{vac}} \,/\, (n\, n_{\text{g}})$

Optical gain is very large!

Einstein, Physikalische Zeitschrift 18, 121 (1917).
Milonni, Journal of Modern Optics 42, 1991 (1995).

Equations are shown for nonmagnetic ($\mu = 1$) materials

- Implications:

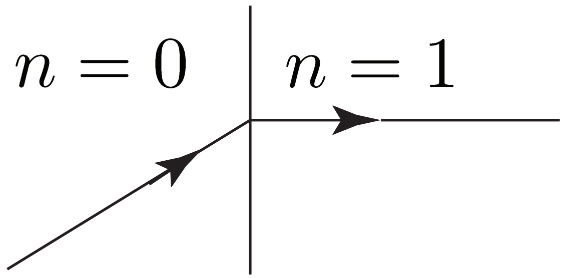
- If we can inhibit spontaneous emission, we can build thresholdless lasers.
- Expect superradiance effects to be pronounced in ENZ materials.

Optics of Zero-Index Materials

- Snell's law leads to intriguing predictions

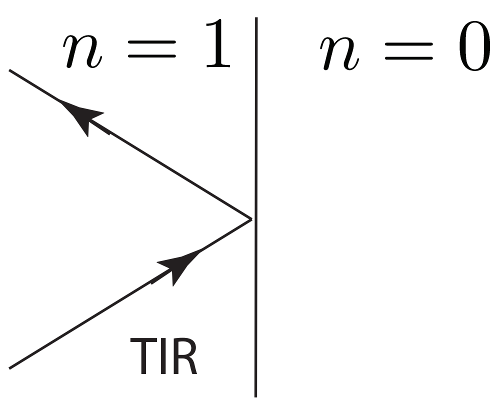
$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

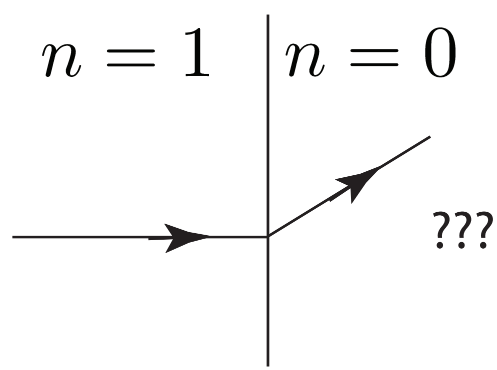
- Light always leaves perpendicular to surface of ENZ material!



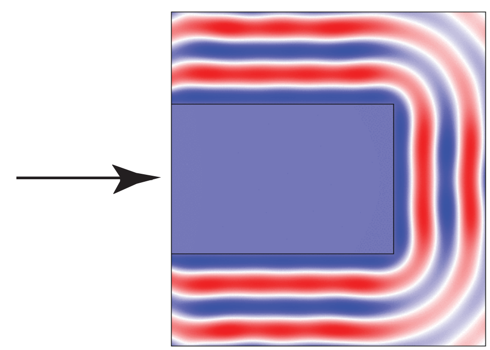
Y. Li, et al., Nat. Photonics 9, 738, 2015; D. I. Vulis, et al., Opt. Express 25, 12381, 2017.

- Thus light can enter an ENZ material only at normal incidence!





Y. Li, et al., Nat. Photonics 9, 738, 2015.



Light enters at normal incidence but leaves in all directions.

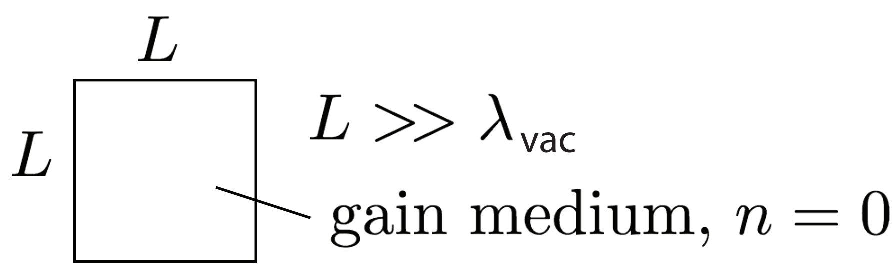
(wave-optics simulation - O. Reshef)

Some Consequences of ENZ Behavior - 1

- "Funny" lenses

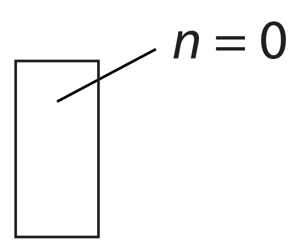
A. Alù et al., Phys. Rev. B 75, 155410, 2007; X.-T. He, ACS Photonics, 3, 2262, 2016.

- Large-area single-transverse-mode surface-emitting lasers



J. Bravo-Abad et al., Proc. Natl. Acad. Sci. USA 109, 976, 2012.

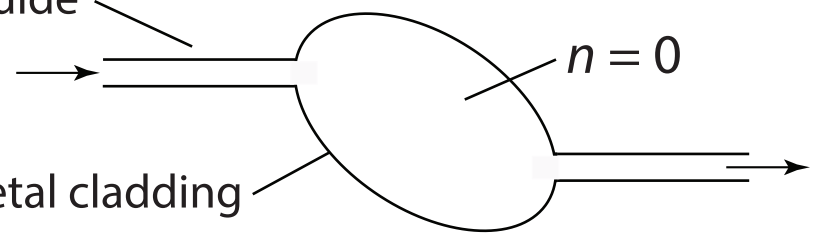
- No Fabry-Perot interference



O. Reshef et al., ACS Photonics 4, 2385, 2017.

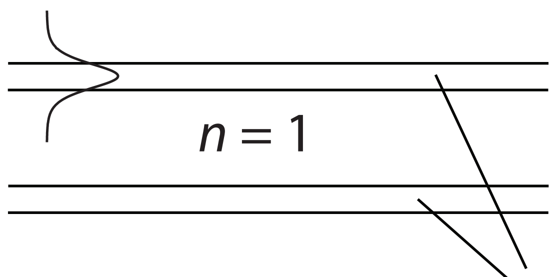
Some Consequences of ENZ Behavior - 2

- Super-coupling (of waveguides)



M. G. Silveirinha and N. Engheta, Phys. Rev. B 76, 245109, 2007; B. Edwards et al., Phys. Rev. Lett. 100, 033903, 2008.

- Coupling between two distant waveguides



Mode of upper waveguide beams into the lower waveguide even for large separation

Recall that $k = n\,\omega / c$ vanishes in an ENZ medium.

- Automatic phase matching of NLO processes

  - Recall that we need $\Delta k = 0$, but when $n=0$ $k = n\omega/c$ vanishes and so does Δk.

  - We have observed this effect in a Dirac-cone, zero-index metamaterial.

  - Usual four-wave mixing process

  - With zero-index materials we can have

H. Suchowski et al., Science 342, 1223, 2013.

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- **Huge NLO Response of ITO and ITO Metastructures**
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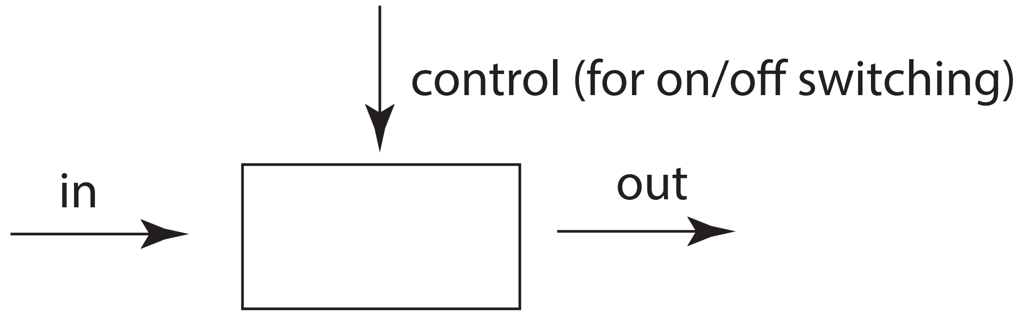
Nonlinear Optics and Optical Switching

- An important application in photonic technologies is optical switching.



- One wants a switch with fast switching times and that operates with weak control fields.
- One needs a nonlinear interaction in order for one optical field to control another field.
- A strong nonlinear response is needed. How does one quantify the strength of a nonlinear response? Two standard methods:

$$n = n_0 + n_2 I$$

$$P^{\mathrm{NL}} = 3\chi^{(3)}|E|^2 E$$

- The nonlinear coefficients are n_2 and $\chi^{(3)}$

Implications of ENZ Behavior for Nonlinear Optics

Here is the intuition for why the ENZ condition is of interest in NLO

Recall the standard relation between n_2 and $\chi^{(3)}$

$$n_2 = \frac{3\chi^{(3)}}{4\epsilon_0 c\, n_0\, \mathrm{Re}(n_0)}$$

Note that under ENZ conditions the denominator becomes very small, leading to a very large value of n_2

Footnote:

Standard notation for perturbative NLO

$$P = \chi^{(1)}E + \chi^{(2)}E^2 + \chi^{(3)}E^3 + \ldots$$

P is the induced dipole moment per unit volume and E is the field amplitude.

Also, the refractive index changes according to

$$n = n_0 + n_2 I + n_4 I^2 + \ldots$$

How to Choose an Epsilon-Near-Zero Materials

- Electrical conductors

  All conductors display ENZ behavior at their (reduced) plasma frequency

  Recall the Drude formula

$$\epsilon(\omega) = \epsilon_\infty - \frac{\omega_p^2}{\omega(\omega + i\gamma)}$$

  Note that $\mathrm{Re}\,\epsilon = 0$ for $\omega = \omega_p/\sqrt{\epsilon_\infty} \equiv \omega_0$.

  ENZ wavelength restricted to a limited range in the visible.

- Electrical insulators (dielectrics)

  Dielectrics can show ENZ behavior at their (optical) phonon resonance.

  ENZ wavelength restricted to a limited range in the mid-IR.

- Metamaterials

  Can design the material so that the ENZ or EMNZ wavelengths are at any desired value.

- Challenge (for any material system). For low loss, we want Im ε as small as possible at the wavelength where Re ε =0.

Nonlinear Optics of Indium Tin Oxide (ITO)

- We recently reported that, at its ENZ wavelength, ITO possesses a nonlinear coefficient n_2 that is 100 times larger than those of previously reported materials [1].
- ITO is a degenerate semiconductor (so highly doped as to be metal-like).
- ITO has a large density of free electrons, and a bulk plasma frequency corresponding to a wavelength of approximately 1.24 μm.
- Dielectric properties of ITO are well described by the Drude formula.

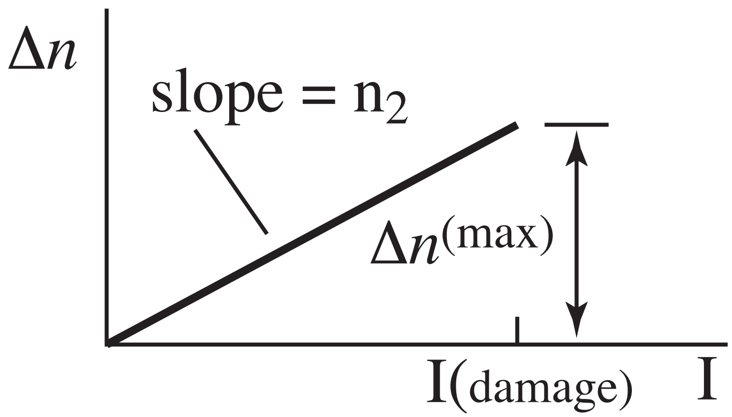
$$\epsilon(\omega) = \epsilon_\infty - \frac{\omega_p^2}{\omega(\omega + i\gamma)}$$

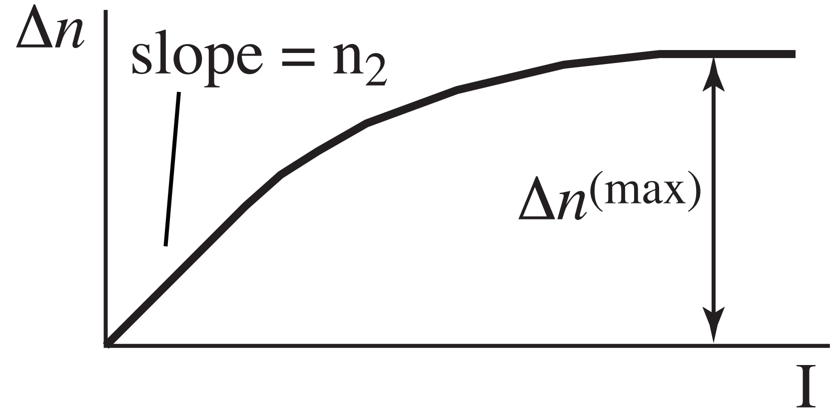
- Note that aluminum-doped zinc oxide (AZO), another transparent conducting oxide, also has strong nonlinear response at its ENZ wavelength [2].

1. Alam, De Leon and Boyd, Science 352, 795–797 (2016)
2. Caspani, Shalaev, Boltasseva, Faccio et al., Phys. Rev. Lett. 116, 233901 (2016).

What Makes a Good (Kerr-Effect) Nonlinear Optical Material?

- We want n_2 large ($\Delta n = n_2 I$). We also want $\Delta n^{(max)}$ large.
  These are distinct concepts! Damage and saturation can limit $\Delta n^{(max)}$





- For ITO at its ENZ wavelength, both n_2 and $\Delta n^{(max)}$ are extremely large:
  ($n_2 = 1.1 \times 10^{-10}$ cm^2/W and $\Delta n^{(max)} = 0.8$)

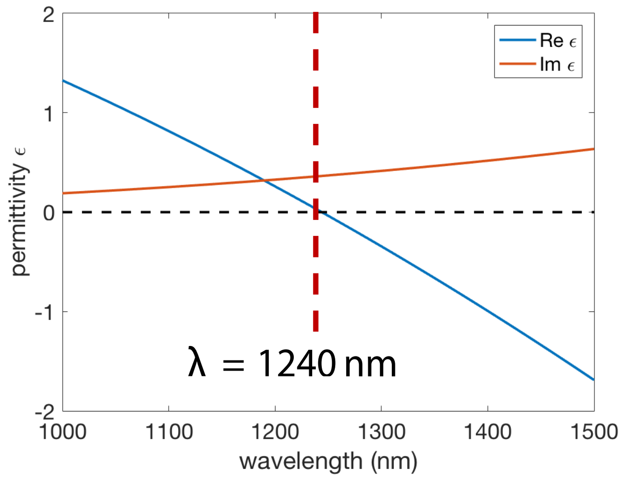
- n_2 is 3.4×10^5 times larger than that of silica glass
  $\Delta n^{(max)}$ is 2700 times larger that that of silica glass

  (For silica glass $n_2 = 3.2 \times 10^{-16}$ cm^2/W, $I_{damage} = 1$ TW/cm^2, and thus $\Delta n_{(max)} = 3 \times 10^{-4}$)

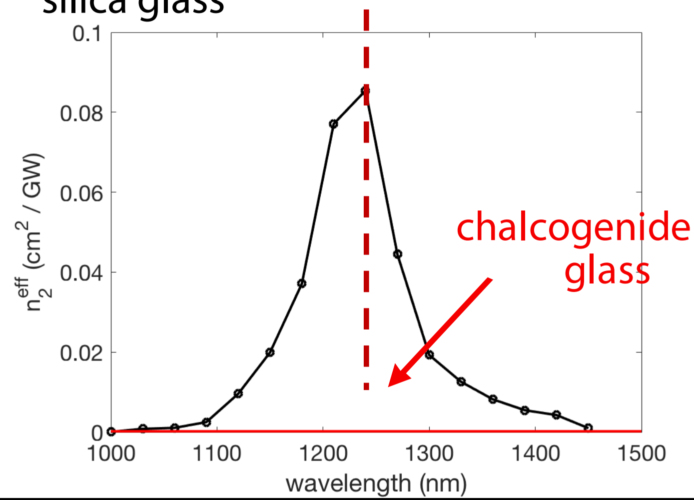
M. Z. Alam, I. De Leon, R. W. Boyd, Science 352, 795 (2016).

Huge, Fast NLO Response of Indium Tin Oxide at its ENZ Wavelength

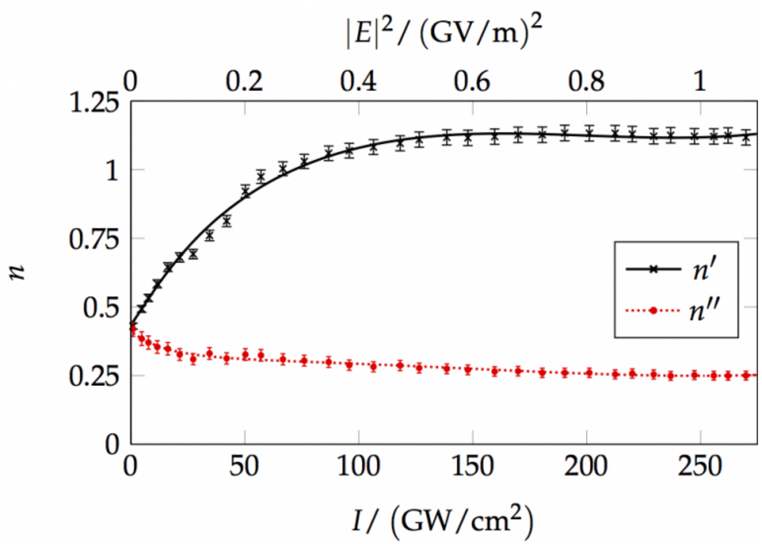
- ellipsometry



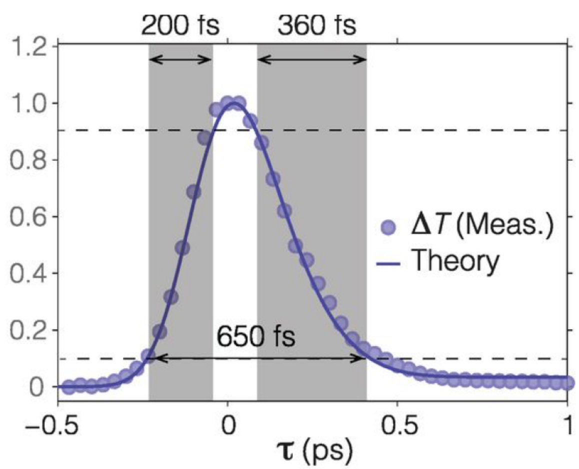
- n_2 can be 3.4 x10^5 times larger than that of silica glass



- overall change in refractive index of 0.8

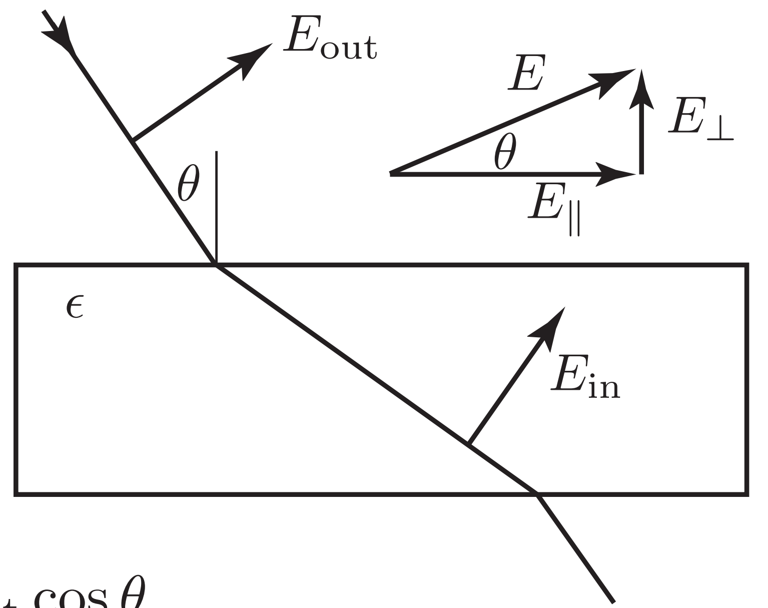


- sub picosecond reponse time



M. Z. Alam et al., Science 352, 795-797 (2016)

The NLO Response Is Larger For Oblique Incidence



Standard boundary conditions show that:

$$E_{\rm in,\parallel} = E_{\rm out,\parallel} = E_{\rm out}\cos\theta$$

$$D_{\rm in,\perp} = D_{\rm out,\perp} \quad \Rightarrow \quad E_{\rm in,\perp} = E_{\rm out,\perp}/\epsilon = E_{\rm out}\cos\theta/\epsilon$$

Thus the total field inside of the medium is given by

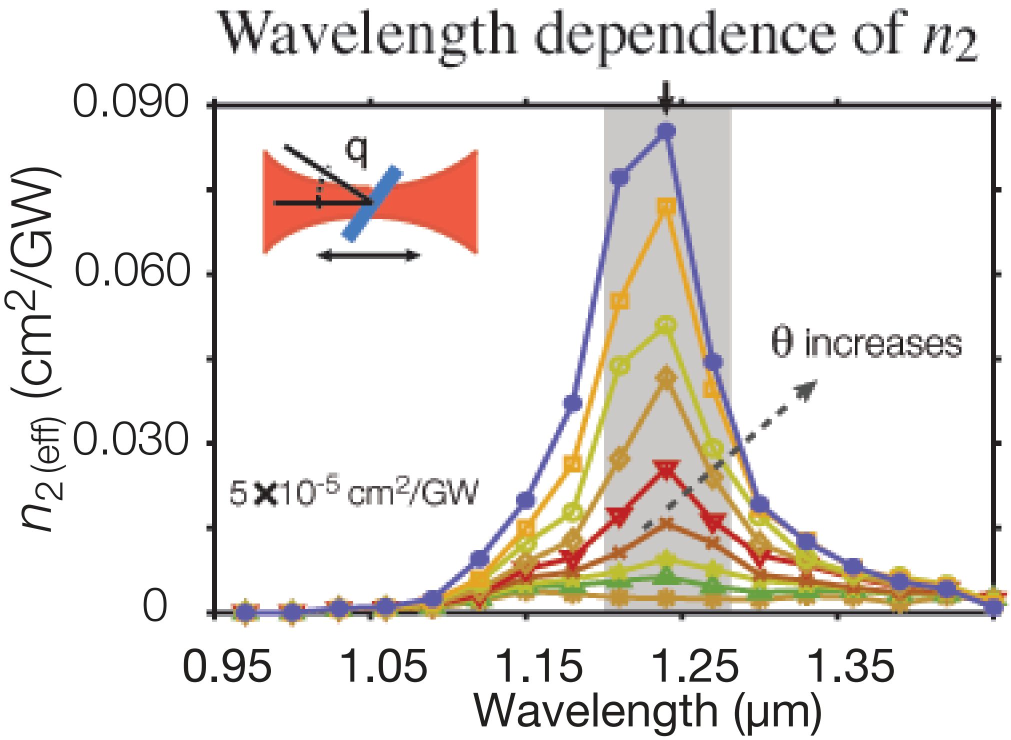
$$E_{\rm in} = E_{\rm out}\sqrt{\cos^2\theta + \frac{\sin^2\theta}{\epsilon}}$$

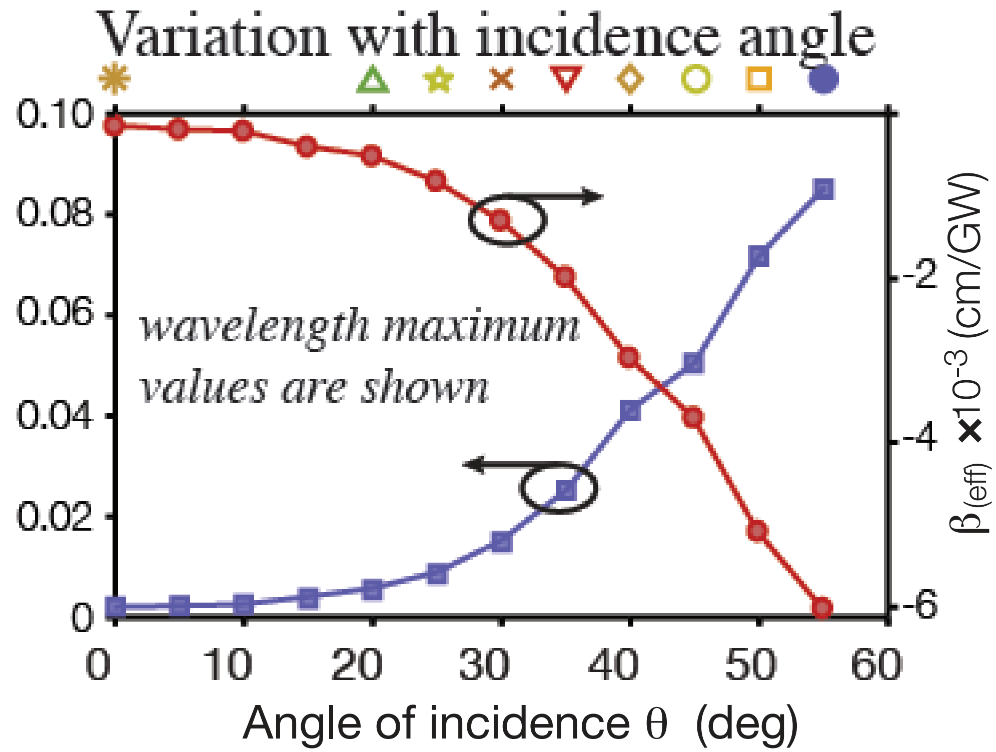
Note that, for $\epsilon < 1$, $E_{\rm in}$ exceeds $E_{\rm out}$ for $\theta \neq 0$.

Note also that, for $\epsilon < 1$, $E_{\rm in}$ increases as θ increases.

Huge Nonlinear Optical Response of ITO

- Z-scan measurements for various angles of incidence





peak laser intensity was 50 GW cm-2

- Note that n_2 is positive (self focusing) and β is negative (saturable absorption).

- Both n_2 and nonlinear absorption increase with angle of incidence

- n_2 shows a maximum value of 0.11 cm^2/GW = 1.1 × 10^{-10} cm^2/W at 1.25 μm and 60 deg. This value is 2000 times larger than that away from ENZ region.

Why is n_2 so large for ITO?

The short-wavelength (away from the ENZ resonance) value of n_2 of ITO is 5×10^{-5} cm^2/GW, which is 150 times larger that of fused silica (3.2×10^{-7} cm^2/GW).

There is a 43 x enhancement from working at the ENZ wavelength and an additional 43 x enhancement from using non-normal incidence.

Thus $n_2 = 0.1$ cm^2/GW, which is 3.4×10^5 times that of fused silica.

Incidentally, for arsenic trisulfide glass, $n_2 = 2.4 \times 10^{-4}$ cm^2/GW. which is 800 times larger than that of fused silica.

R.E. Slusher et al., J. Opt. Soc. Am. B 21, 1146 (2004).

Why Does ENZ Lead to Large NLO Response?

Recall that:

$$n_2 = \frac{3\chi^{(3)}}{4\epsilon_0 c\, n_0 \,\mathrm{Re}(n_0)}$$

It is instructive to see where this result comes from. Let

$$\epsilon = \epsilon_b + \Delta\epsilon \quad \text{(b = “background”)}$$

$$n = \sqrt{\epsilon} = \sqrt{\epsilon_b + \Delta\epsilon}$$

Assume $\Delta\epsilon \ll \epsilon_b$ (this assumption can be violated).

$$n = \sqrt{\epsilon_b}\left(1 + \frac{\Delta\epsilon}{2\epsilon_b} + \cdots\right) = \sqrt{\epsilon_b} + \frac{\Delta\epsilon}{2\sqrt{\epsilon_b}}$$

or

$$n = n_b + \Delta n \quad \text{where} \quad \Delta n = \frac{\Delta\epsilon}{2n_b}$$

Summary: Why Does ENZ Lead to Large NLO Response?

1. From form of n_2 $\quad n_2 = \dfrac{3\chi^{(3)}}{4\epsilon_0 c\, n_0\, \mathrm{Re}(n_0)}$

2. From simple math: $\quad n = n_b + \Delta n \quad \text{where} \quad \Delta n = \dfrac{\Delta\epsilon}{2n_b}$

3. Note behavior of wave equation for $\epsilon = 0$

$$\nabla \times \nabla \times \mathbf{E} + \frac{\epsilon\mu}{c^2}\frac{\partial^2}{\partial t^2}\mathbf{E} = -\mu\frac{\partial^2 \mathbf{P}^{\mathrm{NL}}}{\partial t^2}$$

4. From Maxwell's equations, it is easy to show that the nonlinear response scales as

$$\frac{\left.\dfrac{dH_x}{dz'}\right|_{\mathrm{nl}}}{|H_x|} \propto \sqrt{\frac{\mu_r}{\epsilon_r}}$$

5. Detailed numerical integration confirms this behavior.

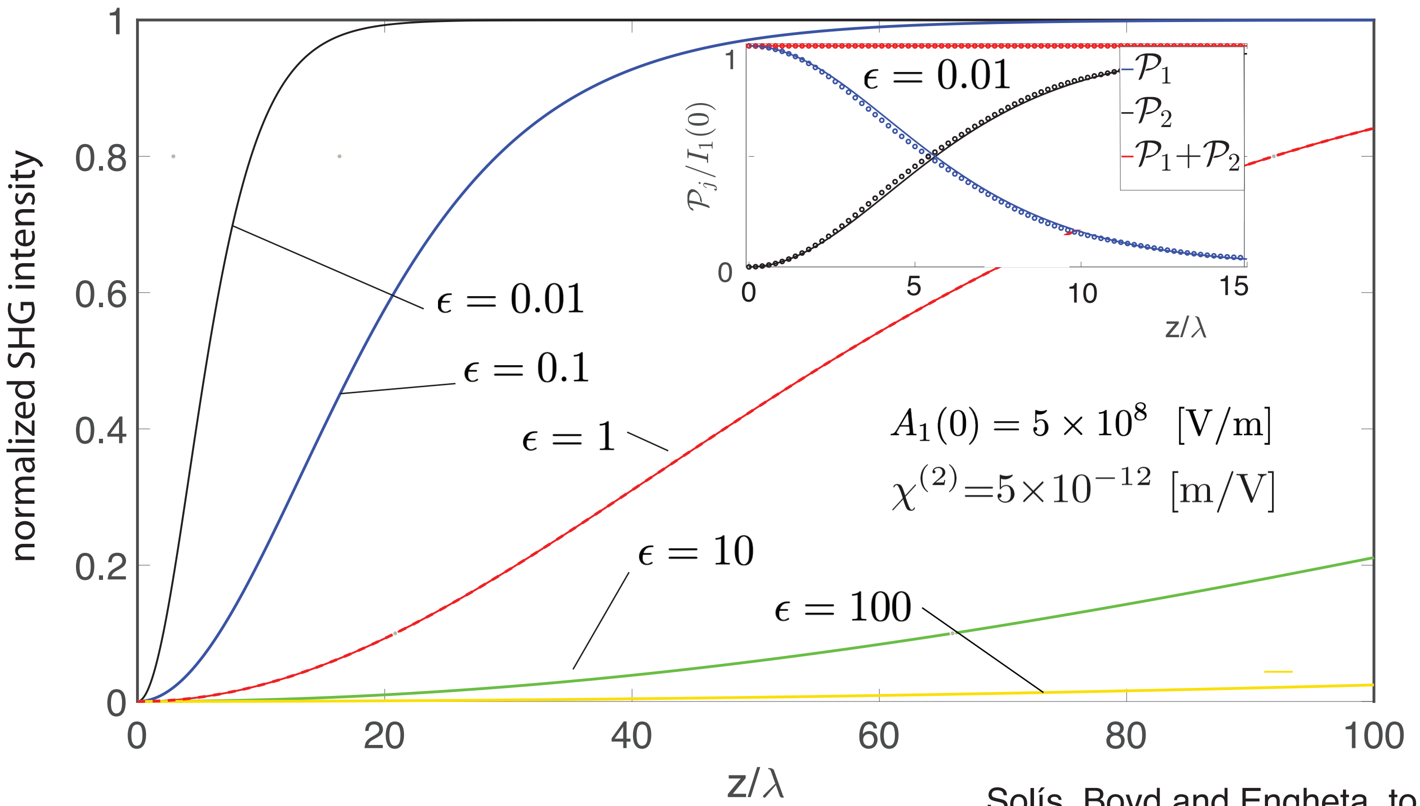
Dependence of Second-Harmonic Generation on the Linear Dielectric Permittivity

- We solve the standard equations for second-harmonic generation

$$\frac{dA_1}{dz} = i\frac{\eta_1\omega_1\chi^{(2)}}{c}A_2(z)A_1^*(z)e^{-i\Delta kz},$$

$$\frac{dA_2}{dz} = i\frac{\eta_2\omega_2\chi^{(2)}}{2c}A_1^2(z)e^{i\Delta kz},$$

- We take $\Delta k = 0$ and plot the solution for various values of the permittivity ε.
- We find that the growth rate increases dramatically as the permittivity is decreased.



Solís, Boyd and Engheta, to be published

See also Solís, CLEO Poster JW2D.15

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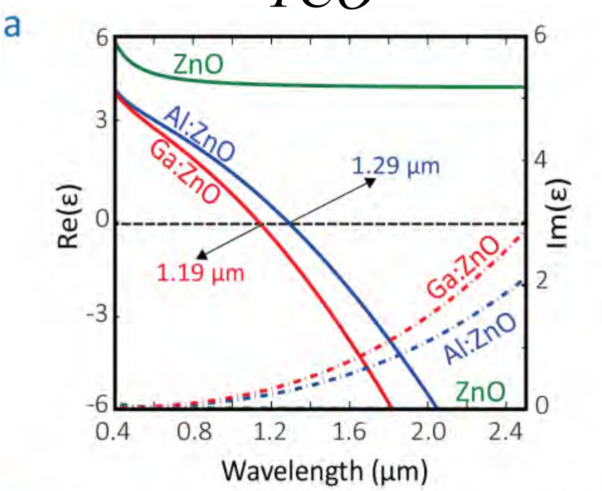
Epsilon-Near-Zero (ENZ) and Near Zero-Index (NZI) Materials

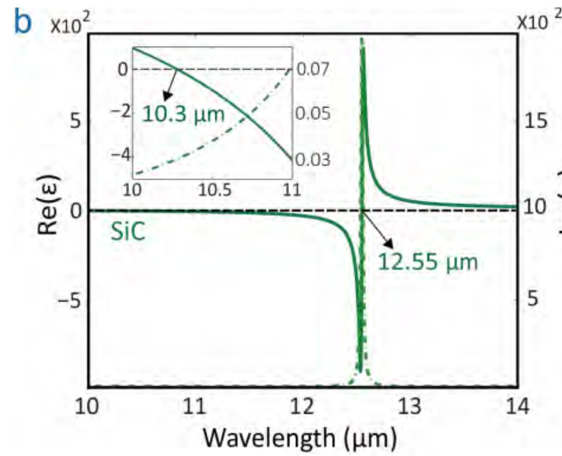
Homogeneous materials

TCO



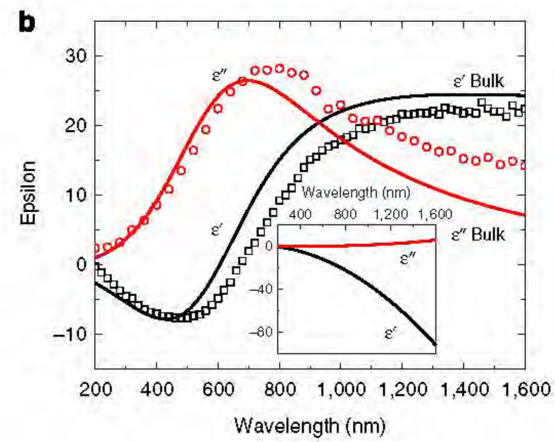
A. Boltasseva (Purdue)
Kim et al., Optica (2016)

SiC



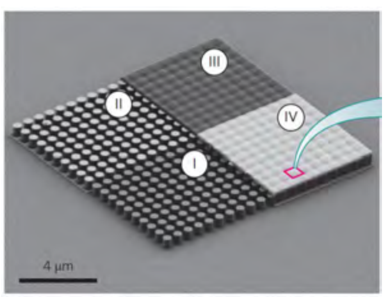
J. Caldwell (Vanderbilt)
Kim et al., Optica (2016)

$Bi_{1.5}Sb_{0.5}Te_{1.8}Se_{1.2}$

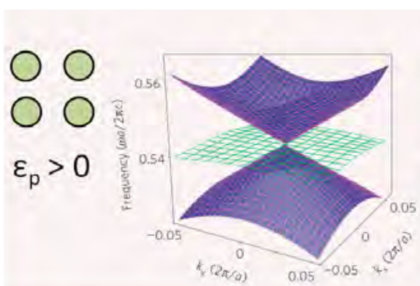


N. Zheludev (Southmapton)
Ou et al., Nat. Commun. (2014)

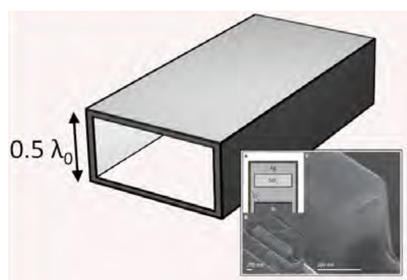
Metamaterials



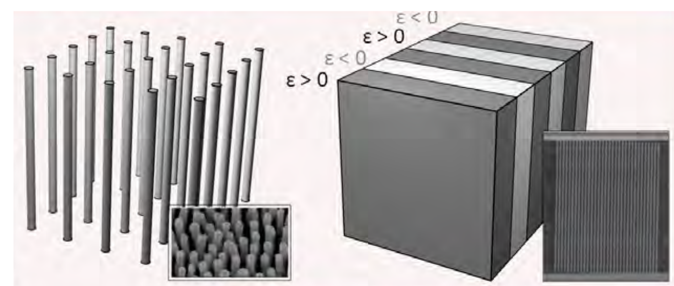
E. Mazur Li et al., Nat. Photon. (2015)



Chan, Huang et al., Nat. Mater. (2011)



SEM from: Polman's & Engheta's
Vesseur et al., PRL (2013)



$\mathrm{Re}(\varepsilon) \cong 0$

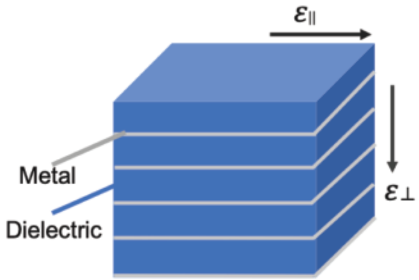
Wire SEM from: Zayat & Podolskiy
Pollard et al., PRL (2009)
Stack SEM from: Polman & Engheta
Mass et al., Nat. Photon. (2013)

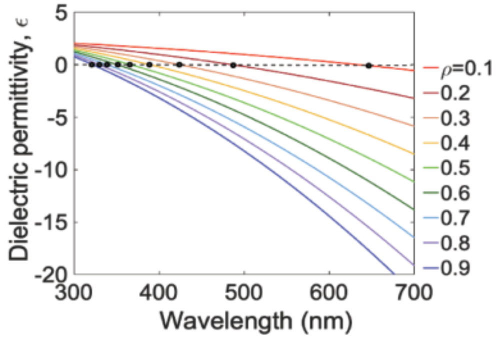
Nonlinear Optical Properties of a Layered Metamaterial in its ENZ Region

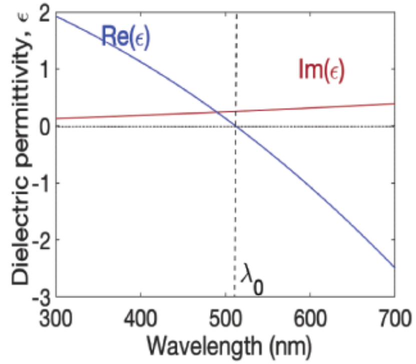
Do layered metamaterials also show enhanced NLO response at ENZ wavelength?

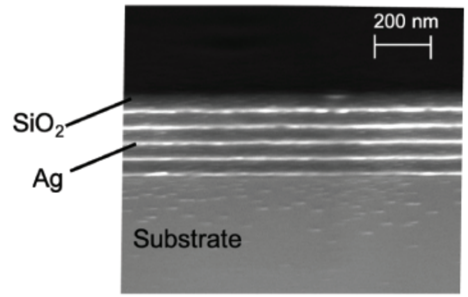
- By controlling the metallic fill fraction ρ, we can set the ENZ wavelength to be anywhere from 300 to 700 nm. We use $\rho = 0.2$, which corresponds to 500 nm. We deposit five layer pairs

- Note that the real part of epsilon vanishes at 508 nm, close to the design wavelength. The SEM shows our structure. Ag thickness = 16 nm; silica thicness = 65 nm



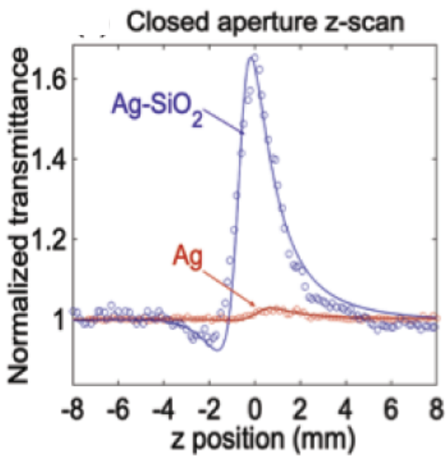


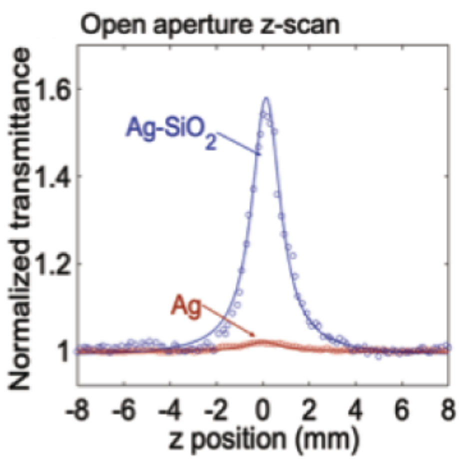


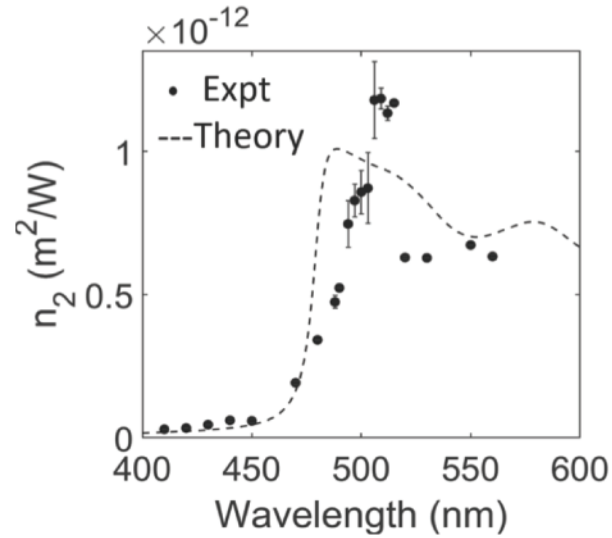


- We perform Z-scan measurements on the sample. Note the enhanced response of the composite as compared to a single layer of silver.

- Note the pronounced peak in the value of n_2 around the ENZ wavelength. We find a good but not perfect agreement with a simple effective medium theory.







Suresh, Reshef, Alam, Upham, Karimi and Boyd

Physics and Applications of Epsilon-Near-Zero Materials

- Physics of ENZ Materials
- Huge NLO Response of ITO and ITO Metastructures
- Materials for ENZ
- **Applications of ENZ Materials**

Giant Nonlinear Response of ENZ Metastructures: Our Team

Nader Engheta

- H. Nedwill Ramsey Professor at the University of Pennsylvania
- B.S. degree from the University of Tehran and his M.S and Ph.D. from Caltech.
- Activities include ENZ, photonics, metamaterials, nano-optics, graphene optics, electrodynamics, microwave and optical antennas, studies of fields and waves.
- Many awards including the Streifer Award of IEEE and the Gold Medal from SPIE

Eric Mazur

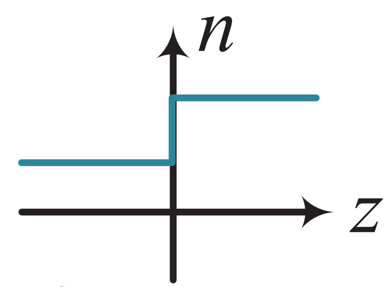
- Balkanski Professor of Physics and Applied Physics at Harvard University
- Ph.D. University of Leiden.
- Activities include light-matter interactions with ultrashort laser pulses, nonlinear optics at the nanoscale, and zero-index dielectric metamaterials.
- Awards include the Beller Award of OSA and the Millikan Medal of the AAPT

Alan Willner

- Steven & Kathryn Sample Chair in Engineering at the University of Southern California.
- Ph.D. Columbia University
- Honors include Member of US National Academy of Engineering; Int'l Fellow of UK Royal Academy of Engineering; President of OSA and of IEEE Photonics Society.
- Activities include using nonlinearity for signal processing and wave manipulation.

Wavelength Conversion by Time Refraction

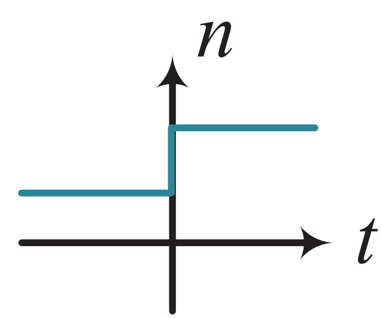
- Recall that in space refraction (normal refraction) frequency is conserved but the wavelength is changed



index vs. position

$$\frac{c}{f} = n \cdot \lambda \longrightarrow n_1 \lambda_1 = n_2 \lambda_2$$

- Time refraction (analog of space refraction)



index vs. time

$$\frac{c}{f} = n \cdot \lambda \longrightarrow \boxed{n_1 f_1 = n_2 f_2}$$

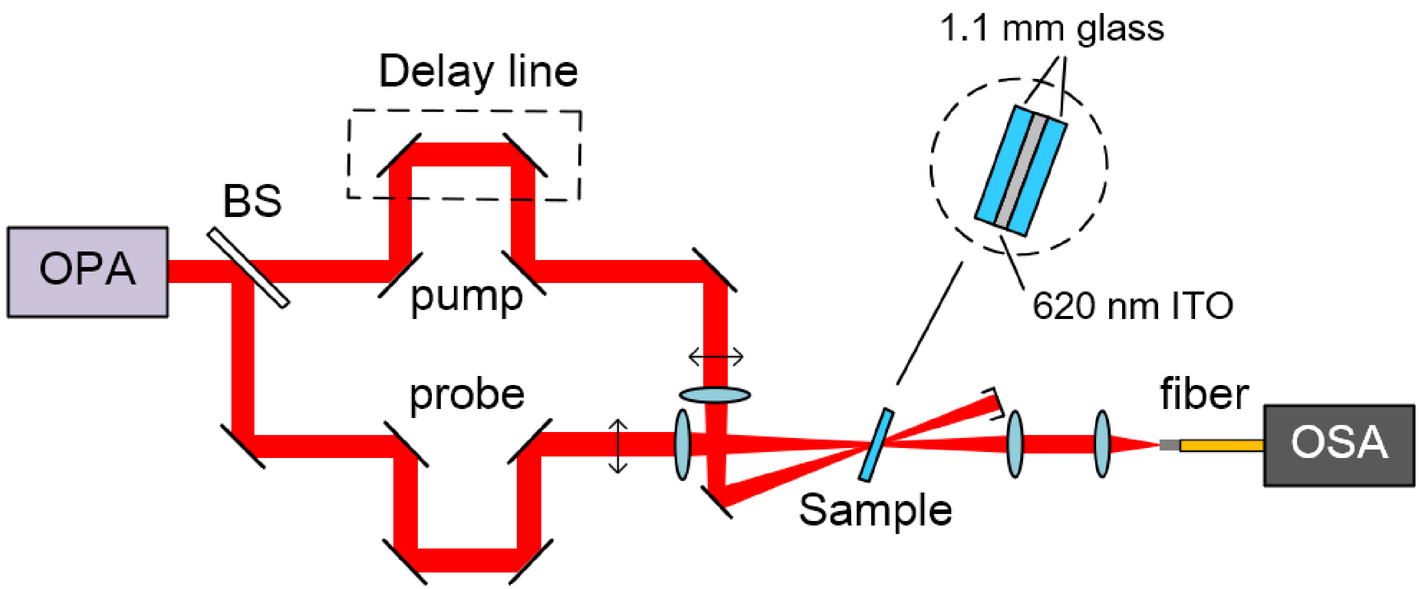
Photon frequency (energy) is changed because of the temporal change in index, but the wavelength (inverse of momentum) is conserved in the absence of any spatial asymmetry

- Time refraction is an alternative way of understanding frequency broadening and shifting by self-phase modulation:

$$\delta\omega(t) = \frac{d}{dt}\phi_{NL} = \frac{d}{dt}[n_2 I(t) L\omega/c]$$

Laboratory Study of Wavelength Conversion by Time Refraction

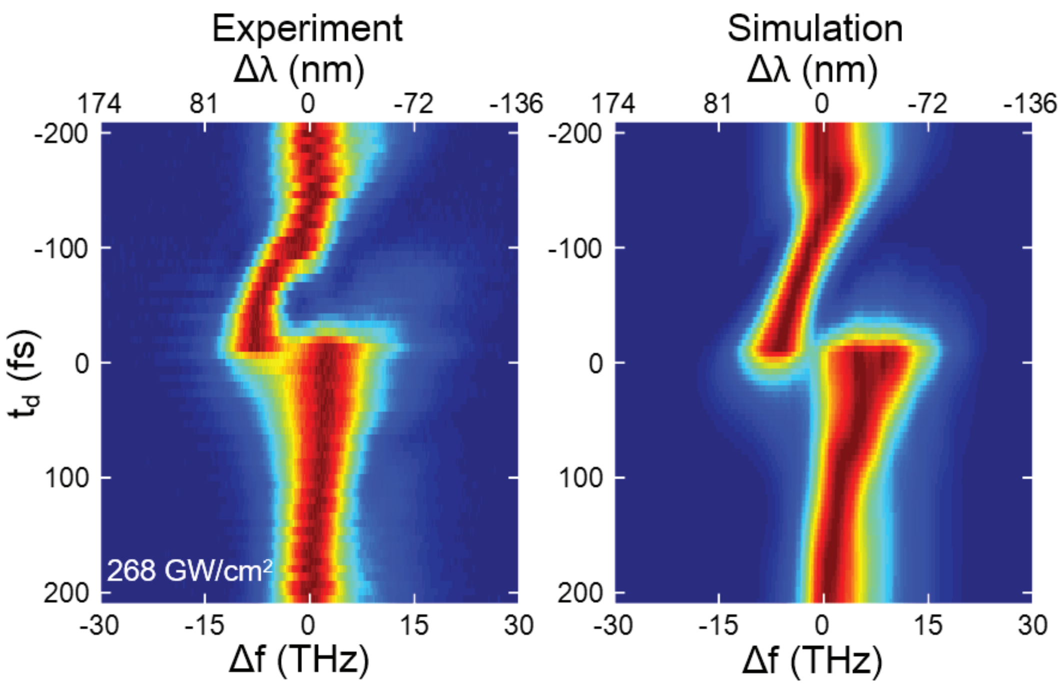
- Pump beam creates a time-varying refractive index in ITO sample
- Frequency of probe beam is thereby modified.

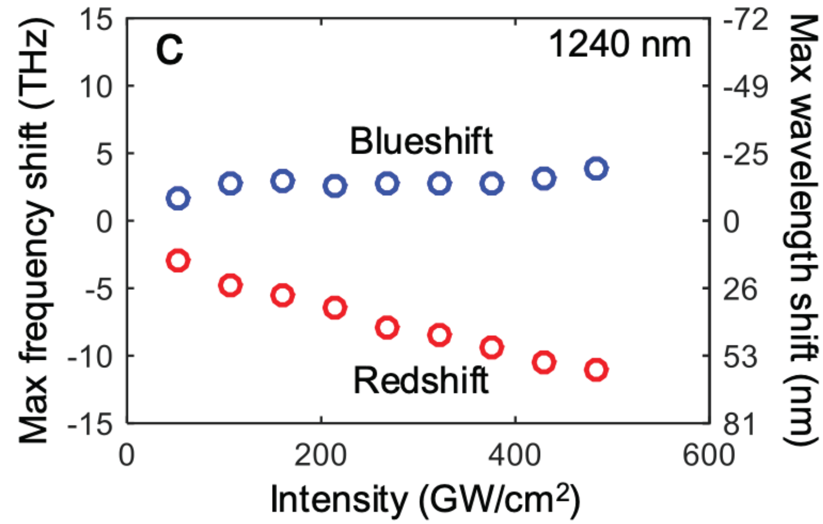


OPA = optical parametric amplifier
  wavelength = 1240 nm
  pulse duration = 120 fs
OSA = optical spectrum analyzer

Results: Adiabatic Wavelength Conversion by Time Refraction

Experimental results at 1240 nm





Probe phase and amplitude are measured by frequency-resolved optical gating (FROG)

The wavelength shift can be controlled by the pump intensity and the sign of the time delay.

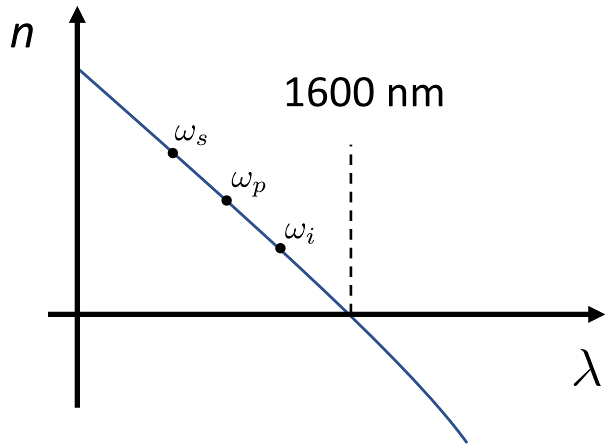
- The observed effect is 100 times larger with almost 100 times smaller propagation distance than previous reports of AWC.
- Application: wavelength-division multiplexing for telecom

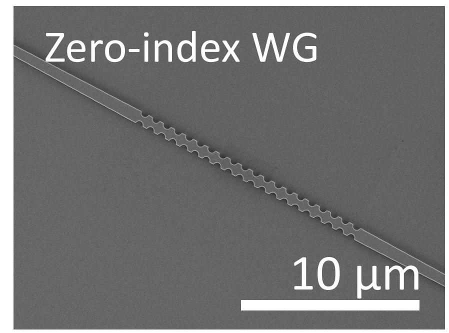
Zhou, Alam, Karimi, Upham, Reshef, Liu, Willner and Boyd, Nature Commun. 11:2180 (2020)

Phase-matching in low-index media

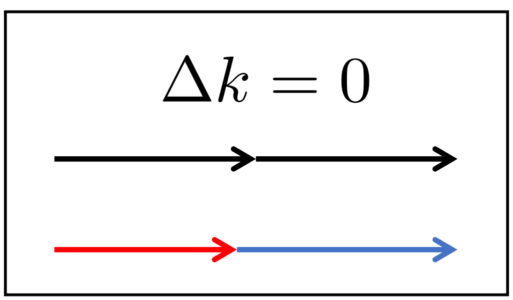
Here is a normal situation; all indices are positive

$$|k| \equiv \frac{2\pi n}{\lambda}$$

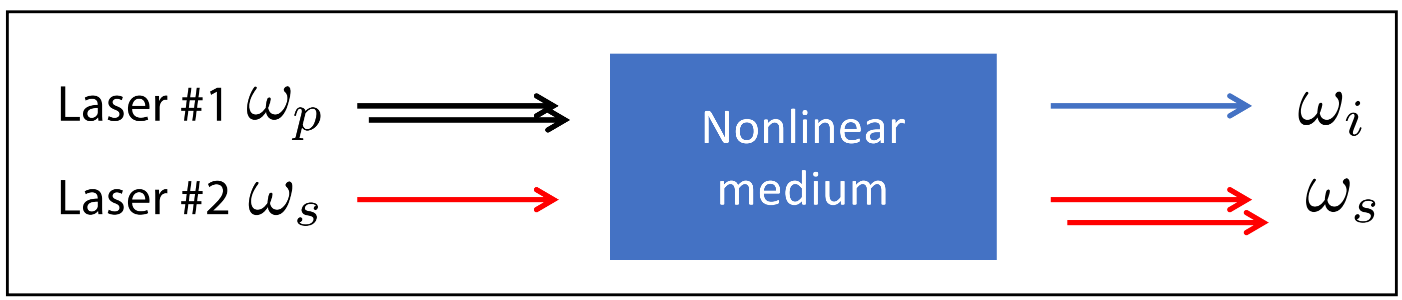




Phase-matching



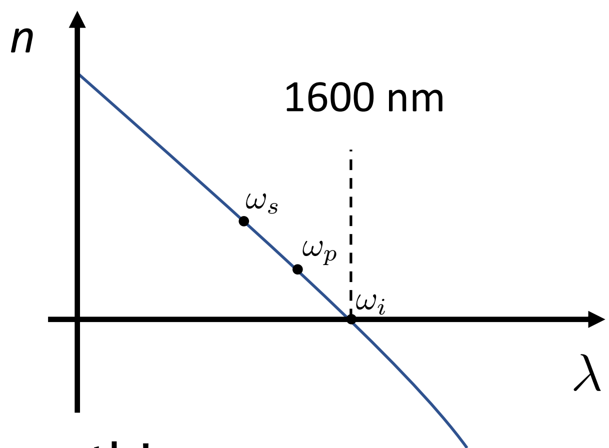
Experimental result

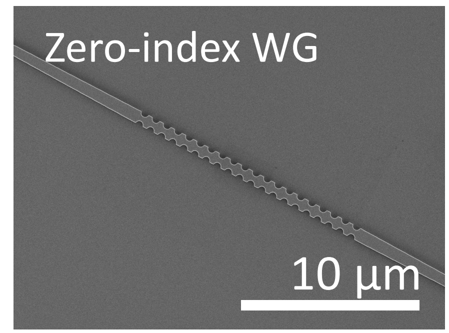


Phase-matching in low-index media

Let's examine what happens as the indices get smaller:

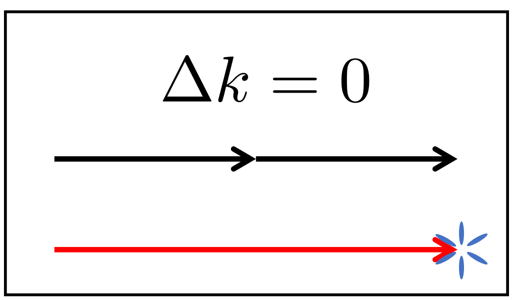
$$|k| \equiv \frac{2\pi n}{\lambda}$$



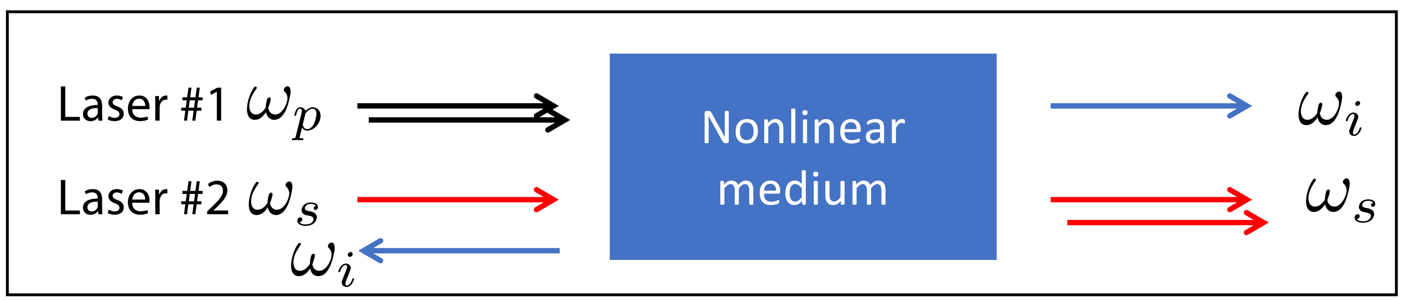


Idler k-vector has zero length!

Phase-matching



Experimental result

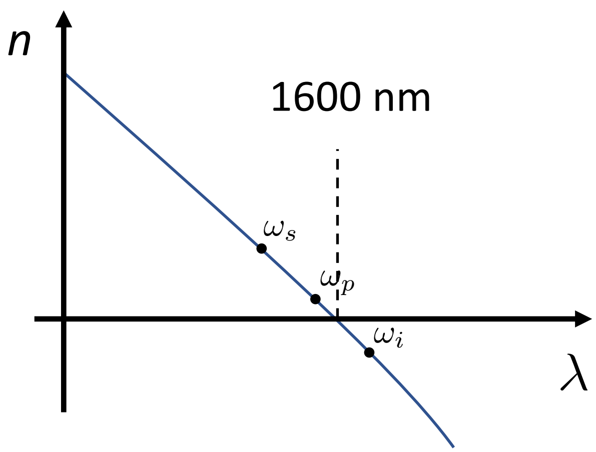


Output: forward AND backward

Phase-matching in low-index media

Let's examine what happens as the indices get still smaller:

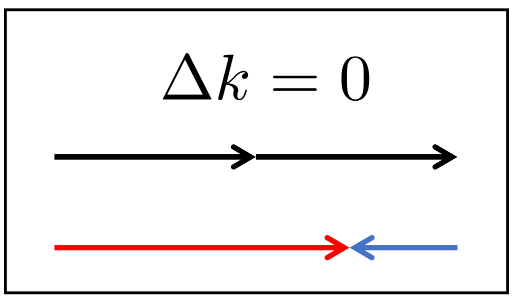
$$|k| \equiv \frac{2\pi n}{\lambda}$$



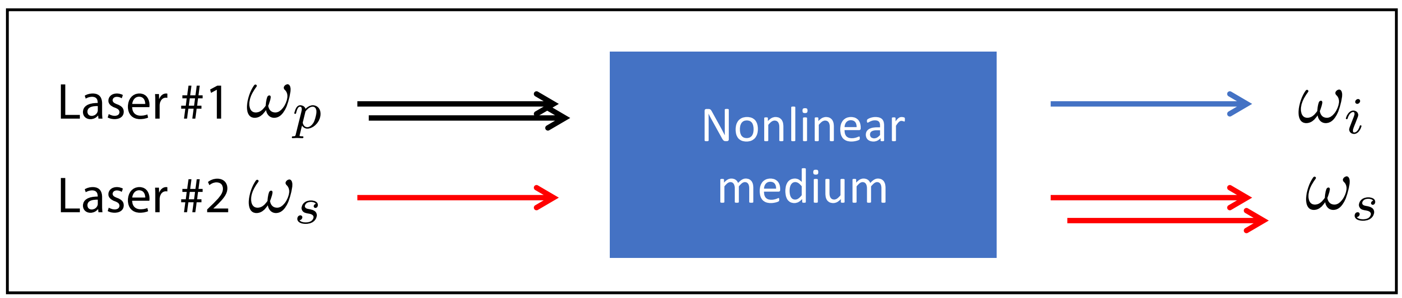
Now the pump and signal indices are positive, but the idler index is negative.

The k-vector and Poynting vector for the idler point in opposite directions

Phase-matching diagram



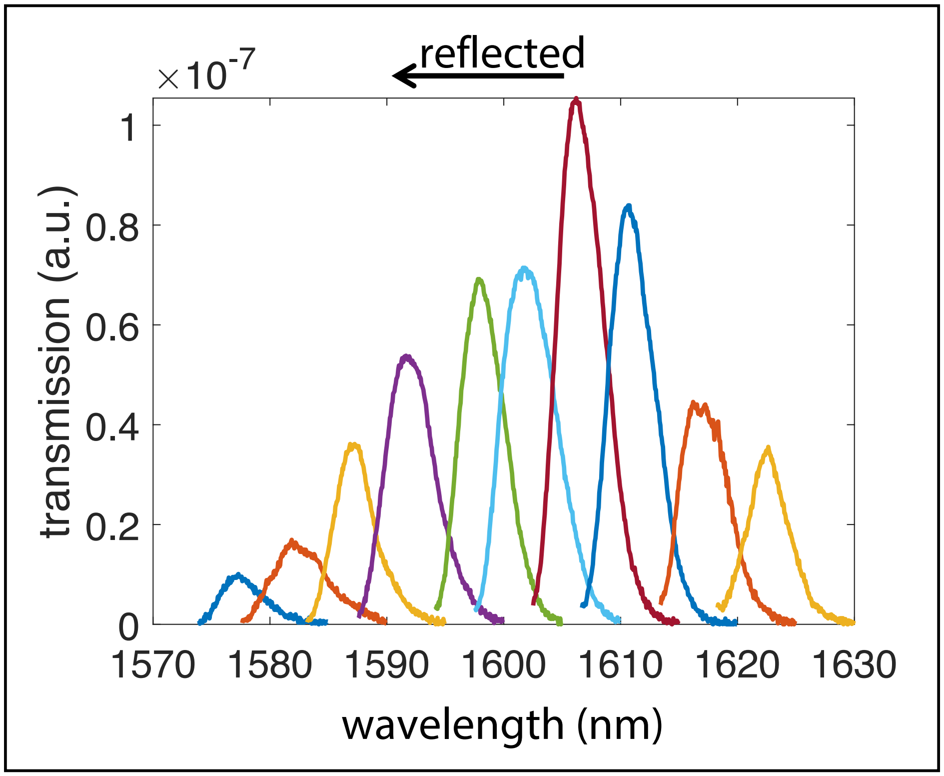
Experimental result

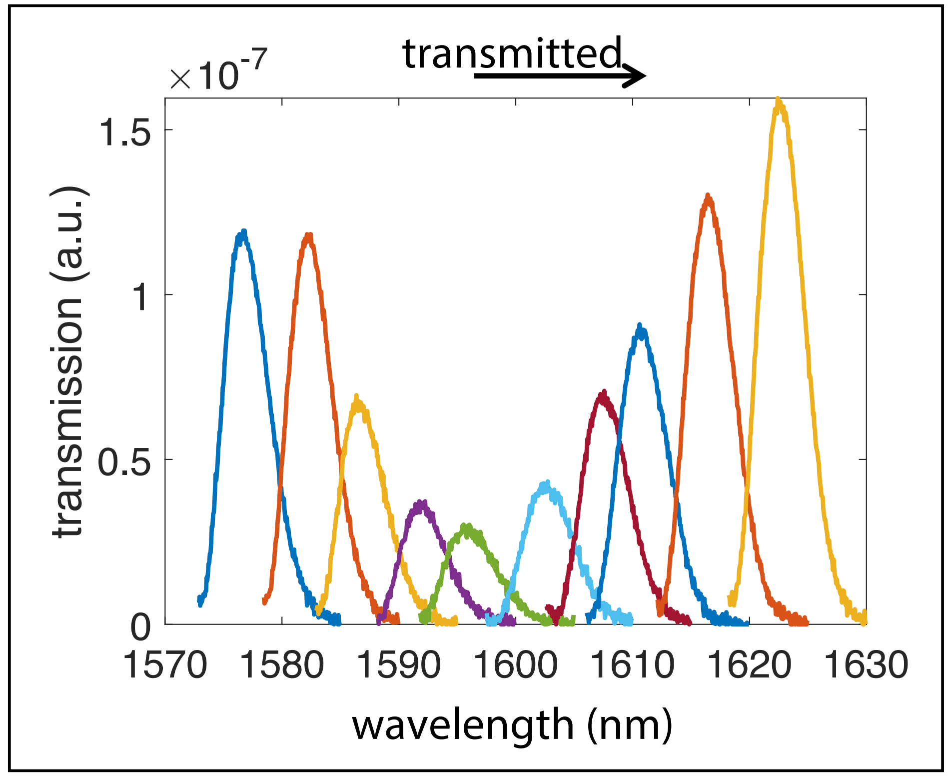


Output: only forward

Phase-matching in low-index media: Laboratory results

We perform this experiment, recording the idler spectrum for various pump and signal wavelengths.

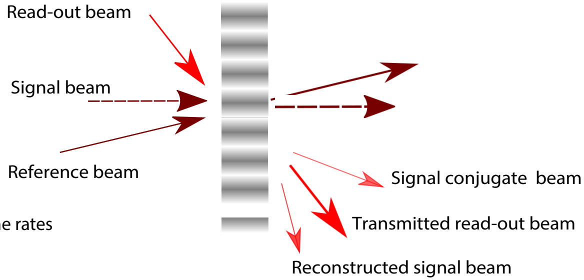




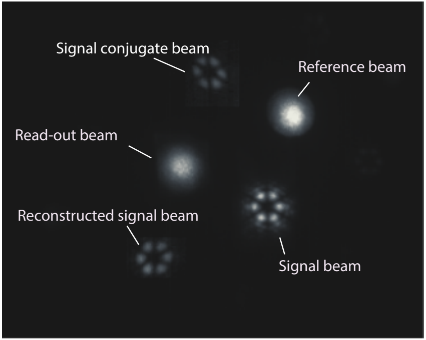
J. Gagnon, O. Reshef* et al. (unpublished results)*

Real-Time Holography with THz Refresh Rates

- Goal: Real-time holography with video or much faster refresh rates.
- The ultrafast response of ITO permits THz refresh rates
- Important applications involve image processing and signal processing
- Current real-time holographic materials cannot even support video frame rates



- Demonstration of image processing (edge enhancement)

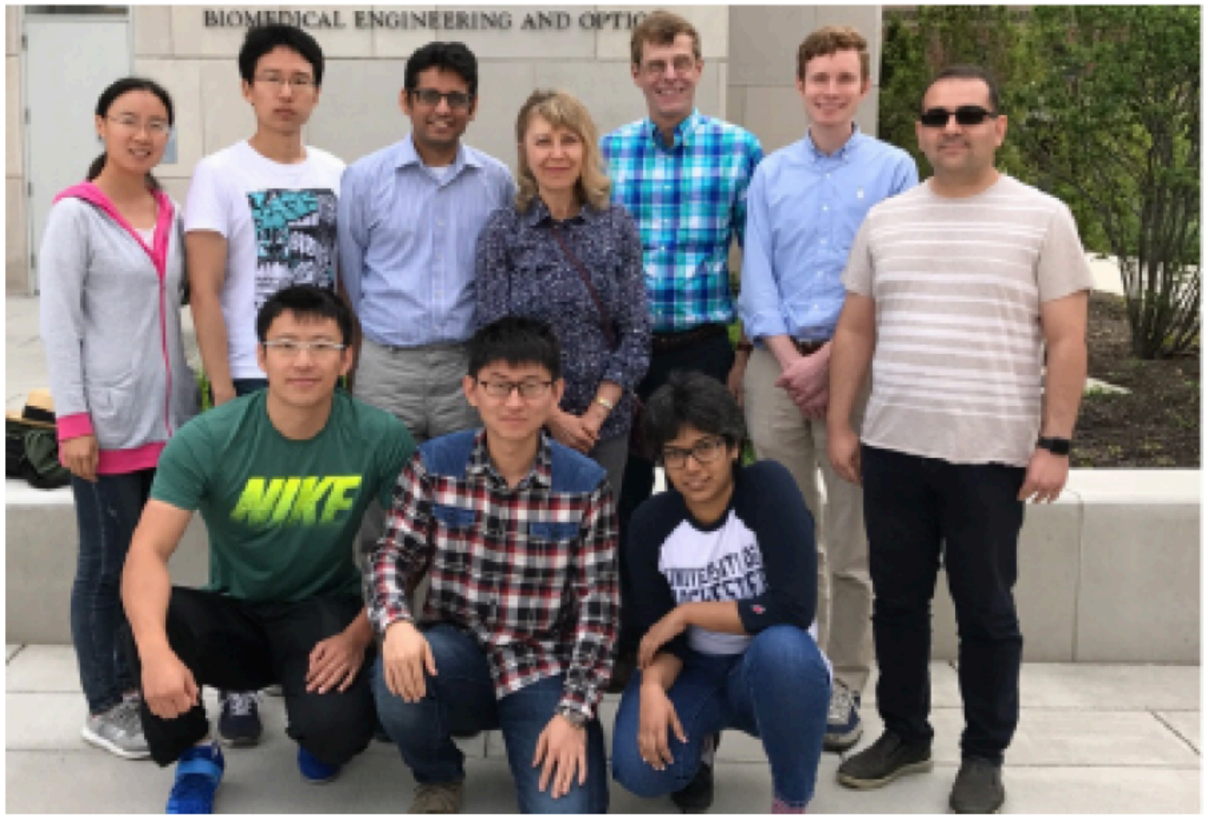


Alam, Fickler, Reshef, Giese, Upham, and Boyd

Special Thanks To My Students and Postdocs!

Ottawa Group

Rochester Group



Summary: Physics and Applications of ENZ Materials

- Extremely interesting physical processes occur in ENZ materials
- ENZ materials, metamaterials, and metastructures display extremely large NLO response
- The huge, ultrafast NLO response of ENZ materials lend themselves to many important applications

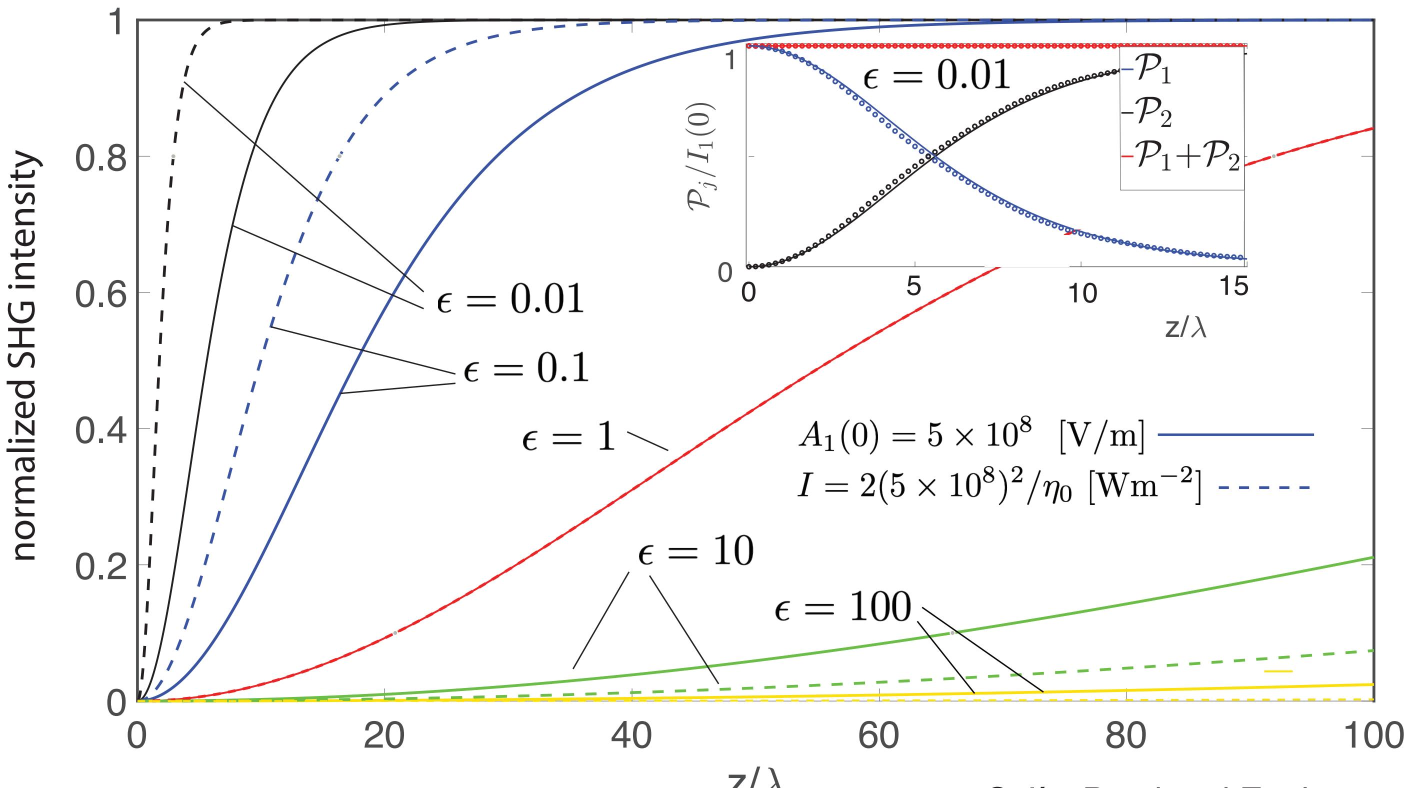
The visuals of this talk are posted at boydnlo.ca/presentations

Dependence of Second-Harmonic Generation on the Linear Dielectric Permittivity

- We solve the standard equations for second-harmonic generation

$$\frac{dA_1}{dz} = i\frac{\eta_1\omega_1\chi^{(2)}}{c}A_2(z)A_1^*(z)e^{-i\Delta kz},$$
$$\frac{dA_2}{dz} = i\frac{\eta_2\omega_2\chi^{(2)}}{2c}A_1^2(z)e^{i\Delta kz},$$

- We take $\Delta k = 0$ and plot the solution for various values of the permittivity ε.
- We find that the growth rate increases dramatically as the permittivity is decreased.



$\chi^{(2)}=5\times10^{-12}$ [m/V]

Solís, Boyd and Engheta, to be published

See also Solís, CLEO Poster JW2D.15