How Light Behaves when the Refractive Index Vanishes

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The visuals of this talk are posted at boydnlo.ca/presentations

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Brief Self-Introduction: Robert Boyd

Born in Buffalo, NY, USA

Bachelor’s Degree in Physics from MIT

PhD from University of California, Berkeley

Professor, University of Rochester, 1977 - present

CERC Professor, University of Ottawa, 2010 - present

• Research interests: optical physics, nonlinear optics, quantum optics
What is the Refractive Index?

- The refractive index determines how much a beam of light bends (or refracts) when it passes from one material to another.

- This relationship is known as Snell’s Law

\[ n_1 \sin \theta_1 = n_2 \sin \theta_2 \]

- The refractive index also determines the phase velocity of light \( v \)

\[ v = c/n \]

- Refraction at the surface of water explains why things look closer when they are under water.
What is the Refractive Index?

- Properties of the refractive index

\[ n_1 \sin \theta_1 = n_2 \sin \theta_2 \quad v = c/n \]

- We can understand why these two properties by an analogy to marching soldiers
What is the Refractive Index?

- Refractive index of some common materials
  
<table>
<thead>
<tr>
<th>Material</th>
<th>Refractive Index</th>
</tr>
</thead>
<tbody>
<tr>
<td>vacuum</td>
<td>1.0</td>
</tr>
<tr>
<td>air</td>
<td>1.0003</td>
</tr>
<tr>
<td>water</td>
<td>1.33</td>
</tr>
<tr>
<td>window glass</td>
<td>1.5</td>
</tr>
<tr>
<td>germanium</td>
<td>4.0</td>
</tr>
</tbody>
</table>

- The refractive index can be less than 1.0 for some extreme circumstances

- But can the refractive index ever vanish or at least be close to $n = 0$? And what would be the properties of light under these conditions?
How Light Behaves when the Refractive Index Vanishes

- Physics of Near-Zero Index (NZI) Materials
- Nonlinear Optical Properties of NZI Materials
- Meta-materials for NZI Studies
- Applications of NZI Materials
Physics of Near-Zero-Index (NZI) Materials

• The wavelength of light is given by
  \[ \lambda = \frac{\lambda_{\text{vac}}}{n} \]
  and is significantly lengthened in a NZI material. The wavelength approaches infinity as \( n \) approaches zero.

  ![Wavelength Image](image)

• The phase velocity of light is given by
  \[ v = \frac{c}{n} \]
  and also approaches infinity as \( n \) approaches zero.

• For \( n \) approaching zero, the field oscillates in time but not in space; oscillations are in phase everywhere

Physics of Near-Zero-Index (NZI) Materials

- Radiative processes are strongly modified in a NZI material
  - Einstein $A$ coefficient (spontaneous decay rate $= 1 / (\text{spontaneous emission lifetime})$)
    \[ A = n A_{\text{vac}} \]
    We can control (inhibit!) spontaneous emission!
  - Einstein $B$ coefficient
    \[ B = B_{\text{vac}} / n^2 \]
    Optical gain is very large!

- Implications:
  - If we can inhibit spontaneous emission, we can build thresholdless lasers.
  - Expect superradiance effects to be pronounced in ENZ materials.

Einstein, Physikalische Zeitschrift 18, 121 (1917).
Physics of Near-Zero-Index (NZI) Materials -- More

• Snell’s law leads to intriguing predictions

\[ n_1 \sin \theta_1 = n_2 \sin \theta_2 \]

• Light always leaves perpendicular to surface of ENZ material!

\[ n_1 = 0 \quad | \quad n_2 = 1 \]


• Thus light can enter an ENZ material only at normal incidence!

Maxwell Equations Prediction

- light enters slab at normal incidence
- but leaves in all directions!
Some Consequences of NZI Behaviour - 1

- Funny lenses

\[ n = 0 \]


- Large-area single-transverse-mode surface-emitting lasers

\[ L \]

\[ L \gg \lambda_{\text{vac}} \]

gain medium, \( n = 0 \)


- No Fabry-Perot interference

\[ n = 0 \]

O. Reshef et al., ACS Photonics 4, 2385, 2017.
Some Consequences of NZI Behavior - 2

- Super-coupling (of waveguides)

- Large evanescent tails for waveguide coupling

- Automatic phase matching of NLO processes

Recall that $k = n \omega / c$ vanishes in an ENZ medium.

For example, the following 4WM process is allowed.


Some Consequences of NZI Behaviour - 3

- How is the theory of self-focusing modified?
- Does the theory of Z-scan need to be modified?
- How is the theory of blackbody radiation modified?
- Do we expect very strong superradiance effects?
- More generally, how is any NLO process modified when \( n_0 = 0 \)?
Some Technical Details from Electromagnetic Theory

- The linear response of any material to electromagnetic radiation can be described by
  
  - The dielectric permittivity (dielectric constant) \( \varepsilon \) define through the relation
    
    \[ \mathbf{D} = \varepsilon \mathbf{E} \]
    
    where \( \mathbf{D} \), known as the dielectric displacement, and \( \mathbf{E} \), known as the electric field, are the two fields that describe the material response to an electric field.

  - The magnetic permeability \( \mu \) define through the relation
    
    \[ \mathbf{B} = \mu \mathbf{H} \]
    
    where \( \mathbf{B} \), known as the magnetic field, and \( \mathbf{H} \), known as the magnetic intensity, are the two fields that describe the magnet response of a material to an applied field.

- It is straightforward to shown from the equations of electromagnetism that
  
  \[ n = \sqrt{\varepsilon \mu} \]

  - Thus, \( n=0 \) when either \( \varepsilon =0 \) or \( \mu=0 \) (or both \( \varepsilon \) and \( \mu \) equal zero).

- Terminology:
  
  ENZ: epsilon near zero
  MNZ: mu near zero
  EMNZ: epsilon and mu near zero
Surface Reflection

- There is a problem getting light into a zero-index material.
- There is always reflection from the boundary between two materials.

- The impedance and surface reflectivity are given by

\[ Z = \sqrt{\frac{\mu}{\varepsilon}} \quad R = \left| \frac{Z - 1}{Z + 1} \right|^2 \]

- Thus the reflectivity will be 100% if \( \varepsilon = 0 \) unless \( \mu = 0 \) as well.

- This is one reason for the interest in developing EMNZ materials (epsilon and mu near zero materials).
How Light Behaves when the Refractive Index Vanishes

• Physics of Near-Zero Index (NZI) Materials

• Nonlinear Optical Properties of NZI Materials

• Meta-materials for NZI Studies

• Applications of NZI Materials
An important application in photonic technologies is optical switching.

One wants a switch with fast switching times and that operates with weak control fields.

One needs a nonlinear interaction in order for one optical field to control another field.

A strong nonlinear response is needed. How does one quantify the strength of a nonlinear response? Two standard methods:

\[ n = n_0 + n_2 I \]

\[ P^{NL} = 3\chi^{(3)}|E|^2 E \]

The nonlinear coefficients are \( n_2 \) and \( \chi^{(3)} \).
Implications of ENZ Behavior for Nonlinear Optics

Here is the intuition for why the ENZ condition is of interest in NLO

Recall the standard relation between $n_2$ and $\chi^{(3)}$

$$n_2 = \frac{3\chi^{(3)}}{4\varepsilon_0 c n_0 \text{Re}(n_0)}$$

Note that under ENZ conditions the denominator becomes very small, leading to a very large value of $n_2$

Footnote:

Standard notation for perturbative NLO

$$P = \chi^{(1)} E + \chi^{(2)} E^2 + \chi^{(3)} E^3 + ...$$

P is the induced dipole moment per unit volume and $E$ is the field amplitude.

Also, the refractive index changes according to

$$n = n_0 + n_2 I + n_4 I^2 + ...$$
Epsilon-Near-Zero Materials

• Metamaterials
  Materials tailor-made to display ENZ behaviour

• Homogeneous materials
  All materials display ENZ behaviour at their (reduced) plasma frequency
  Recall the Drude formula
  \[ \varepsilon(\omega) = \varepsilon_{\infty} - \frac{\omega_p^2}{\omega(\omega + i\gamma)} \]
  Note that Re \( \varepsilon = 0 \) for \( \omega = \omega_p/\sqrt{\varepsilon_{\infty}} \equiv \omega_0 \).

• Challenge: Obtain low-loss ENZ materials
  Want Im \( \varepsilon \) as small as possible at the frequency where Re \( \varepsilon = 0 \).

• We are examining a several materials
  ITO: indium tin oxide
  AZO: aluminum zinc oxide
  FTO: fluorine tin oxide
New Nonlinear Optical Material for Quantum Photonics

- We want all-optical switches that work at the single-photon level
- We need photonic materials with a much larger NLO response
- We recently reported a new NLO material with an $n_2$ value 100 times larger than those previously reported (but with some background absorption).
- Material makes use of strong enhancement that occurs in the epsilon-near-zero (ENZ) spectral region.
- A potential game changer for the field of photonics

What Makes a Good (Kerr-Effect) Nonlinear Optical Material?

• We want $n_2$ large ($\Delta n = n_2 I$). We also want $\Delta n^{(\text{max})}$ large.
  These are distinct concepts! Damage and saturation can limit $\Delta n^{(\text{max})}$

- For ITO at ENZ wavelength, both $n_2$ and $\Delta n^{(\text{max})}$ are extremely large
  ($n_2 = 1.1 \times 10^{-10} \text{ cm}^2/\text{W}$ and $\Delta n^{(\text{max})} = 0.8$)

- $n_2$ is $3.4 \times 10^5$ times larger than that of silica glass
  $\Delta n^{(\text{max})}$ is 2700 times larger that that of silica glass
  (For silica glass $n_2 = 3.2 \times 10^{-16} \text{ cm}^2/\text{W}$, $I_{\text{damage}} = 1 \text{ TW/cm}^2$, and thus $\Delta n^{(\text{max})} = 3 \times 10^{-4}$)

Optical Properties of Indium Tin Oxide (ITO)

- ITO is a degenerate semiconductor (so highly doped as to be metal-like).
- It has a large density of free electrons, and a bulk plasma frequency corresponding to a wavelength of approximately 1.24 μm.
- Recall the Drude formula

\[ \varepsilon(\omega) = \varepsilon_\infty - \frac{\omega_p^2}{\omega(\omega + i\gamma)} \]

Note that \( \text{Re} \varepsilon = 0 \) for \( \omega = \omega_p/\sqrt{\varepsilon_\infty} \equiv \omega_0 \).

The region near \( \omega_0 \) is known as the epsilon-near-zero (ENZ) region.

- There has been great recent interest in studies of ENZ phenomena:

Huge, Fast NLO Response of Indium Tin Oxide at its ENZ Wavelength

- ellipsometry

\[ \lambda = 1240 \text{ nm} \]

- overall change in refractive index of 0.8

- sub picosecond response time

\[ n_2 \text{ can be } 3.4 \times 10^5 \text{ times larger than that of silica glass} \]

\[ \text{Figures and graphs demonstrating the NLO response and properties of chalcogenide glass.} \]

Huge Nonlinear Optical Response of ITO

- Z-scan measurements for various angles of incidence

![Wavelength dependence of $n_2$](image1)

<table>
<thead>
<tr>
<th>Wavelength (μm)</th>
<th>$n_2$ (eff) (cm$^2$/GW)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.95</td>
<td>5 x 10^{-5}</td>
</tr>
<tr>
<td>1.05</td>
<td>0.03</td>
</tr>
<tr>
<td>1.15</td>
<td>0.06</td>
</tr>
<tr>
<td>1.25</td>
<td>0.09</td>
</tr>
<tr>
<td>1.35</td>
<td>0.08</td>
</tr>
</tbody>
</table>

Note that $n_2$ is positive (self focusing) and $\beta$ is negative (saturable absorption).

- Both $n_2$ and nonlinear absorption increase with angle of incidence

- $n_2$ shows a maximum value of 0.11 cm$^2$/GW = 1.1 x 10^{-10} cm$^2$/W at 1.25 μm and 60 deg. This value is 2000 times larger than that away from ENZ region.

- Variation with incidence angle

![Variation with incidence angle](image2)

peak laser intensity was 50 GW cm$^{-2}$
An ENZ Metasurface

- We functionalize ITO by creating a photonic metasurface
- We obtain an even larger NLO response by placing a gold antenna array on top of ITO.
  - Lightning rod effect: antennas concentrate the field within the ITO
  - Coupled resonators: ENZ resonance and nano-antennas

Concept:  
SEM:
A thin ENZ medium supports a bulk plasma mode.

A thin layer of ITO supports two modes
- the bulk plasma mode, also called the ENZ or long-range SPP mode
- the short range surface plasmon polariton (SPP) mode
The material exhibits extremely large $n_2$ over a broad spectral range. The magnitude of the on-resonance value is 7 orders of magnitude larger than that of SiO$_2$.

Physics and Applications of Epsilon-Near-Zero Materials

- Physics of ENZ Materials
- Huge NLO Response of ITO and ITO Metastructures
- Materials for ENZ
- Applications of ENZ Materials
Epsilon-Near-Zero (ENZ) and Near Zero-Index (NZI) Examples

Homogeneous materials

**TCO**

- A. Boltasseva (Purdue)
- Kim et al., Optica (2016)

**SiC**

- J. Caldwell (Vanderbilt)
- Kim et al., Optica (2016)

**Bi$_{1.5}$Sb$_{0.5}$Te$_{1.8}$Se$_{1.2}$**

- N. Zheludev (Southampton)
- Ou et al., Nat. Commun. (2014)

Metamaterials

- Chan, Huang et al., Nat. Mater. (2011)
- SEM from: Polman’s & Engheta’s
- Vesseur et al., PRL (2013)

- Re($\varepsilon$) = 0
- Wire SEM from: Zayat & Podolskiy
- Pollard et al., PRL (2009)
- StackSEM from: Polman & Engheta
Nader Engheta
- H. Nedwill Ramsey Professor at the University of Pennsylvania
- B.S. degree from the University of Tehran and his M.S and Ph.D. from Caltech.
- Activities include ENZ, photonics, metamaterials, nano-optics, graphene optics, electrodynamics, microwave and optical antennas, studies of fields and waves.
- Many awards including the Streifer Award of IEEE and the Gold Medal from SPIE

Eric Mazur
- Balkanski Professor of Physics and Applied Physics at Harvard University
- Ph.D. University of Leiden.
- Activities include light-matter interactions with ultrashort laser pulses, nonlinear optics at the nanoscale, and zero-index dielectric metamaterials.
- Awards include the Beller Award of OSA and the Millikan Medal of the AAPT

Alan Willner
- Steven & Kathryn Sample Chair in Engineering at the University of Southern California.
- Ph.D. Columbia University
- Honors include Member of US National Academy of Engineering; Int’l Fellow of UK Royal Academy of Engineering; President of OSA and of IEEE Photonics Society.
- Activities include using nonlinearity for signal processing and wave manipulation.
Three Material Platforms Under Investigation

- Nanoantennas coupled to ENZ substrate (out of plane; free-space coupling) (Rochester)

- Dirac cone metamaterials (in plane; compatible with integrated optics) (Harvard)

- Photonically doped metamaterials (out of plane; free-space coupling) (Penn)
Dirac Cone Metamaterials

It is also a ZIM (zero index material)

An EMNZ (epsilon and mu near zero) metamaterial

Opt Express 25, 8326 (2017)

\[ n = \sqrt{\varepsilon \mu} \]

\[ Z = \sqrt{\frac{\mu}{\varepsilon}} \]

\[ R = \frac{1 - Z}{1 + Z} \]
Photonic Doping of ENZ

\[ \varepsilon_{\text{eff}} \approx 0 \]

\[ \mu_{\text{eff}} = \frac{1}{A} \left[ A_h + \frac{2\pi r_p}{k_p} \frac{J_1(k_p r_p)}{J_0(k_p r_p)} \right] \]

I. Liberal, A. Mahmoud, Y. Li, B. Edwards and N. Engheta, Science, 355, March 10, 2017
Physics and Applications of Epsilon-Near-Zero Materials

- Physics of ENZ Materials
- Huge NLO Response of ITO and ITO Metastructures
- Materials for ENZ
- Applications of ENZ Materials
Some Potential Applications of ENZ Behavior

(a) Non-magnetic isolation
- Forward direction
  - Input beam
  - Output beam
- Backward direction
  - No output beam
  - Intense input beam

- Geometry mismatch.
- Non-uniform power distribution.
- Breaking reciprocity.

(b) Full-band shifting and conjugation
- Wideband input beam + pump
- Output beam: shifted band (forward FWM)
- Output beam: shifted band (backward FWM)

(c) High-speed tunable interferometers
- Pump
- Mach-Zehnder interferometer
- Interference Output beam

(d) On-demand quantum emitter
- Short pulse laser
- Quantum emitter embedded in LNO-ENZ
- Output photons
- Single photon detector
Space Refraction and Time Refraction

- **Space refraction**
  \[
  \frac{c}{f} = n \cdot \lambda \quad \rightarrow \quad n_1 \lambda_1 = n_2 \lambda_2
  \]

- **Time refraction** (analog of space refraction)
  \[
  \frac{c}{f} = n \cdot \lambda \quad \rightarrow \quad n_1 f_1 = n_2 f_2
  \]

  Photon frequency (energy) is changed because of the temporal change in index, but the wavelength (inverse of momentum) is conserved in the absence of any spatial asymmetry.

- **Time refraction** is an alternative way of understanding frequency broadening and shifting by self-phase modulation:
  \[
  \delta \omega(t) = \frac{d}{dt} \phi_{NL} = \frac{d}{dt} [n_2 I(t) \omega / c]
  \]
Laboratory Study of Wavelength Conversion by Time Refraction

- Pump beam creates a time-varying refractive index in ITO sample
- Frequency of probe beam is thereby modified.

OPA = optical parametric amplifier
wavelength = 1240 nm
pulse duration = 120 fs
OSA = optical spectrum analyzer
Results: Adiabatic Wavelength Conversion by Time Refraction

Experimental results at 1240 nm

Probe phase and amplitude are measured by frequency-resolved optical gating (FROG)

- The observed effect is 100 times larger with almost 100 times smaller propagation distance than previous reports of AWC.
- Application: wavelength-division multiplexing for telecom
Ultrafast Real-Time Holography with an Epsilon-Near-Zero Material

M. Zahirul Alam, Robert Fickler, Orad Reshef, Enno Giese, Jeremy Upham, Robert W. Boyd
May 19, 2019

Department of Physics, University of Ottawa, Ottawa, Canada
Motivation:
In optical holography an interference pattern is stored in a material to be read later by another light beam. Gabor (1948); Leith and Upatnieks (1964). Photorefractive and photochromatic materials are typical holographic materials. See also works by Lohman, Goodman, Yariv, and Peyghambarian.

**Standard holographic methods have very slow write rates.**
Holography using ITO

- The interference pattern gets written on the refractive index variation in ITO due to its large intensity-dependent changes in refractive index.
- A second gaussian beam of same or different wavelength can be used to read out the transient hologram.
Holography using ITO: experimental results.
• The large nonlinear response of ENZ material can be exploited for real-time holography with an efficiency of 25%.
• The material is four orders of magnitude thinner (310-nm-thick) than a conventional holographic material.
• **9-12 orders of magnitude larger refresh rate.** Limited by the sub-ps recovery time.
• Broadband response. 1000 – 1500 nm wavelength range with larger than 1 % diffraction efficiency.
• Might find applications in multimode communications and real-time signal processing such as edge detection, convolution, correlation, etc.
• Essentially we use structured light beams to temporally structure a surface to perform certain mathematical operations.
Special Thanks To My Students and Postdocs!

Ottawa Group

Rochester Group
• Extremely interesting physical processes occur in ENZ materials

• ENZ materials, metamaterials, and metastructures display extremely large NLO response

• The huge, ultrafast NLO response of ENZ materials lend themselves to many important applications

The visuals of this talk are posted at boydnlo.ca/presentations
All-Optical, Nanoscale, Sub-Picosecond Beam Steering

• Concept

Vary output direction by +/- 20 degrees under all-optical control

Sub-picosecond response time

Beam steerer made of one or many cells

Application: Mode-division multiplexing for telecommunications

• Design

Top View of Cell

Nanoantennas

Phase ramp on reflected beam

Nonlinear response depends on antenna length

• Characterization

We have fabricated this design and are currently testing it