How to Sort OAM States of Light

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The visuals of this talk will be available at boydnlo.ca/presentations/

Presented at the International Conference on Optical Angular Momentum, Ottawa, June 17, 2019.

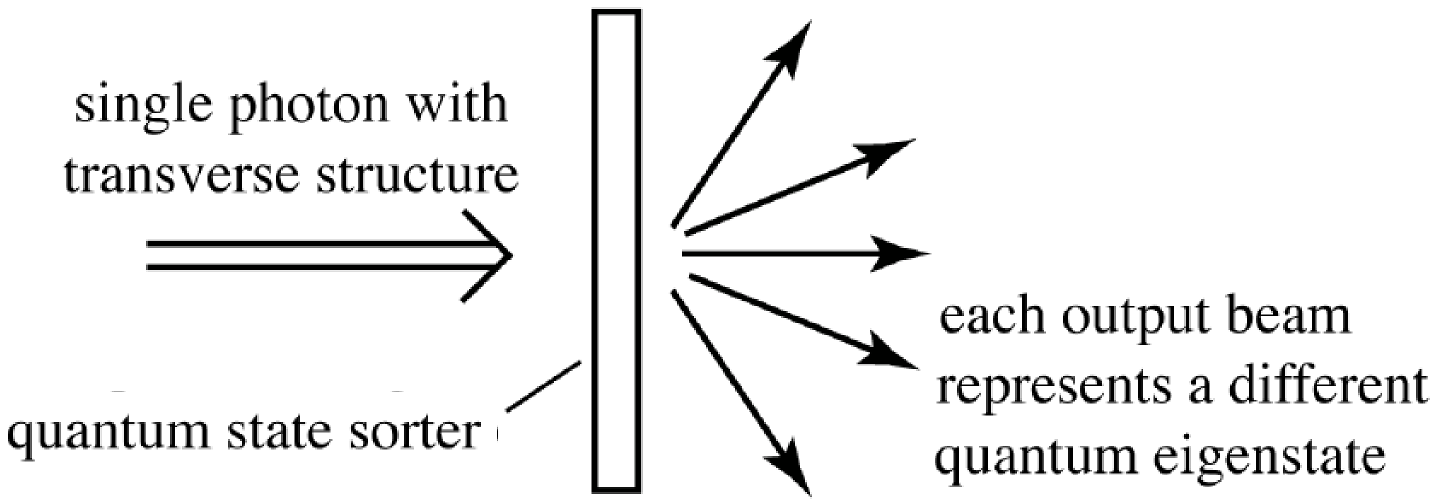
How to Sort OAM States of Light

Prospective

1. OAM Sorters
   Why they are important
   How to construct them
2. Use of OAM Sorters in Quantum Key Distribution
3. Use of Sorters in Superresolution Imaging
4. Summary

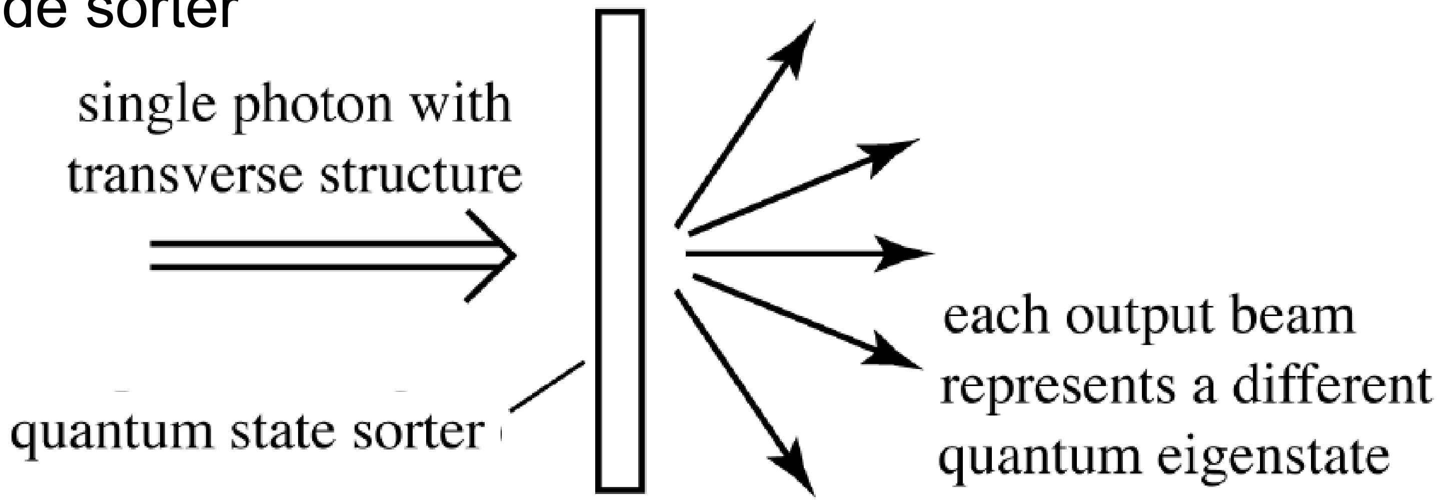
What is a Mode Sorter?

Mode sorting

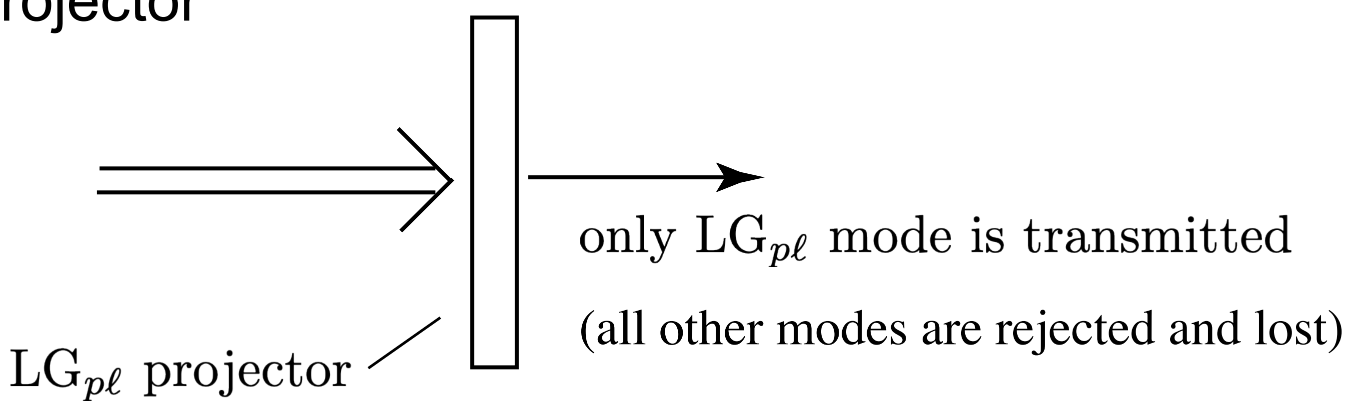


Mode Sorter and Mode Projector

- A mode sorter



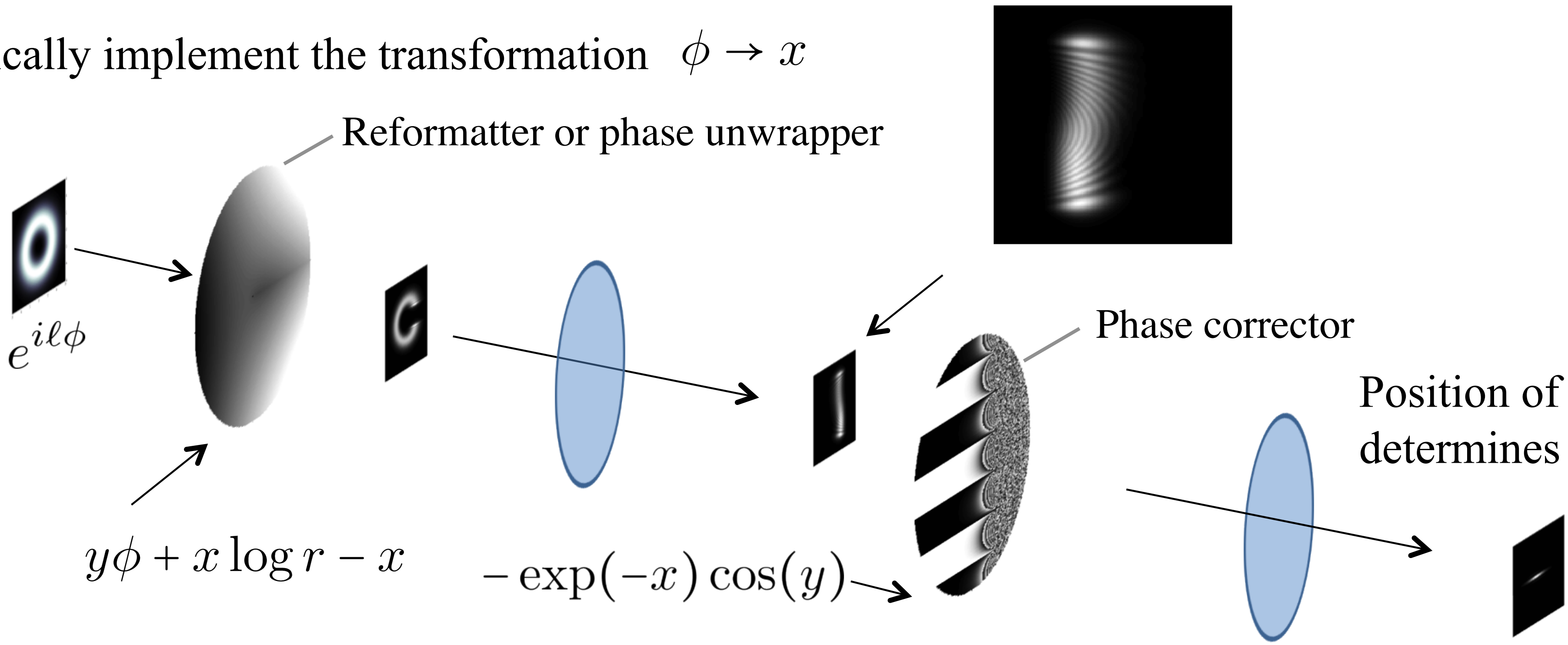
- A mode projector



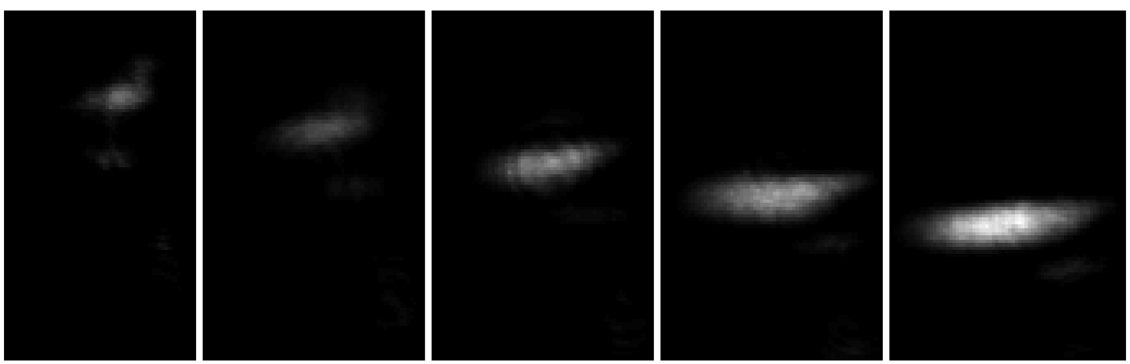
Inefficient: Need to make d measurements to sample a d-dimensional state space
Lose the multiplex advantage of the large state space

The Glasgow (Padgett) OAM Sorter

Optically implement the transformation $\phi \to x$



Experimental Results (CCD images in output plane)



- Resolution limited by the overlap of neighboring states.

*Berkhout *et al. PRL* **105,** 153601 (2010).
O. Bryngdahl, *J. Opt. Soc. Am.* **64**, 1092 (1974).

Laguerre-Gauss Modes

The Laguerre-Gauss modes are a well-known family of modes that carry orbital angular momemtum (OAM).

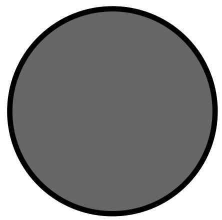
$$LG_{pl}(r,\theta) = \sqrt{\frac{2p!}{\pi(p+|l|)!}}\frac{1}{w_0}\left(\frac{\sqrt{2}r}{w_0}\right)^{|l|} \times \exp\left(-\frac{r^2}{w_0^2}\right) L_p^{|l|}\left(\frac{2r^2}{w_0^2}\right) e^{il\theta}$$

Here *p* is the radial index and *l* is the azimuthal (OAM) index.

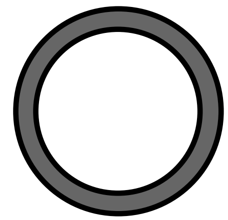
Why We Need to Encode in Azimuth and Radius

• Large telescopes are expensive; we want to make full use of our resources

This?



Or this?



• In concept we want to encode in communication modes, but for large Fresnel numbers they approximate Laguerre-Gauss modes.

• Laguerre-Gauss modes are described by two mode indices, one (l) for azimuthal variation and one (p) for radial variation.

• We can roughly square the size of Hilbert space by encoding in both l and p.

• Miles Padgett showed earlier how to sort in l . Only recently have people shown how to sort in p.

RAPID COMMUNICATIONS

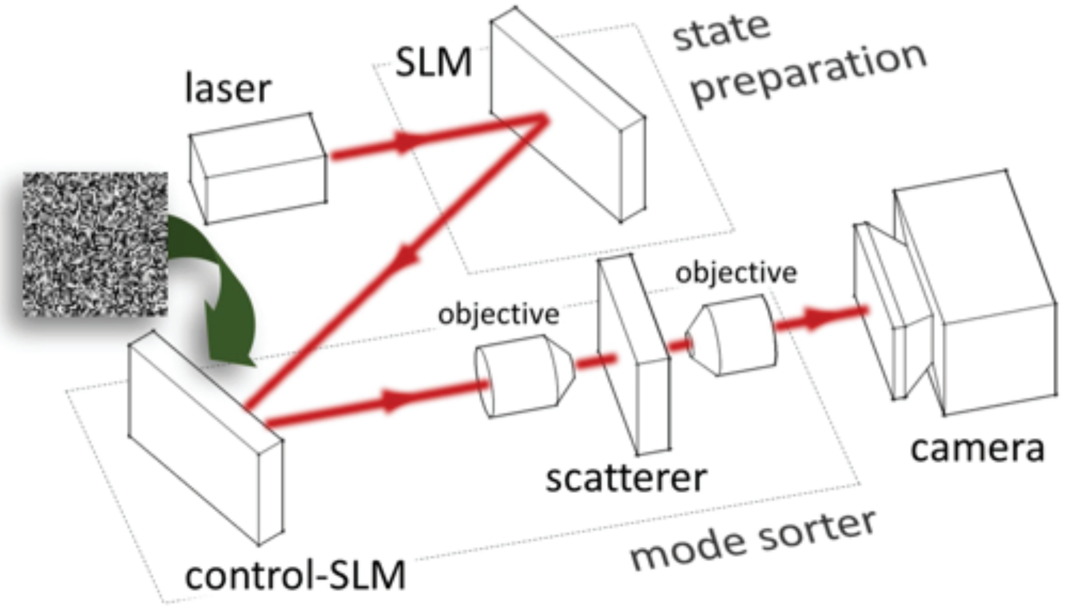
PHYSICAL REVIEW B **95**, 161108(R) (2017)

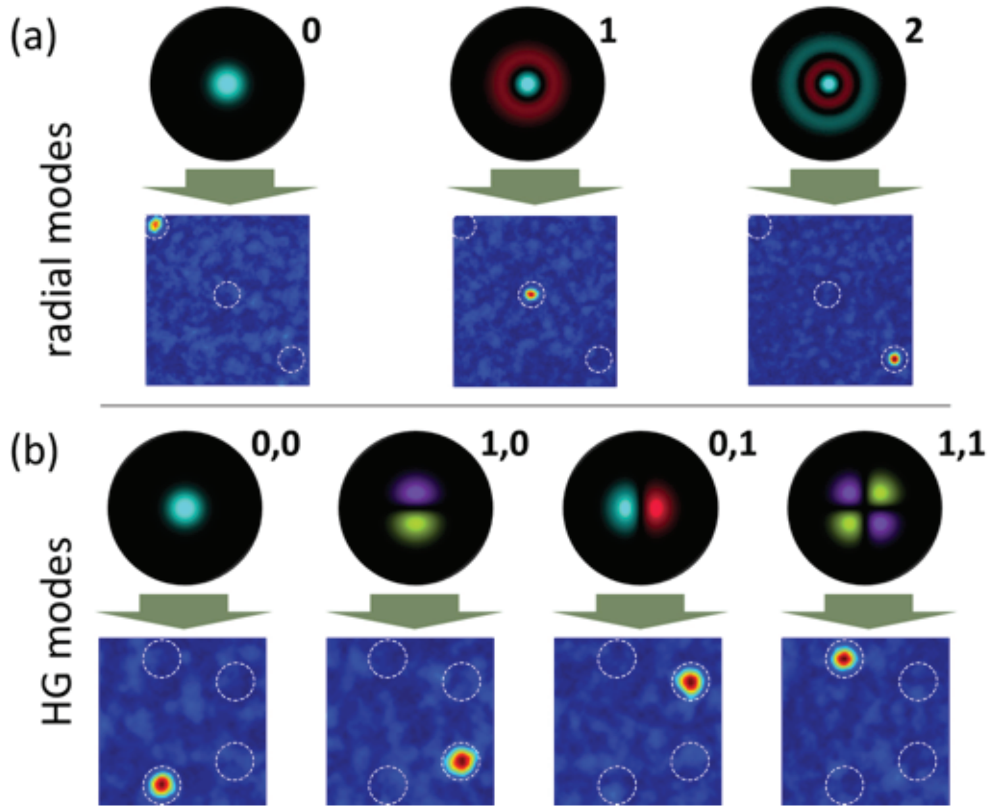
Custom-tailored spatial mode sorting by controlled random scattering

Robert Fickler,[1] Manit Ginoya,[1] and Robert W. Boyd[1,2]
[1]*Department of Physics, University of Ottawa, Ottawa, Canada K1N 6N5*
[2]*Institute of Optics, University of Rochester, Rochester, New York 14620, USA*
(Received 18 January 2017; revised manuscript received 17 March 2017; published 14 April 2017)

fidelity = 97%
efficiency = 0.2%

A genetic algorithm determines the pattern on the control SLM





Letter Vol. 43, No. 24 / 15 December 2018 / *Optics Letters* 6101

Optics Letters

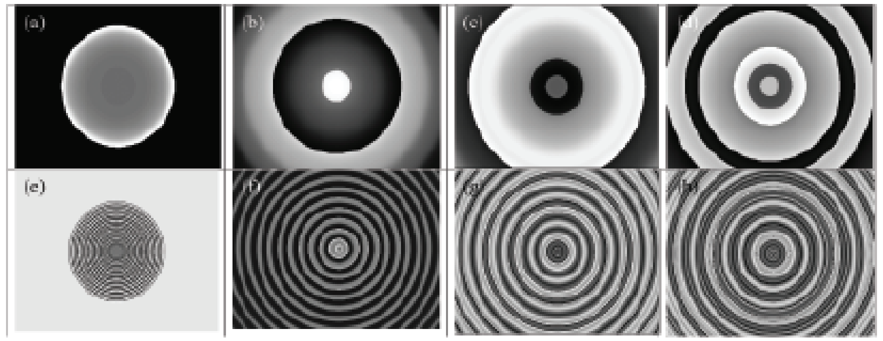
Measurement of the radial mode spectrum of photons through a phase-retrieval method

SAUMYA CHOUDHARY,[1,*] RACHEL SAMPSON,[2] YOKO MIYAMOTO,[3] OMAR S. MAGAÑA-LOAIZA,[4] SEYED MOHAMMAD HASHEMI RAFSANJANI,[5] MOHAMMAD MIRHOSSEINI,[6] AND ROBERT W. BOYD[1,7]

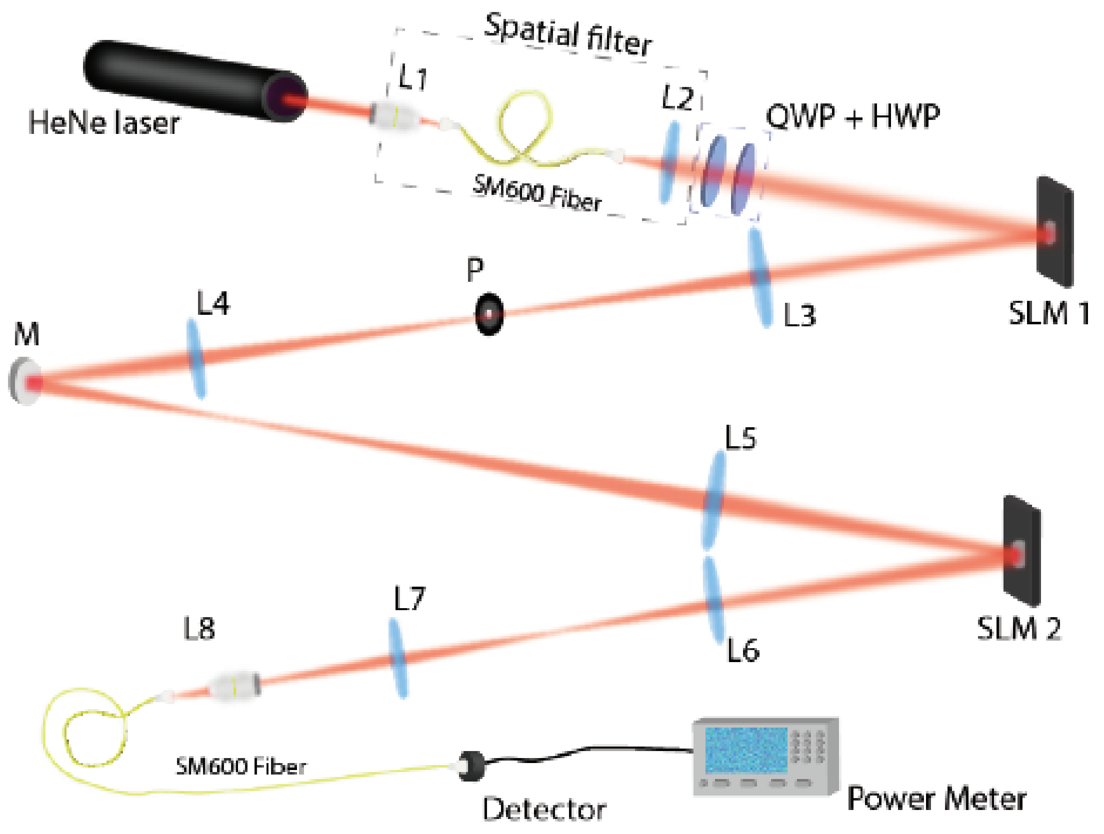
- Phase screens on SLMs 1 and 2 convert a specified input mode to the near-Gaussian fundamental mode of a single mode fiber.
- Uses Gerchberg-Saxton phase-retrieval algorithm to determine phase screens
- Generalizes to any basis set
- Uses projective measurement

Phase Screens

LG (l=0) p=0 p=1 p=2 p=3

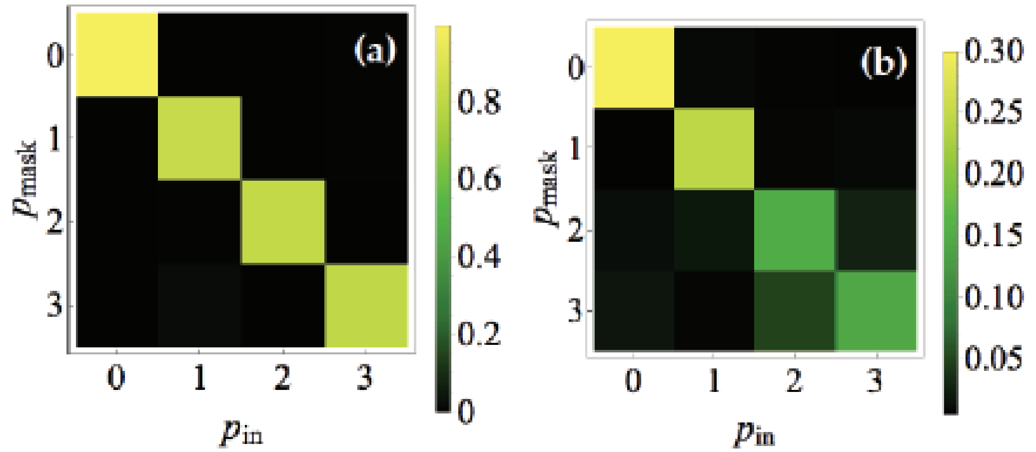


Laboratory Setup



Crosstalk Matrices

numerical experiment

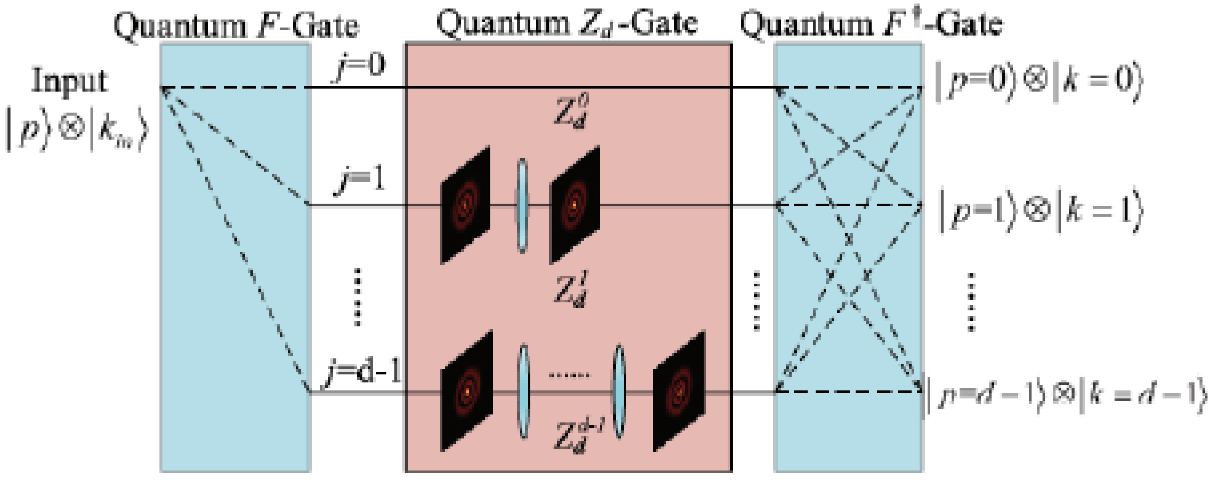


PRL 119, 263602 (2017) PHYSICAL REVIEW LETTERS week ending 29 DECEMBER 2017

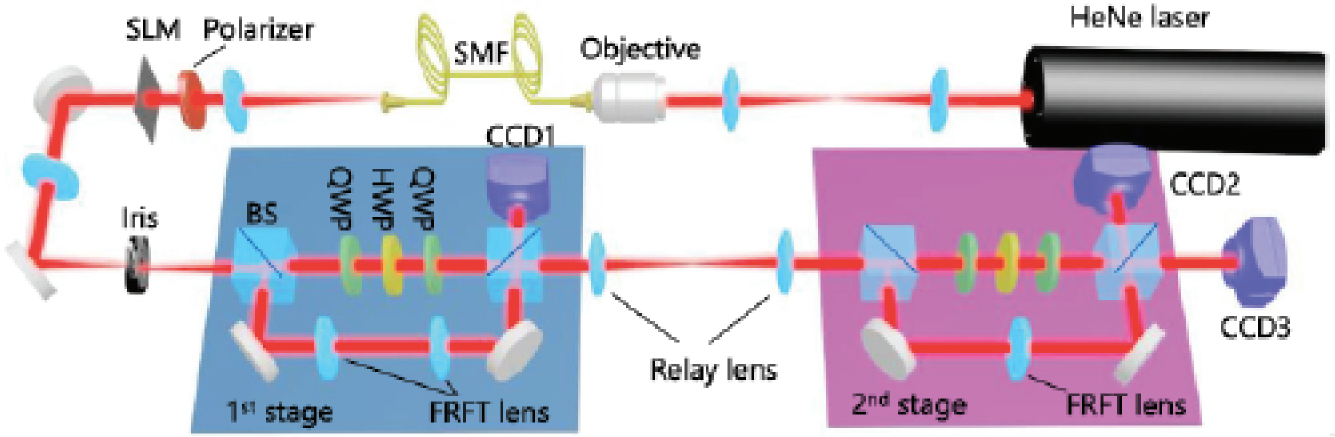
Sorting Photons by Radial Quantum Number

Yiyu Zhou,[1] Mohammad Mirhosseini,[1,*] Dongzhi Fu,[1,2] Jiapeng Zhao,[1] Seyed Mohammad Hashemi Rafsanjani,[1] Alan E. Willner,[3] and Robert W. Boyd[1,4]

- We implement a theoretical proposal of R. Ionicioiu, Sci. Rep. 6, 25356 (2016).

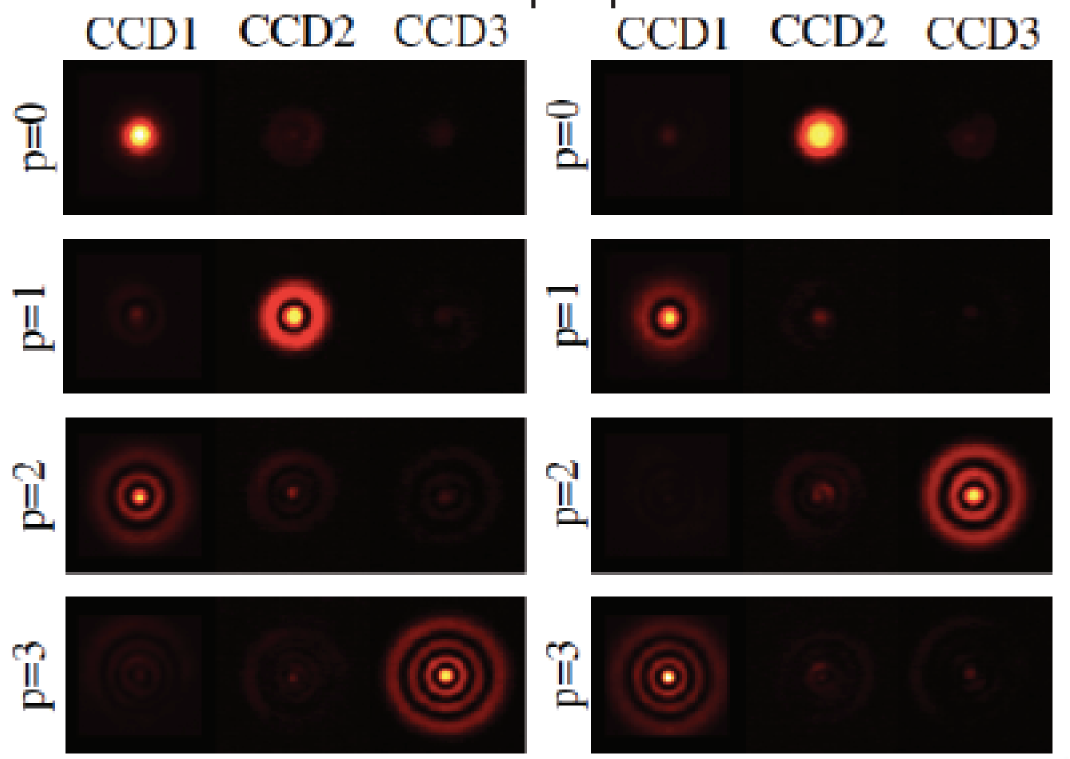


F-gate is a Fourier transform; Z_d-gate is a mode-dependent phase shifter implement by a fractional fourier transform.



- Results

Three output ports



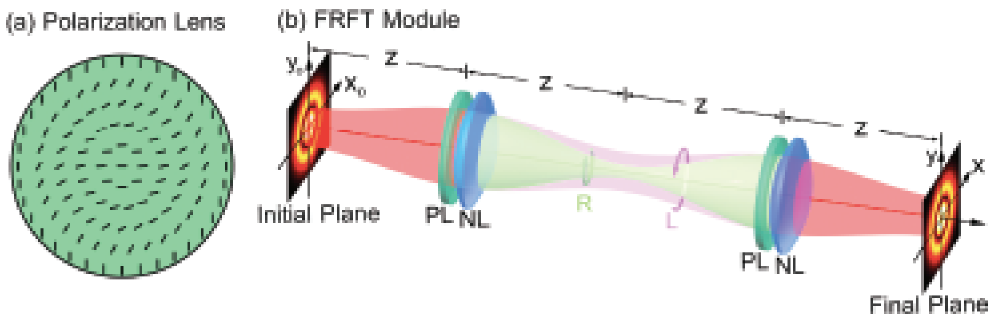
- Output of first Z_d-gate is injected into the second stage to remove the p=0 and p=2 ambiguity
- Intuition: the Gouy phase depends on the radial quantum number which determines the output of the interferometer––

Realization of a scalable Laguerre–Gaussian mode sorter based on a robust radial mode sorter

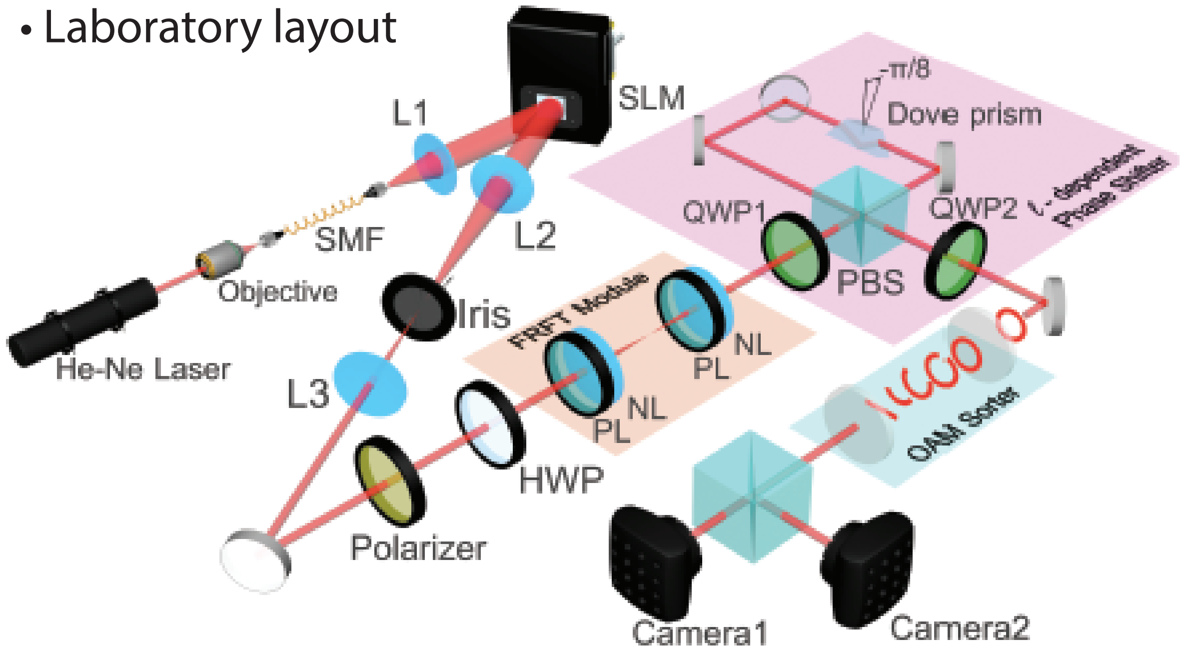
Research Article Vol. 26, No. 25 | 10 Dec 2018 | OPTICS EXPRESS 33057
Optics EXPRESS

DONGZHI FU,[1,2] YIYU ZHOU,[2,6] RUI QI,[2] STONE OLIVER,[3] YUNLONG WANG,[1] SEYED MOHAMMAD HASHEMI RAFSANJANI,[1] JIAPENG ZHAO,[2] MOHAMMAD MIRHOSSEINI,[2] ZHIMIN SHI,[4] PEI ZHANG,[1,*] AND ROBERT W. BOYD[2,5]

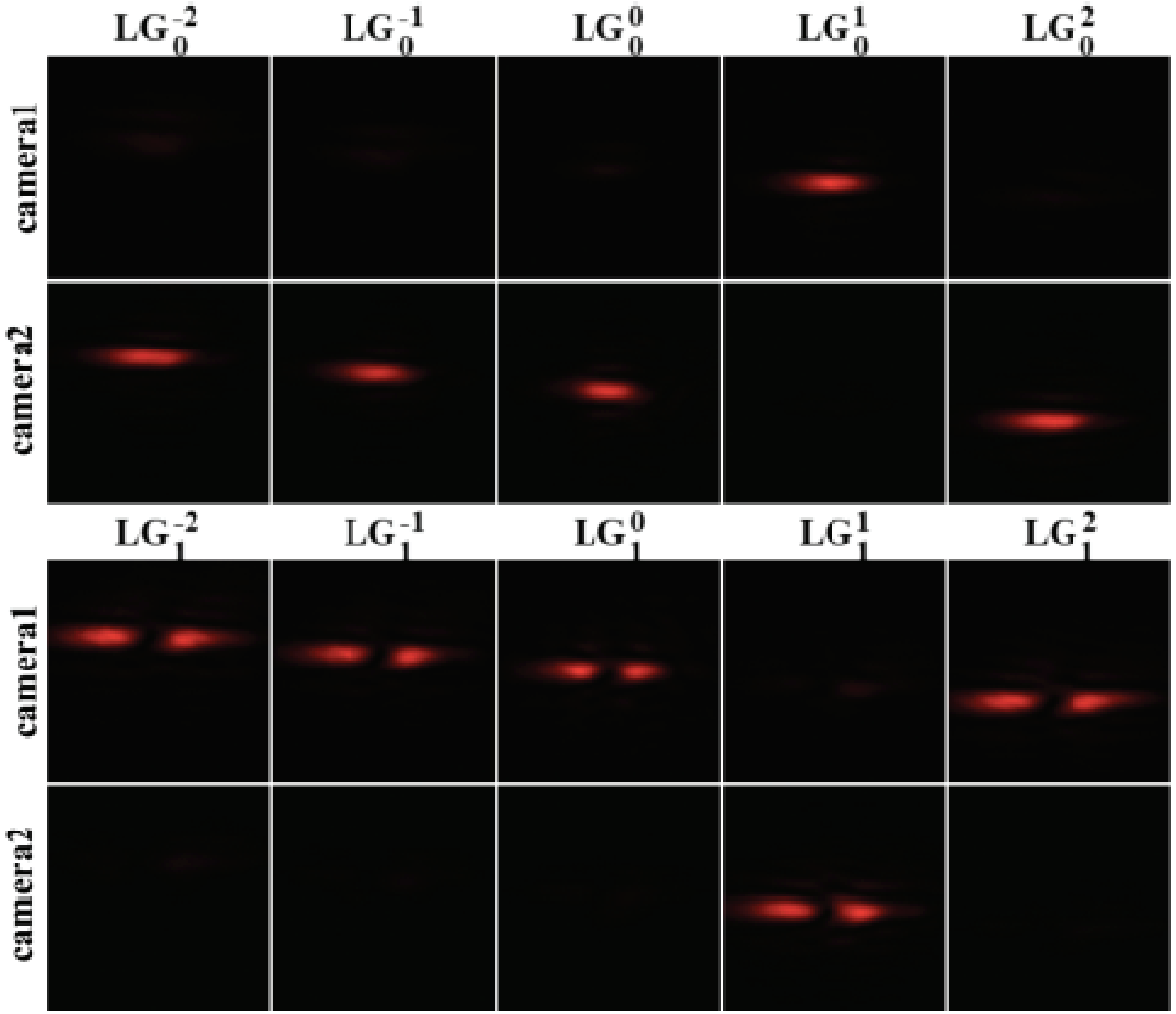
- We know how to build an azimuthal sorter, and we know how to build a radial sorter; we just need to combine them
- We use a polarization lens to create a common-path FRFT (fractional Fourier transform) module



- Laboratory layout



Results for sorting both azimuthal and radial dependence



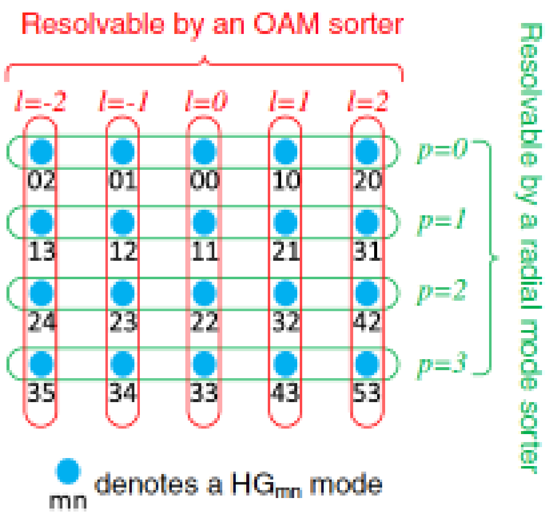
Letter Vol. 43, No. 21 / 1 November 2018 / Optics Letters 5263

Optics Letters

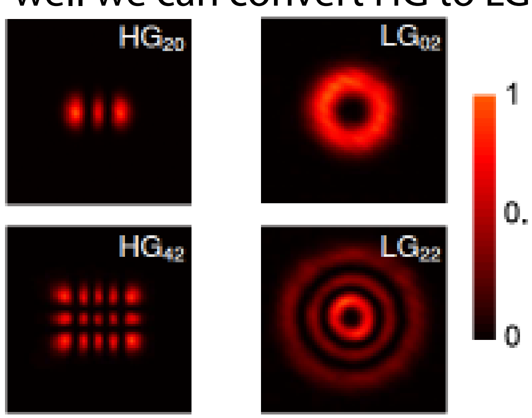
Hermite–Gaussian mode sorter

Yiyu Zhou,[1,*] Jiapeng Zhao,[1] Zhimin Shi,[2,7] Seyed Mohammad Hashemi Rafsanjani,[1,3] Mohammad Mirhosseini,[1,4] Ziyi Zhu,[2] Alan E. Willner,[5] and Robert W. Boyd[1,6,8]

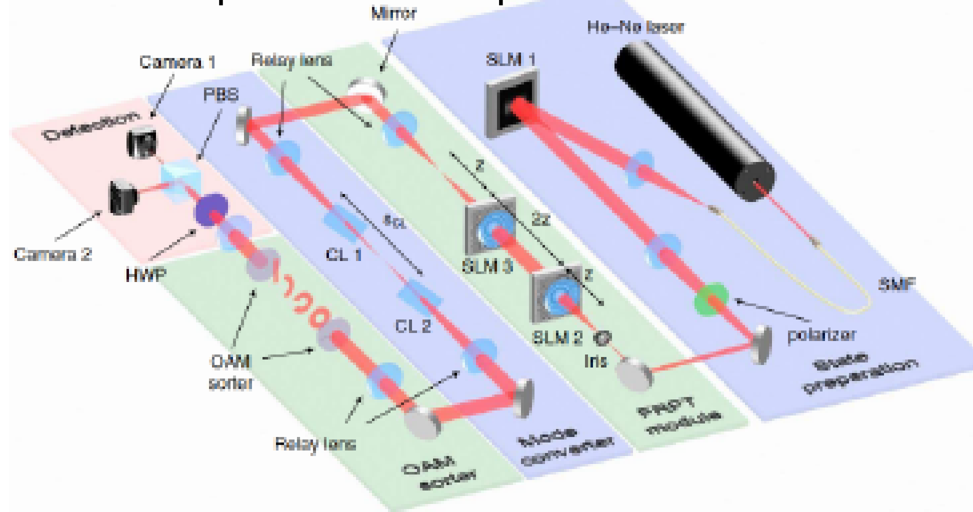
- It is surprisingly difficult to construct an HG mode sorter
- But we know how to construct an LG mode sorter
- And we know how to convert HG modes to LG modes
- Here is the conversion table between LG and HG modes:



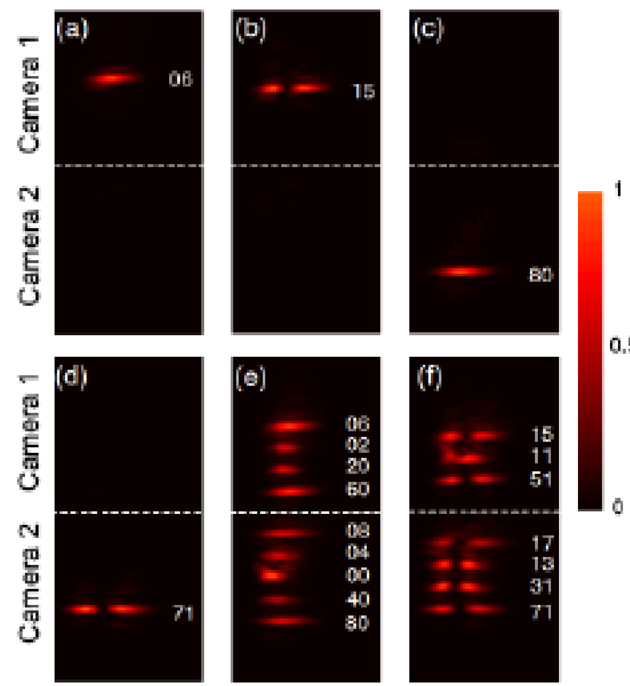
- Here is how well we can convert HG to LG



- Experimental setup



- Laboratory results



What to do with this new generation of sorters?

1. Use in for quantum key distribution (QKD) systems.

2. Use for superresolution imaging

How to Sort OAM States of Light

Prospective

1. OAM Sorters
   - Why they are important
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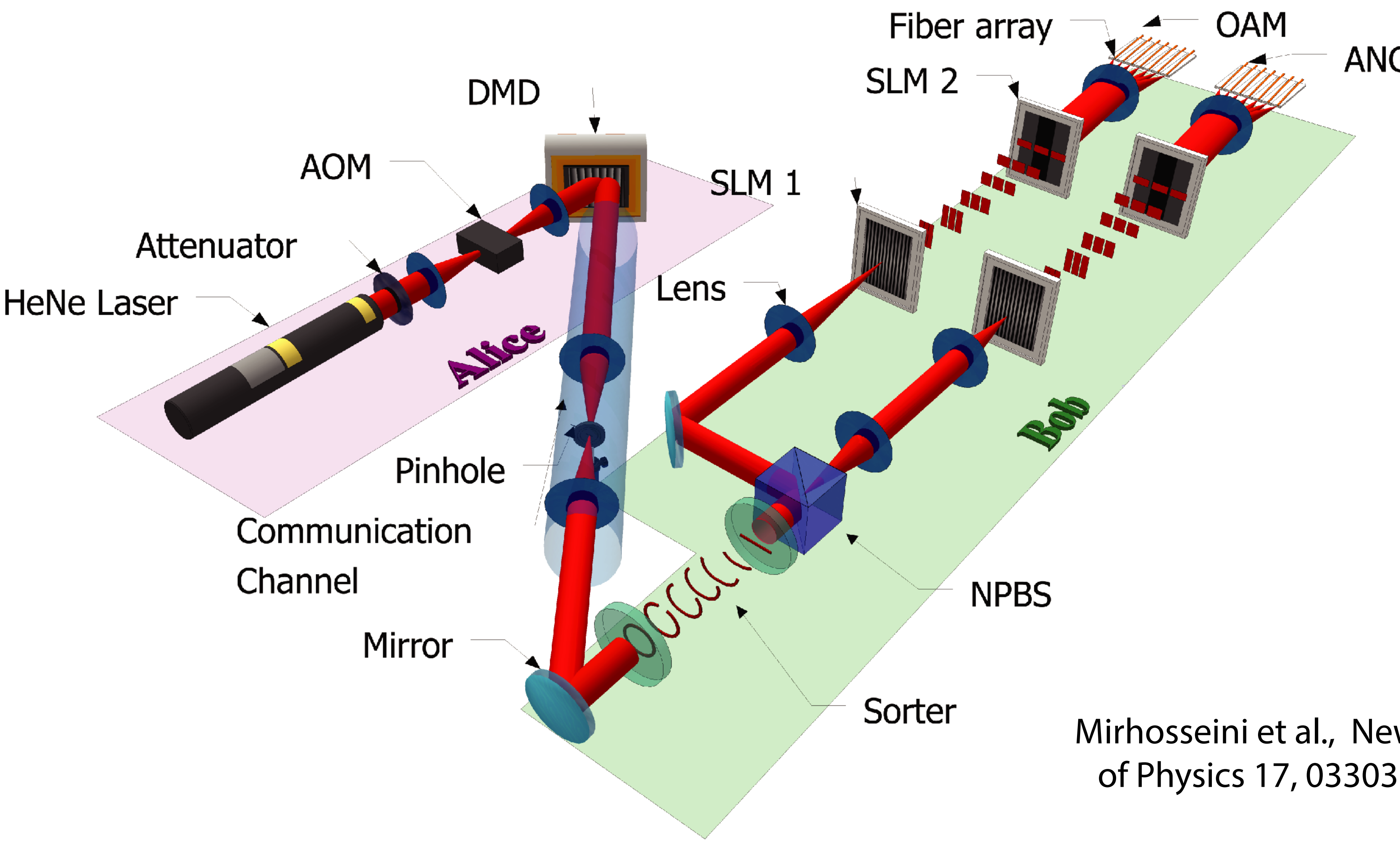
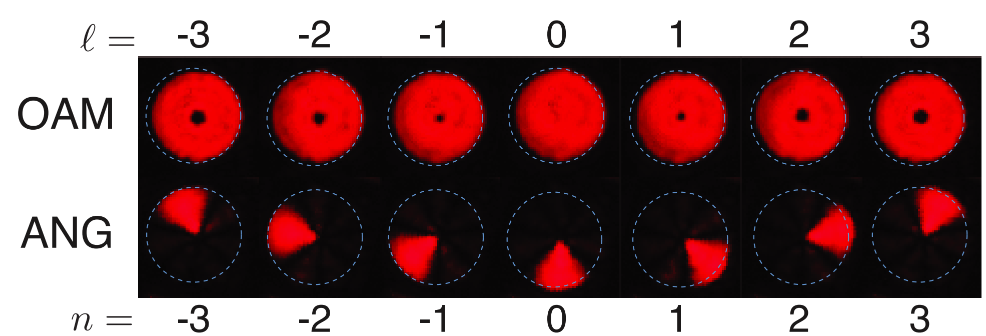
4. Summary

Our OAM Quantum Key Distribution Laboratory Setup



Mirhosseini et al., New Journal of Physics 17, 033033 (2015).

We use a seven-dimensional state space.



Laboratory Results - OAM-Based QKD

Alice	Bob
063133602132045444456141026645	063033601132021444456141026645
545050363603025261643215524164	545022353603025261643215524164
230146602513401613222451551026	230146602515403613422451551026

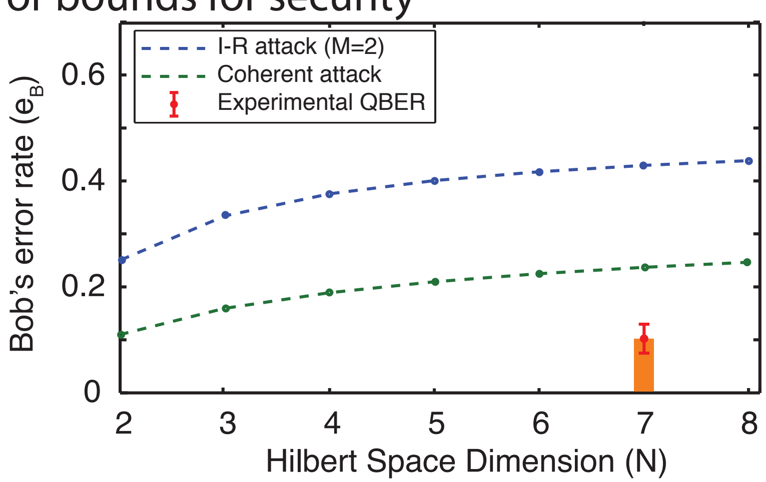
Error Correction

0000 0100 0111 0100 0101 0001 1100 0010 1100 1001 0010 0100 1101 0001 0000 0001 0011 0100 0000 0100 0010 0001 0001 0011 0000 0101 0010 0111 1000 0101 1101 0011 0001 1001 1011 0101 0000 0100 0111 0100 0011 0110 0101 0000 1101 0101 0010 1111 1010 1111 0010 1000

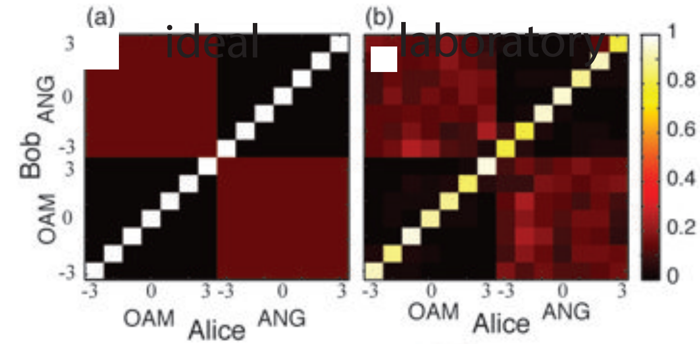
Privacy Amplification

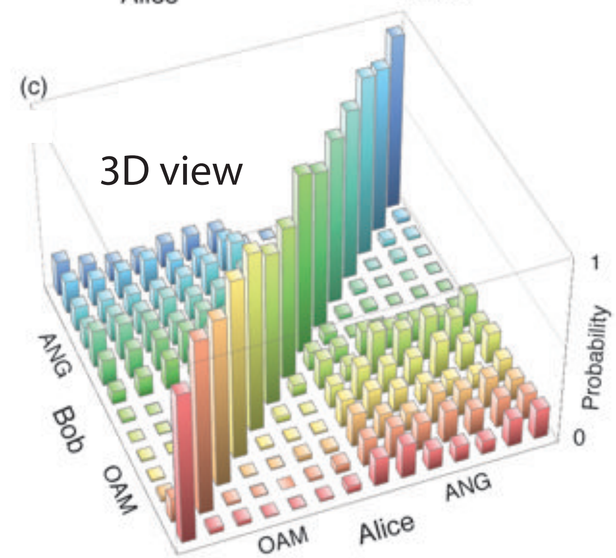
0001 0100 1011 1111 1111 1010 1010 0111 1010 1011 1111 0001 0000 1101 1111 1110 1110 1101 1110 0001 0011 1000 0101 1101 1100 0011 0111 1101 0001 1001 0100 0110 1011 0000 0110 1010 1

- error bounds for security



cross-talk matrices





We use a 7-letter alphabet, and achieve a channel capacity of 2.1 bits per sifted photon.

We do not reach the full 2.8 bits per photon for a variety of reasons, including dark counts in our detectors and cross-talk among channels resulting from imperfections in our sorter.

Nonetheless, our error rate is adequately low to provide full security,

M. Mirhosseini et al., New J. Phys. 17, 033033 (2015).

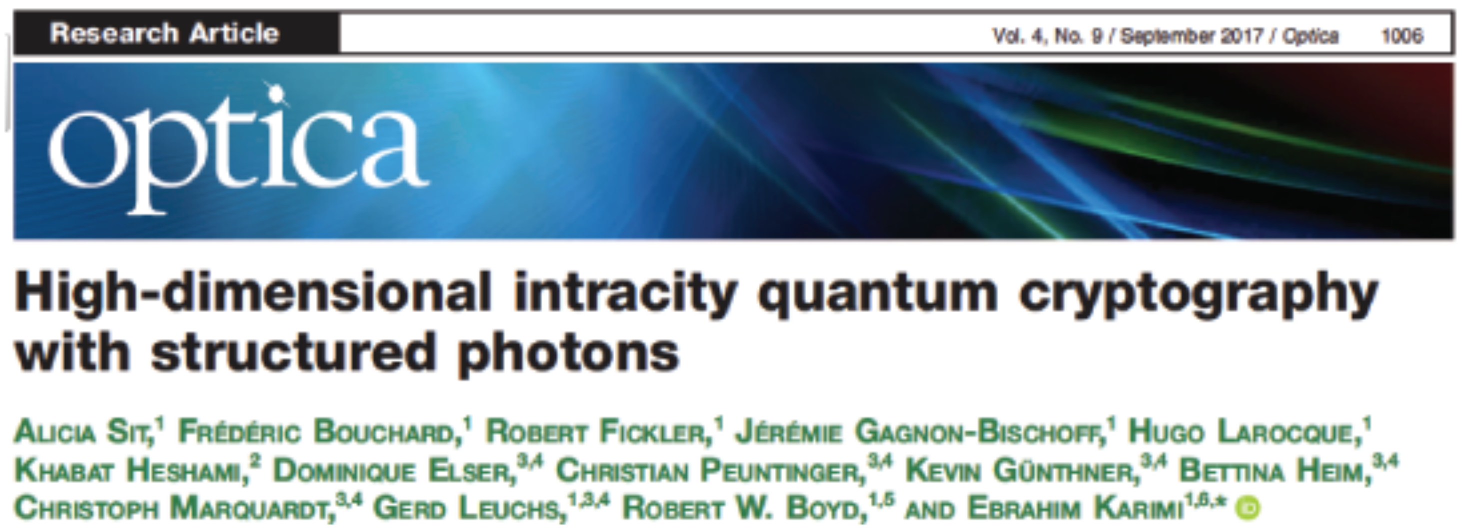
ARTICLES
PUBLISHED ONLINE: 24 JUNE 2012 | DOI: 10.1038/NPHOTON.2012.138

nature photonics

Terabit free-space data transmission employing orbital angular momentum multiplexing

Jian Wang[1,2]*, Jeng-Yuan Yang[1], Irfan M. Fazal[1], Nisar Ahmed[1], Yan Yan[1], Hao Huang[1], Yongxiong Ren[1], Yang Yue[1], Samuel Dolinar[3], Moshe Tur[4] and Alan E. Willner[1]*

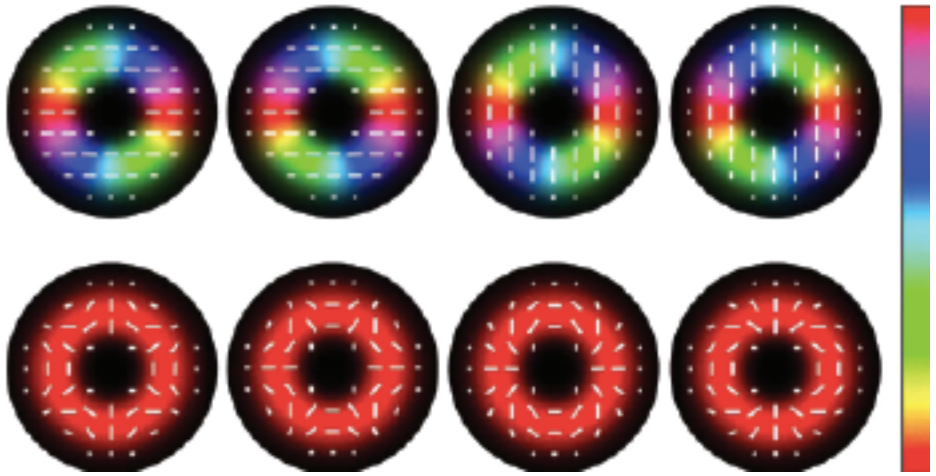
The recognition in the 1990s that light beams with a helical phase front have orbital angular momentum has benefited applications ranging from optical manipulation to quantum information processing. Recently, attention has been directed towards the opportunities for harnessing such beams in communications. Here, we demonstrate that four light beams with different values of orbital angular momentum and encoded with 42.8 × 4 Gbit s^{-1} quadrature amplitude modulation (16-QAM) signals can be multiplexed and demultiplexed, allowing a 1.37 Tbit s^{-1} aggregated rate and 25.6 bit s^{-1} Hz^{-1} spectral efficiency when combined with polarization multiplexing. Moreover, we show scalability in the spatial domain using two groups of concentric rings of eight polarization-multiplexed 20 × 4 Gbit s^{-1} 16-QAM-carrying orbital angular momentum beams, achieving a capacity of 2.56 Tbit s^{-1} and spectral efficiency of 95.7 bit s^{-1} Hz^{-1}. We also report data exchange between orbital angular momentum beams encoded with 100 Gbit s^{-1} differential quadrature phase-shift keying signals. These demonstrations suggest that orbital angular momentum could be a useful degree of freedom for increasing the capacity of free-space communications.


Research Article Vol. 4, No. 9 / September 2017 / Optica 1006
optica

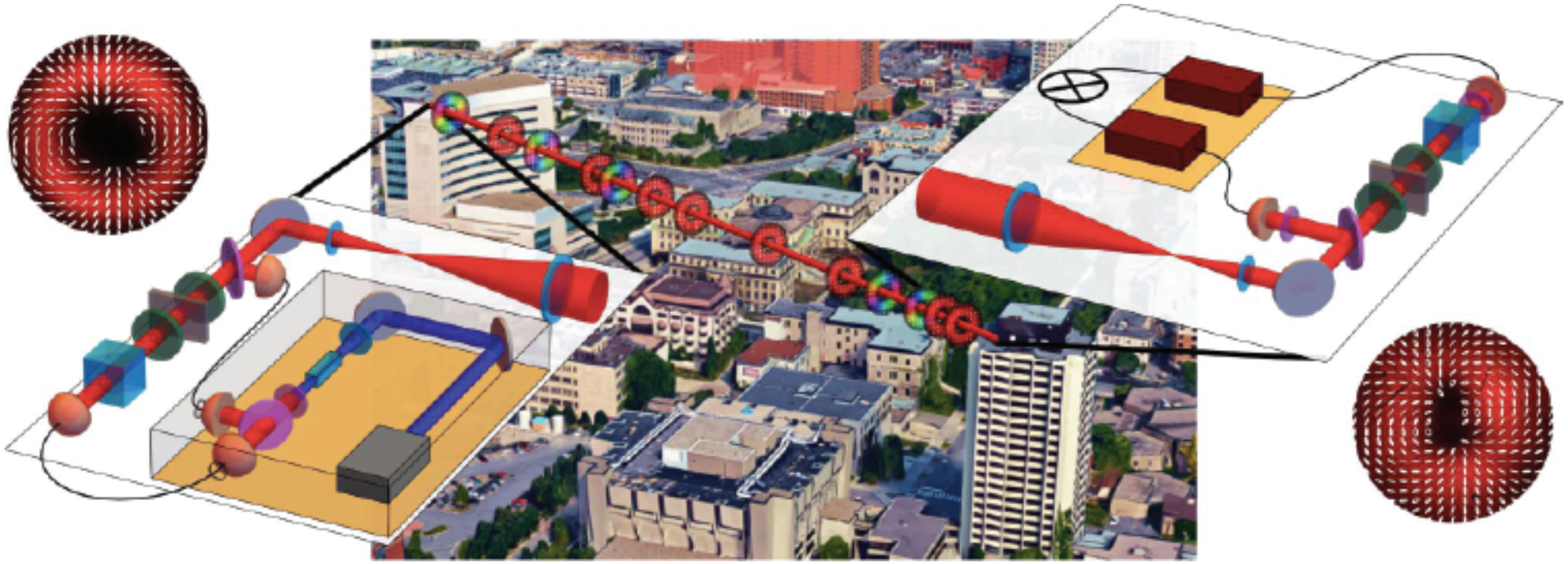
High-dimensional intracity quantum cryptography with structured photons

Alicia Sit,[1] Frédéric Bouchard,[1] Robert Fickler,[1] Jérémie Gagnon-Bischoff,[1] Hugo Larocque,[1] Khabat Heshami,[2] Dominique Elser,[3,4] Christian Peuntinger,[3,4] Kevin Günthner,[3,4] Bettina Heim,[3,4] Christoph Marquardt,[3,4] Gerd Leuchs,[1,3,4] Robert W. Boyd,[1,5] and Ebrahim Karimi[1,6,*]

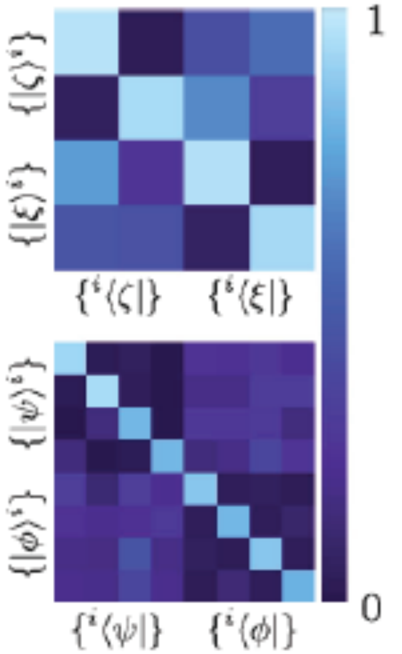
- Encode in a 4-D space of vector (polarization and OAM) modes



The link (300 m pathlength)



Results



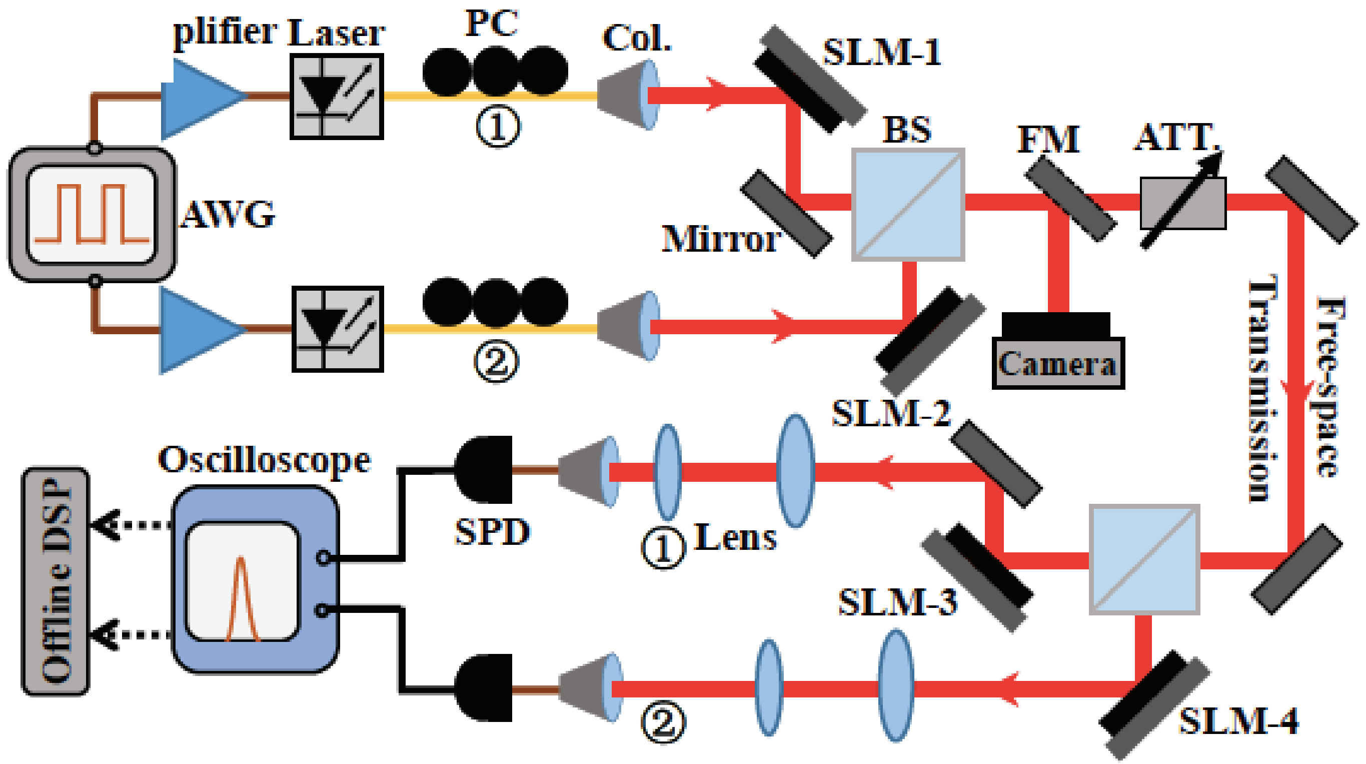
0.65 bits per sifted photon

Letter
Vol. 43, No. 22 / 15 November 2018 / Optics Letters 5639
Optics Letters

Demonstration of a 10 Mbit/s quantum communication link by encoding data on two Laguerre–Gaussian modes with different radial indices

Kai Pang,[1,*] Cong Liu,[1] Guodong Xie,[1] Yongxiong Ren,[1] Zhe Zhao,[1] Runzhou Zhang,[1] Yinwen Cao,[1] Jiapeng Zhao,[2] Haoqian Song,[1] Hao Song,[1] Long Li,[1] Ari N. Willner,[1] Moshe Tur,[3] Robert W. Boyd,[2] and Alan E. Willner[1]

- Each LG channel is driven by a separate laser diode
- High transmission rate is achieved by modulating the laser diode
- Uses projective measurement at receiver



Optics Letters

Spatially multiplexed orbital-angular-momentum-encoded single photon and classical channels in a free-space optical communication link

YONGXIONG REN,[1,4,*] CONG LIU,[1] KAI PANG,[1] JIAPENG ZHAO,[2] YINWEN CAO,[1] GUODONG XIE,[1] LONG LI,[1] PEICHENG LIAO,[1] ZHE ZHAO,[1] MOSHE TUR,[3] ROBERT W. BOYD,[2] AND ALAN E. WILLNER[1]

[1]Department of Electrical Engineering, University of Southern California, Los Angeles, California 90089, USA
[2]Department of Physics and Astronomy, The Institute of Optics, University of Rochester, Rochester, New York 14627, USA
[3]School of Electrical Engineering, Tel Aviv University, Ramat Aviv 69978, Israel
[4]Currently at FutureWei Technologies Inc., Santa Clara, California 95050, USA
*Corresponding author: yongxior@usc.edu

Received 11 September 2017; revised 15 October 2017; accepted 16 October 2017; posted 23 October 2017 (Doc. ID 303043); published 22 November 2017

We experimentally demonstrate spatial multiplexing of an orbital angular momentum (OAM)-encoded quantum channel and a classical Gaussian beam with a different wavelength and orthogonal polarization. Data rates as large as 100 MHz are achieved by encoding on two different OAM states by employing a combination of independently modulated laser diodes and helical phase holograms. The influence of OAM mode spacing, encoding bandwidth, and interference from the co-propagating Gaussian beam on registered photon count rates and quantum bit error rates is investigated. Our results show that the deleterious effects of intermodal crosstalk effects on system performance become less important for OAM mode spacing Δ ≥ 2 (corresponding to a crosstalk value of less than −18.5 dB). The use of OAM domain can additionally offer at least 10.4 dB isolation besides that provided by wavelength and polarization, leading to a further suppression of interference from the classical channel.

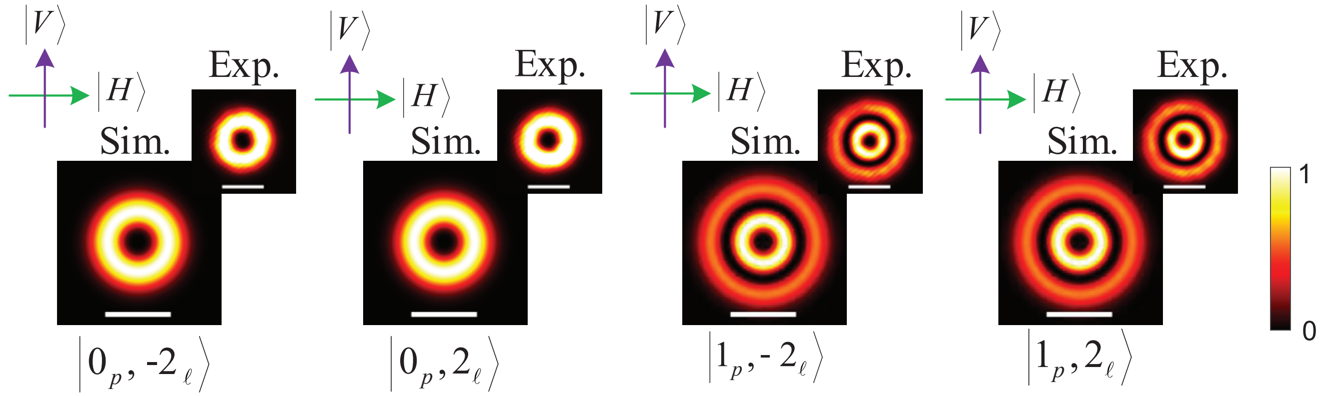
robustness against eavesdropping [6–8]. One example of enabling encoding by multilevel states is employing a set of orthogonal spatial modes for which the photon can occupy one of many states at a given time slot [8–10]. A possible spatial basis set that has recently received increasing interest is orbital angular momentum (OAM) modes [11]. A light beam with a helical wavefront carries an OAM corresponding to $\ell\hbar$ per photon, where $\hbar$ is the reduced Planck constant and ℓ is an unbounded integer [11]. OAM modes with different ℓ values are mutually orthogonal [12], which allows for the simultaneous transmission of multiple data channels [13,14]. Recent advances have shown the use of OAM modes for terabit/s classical optical links and for up to 143-km free-space transmission [13,15,16].

OAM states span a large Hilbert space and can be utilized for high-dimensional quantum encoding based on their orthogonality [10,17]. Moreover, quantum OAM encoding is in principle compatible with data encoding in other domains, such as polarization encoding [17,18]. A proof-of-concept OAM encoding-based quantum link has been recently demonstrated

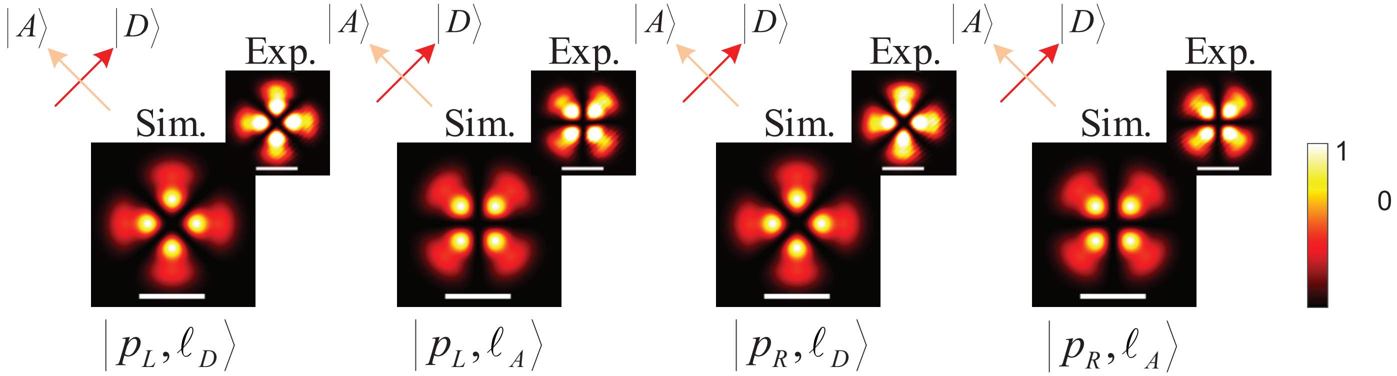
Quantum Key Distribution Using All Degrees of Freedom

- Perform QKD in an 8-dimensional state space.
- Encode in the transverse variation of amplitude, phase, and polarization
- Transfer 2.15 bits per detected photon

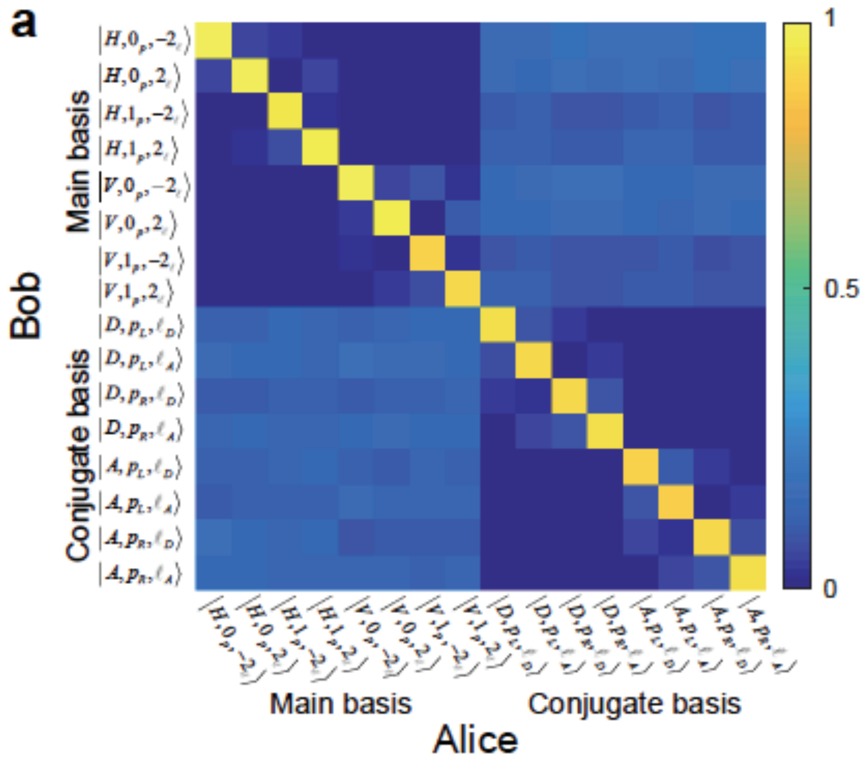
Main basis



Conjugate basis



- Crosstalk matrix



- Secure image transmission

Zhou et al, Opt. Express 27, 10383 (2019)

How to Sort OAM States of Light

Prospective

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   - Why they are important
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2. Use of OAM Sorters in Quantum Key Distribution
3. Use of Sorters in Superresolution Imaging
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What to do with this new generation of sorters?

1. Use in for quantum key distribution (QKD) systems.

2. Use for superresolution imaging

Mode Sorting and Imaging

1. It is most natural to perform imaging in the position basis.
2. However, one can alternatively decompose a (coherent) image into any orthogonal basis set, such as the Hermite-Gauss (HG) or Laguerre-Gauss (LG) modes.
3. There can be advantages in decomposing images into orthogonal mode sets
   a) a small number of parameters can characterize an image[1]
   b) techniques are available manipulate and characterize LG and HG modes
4. We are developing a mode sorter to allow us to decompose an arbitrary coherent optical image into the LG basis.

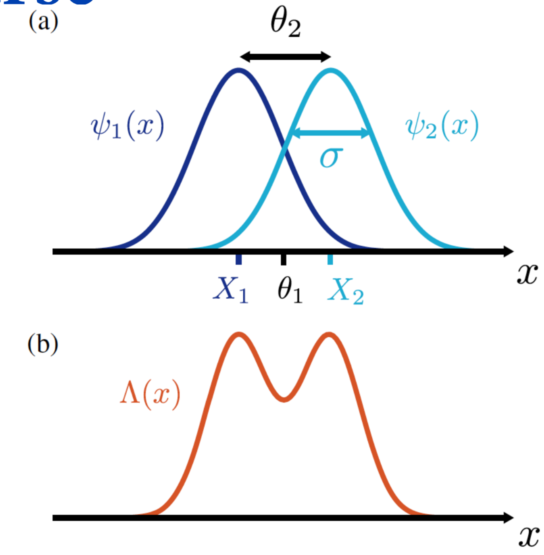
$$LG_{pl}(r,\theta) = \sqrt{\frac{2p!}{\pi(p+|l|)!}}\frac{1}{w_0}\left(\frac{\sqrt{2}r}{w_0}\right)^{|l|} \times \exp\left(-\frac{r^2}{w_0^2}\right) L_p^{|l|}\left(\frac{2r^2}{w_0^2}\right) e^{il\theta}$$

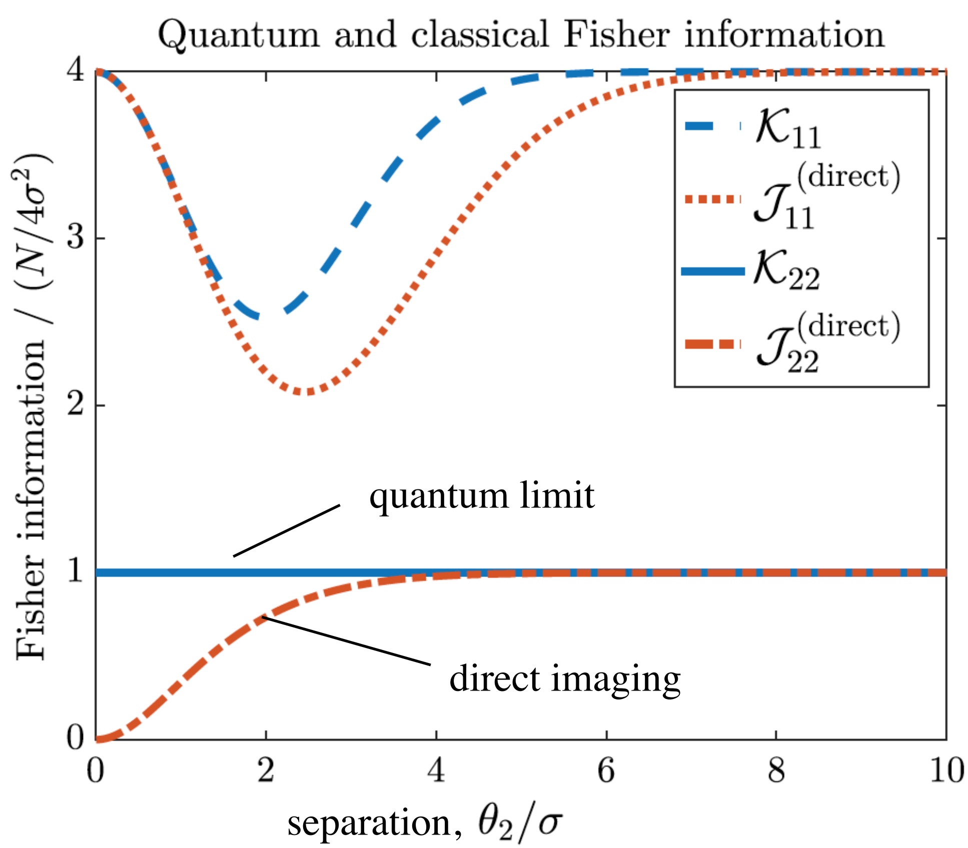
5. We had earlier (with Miles Padgett) developed a sorter for the azimuthal LG modes (OAM modes).
6. However, there has been no demonstrated means to sort on the radial degree of freedom.
7. We are developing such a device, based on a procedure suggested by Radu Ionicioiu[2]

[1] Z. Yang et al., Light: Science & Applications (2017) 6, e17013; doi:10.1038/lsa.2017.13
[2] R. Ionicioiu, "Sorting quantum systems efficiently," Sci. Reports 6, 25356 (2016).

Mankei Tsang and Rayleigh's Curse

- Mankei Tsang and coworkers note that according to classical reasoning the ability to determine the separation between two point sources degrades significantly when this distance becomes smaller than the Rayleigh criterion.
- They call this effect Rayleigh's curse. This conclusion is confirmed by consideration of the Fisher Information (FI) and its inverse, the Cramér-Rao Bound. Specifically, the FI is shown to drop to zero for zero separation.
- These conclusions are based on the assumption that the separation is measured by direct imaging, that is, by measuring the intensity of the focal image of the sources. However, if the full field, amplitude and phase, were to be measured, the FI would remain constant as the separation drops to zero. The same conclusion holds if one measures the image by means of a mode decomposition. By these procedures, one can entirely circumvent Rayleigh's curse.





M. Tsang, R. Nair, and X.-M. Lu, "Quantum theory of superresolution for two incoherent optical point sources," Phys. Rev. X 6, 031033 (2016).

Mankei Tsang and Rayleigh's Curse -- II

-

Mankei Tsang's predicted superresolution has been confirmed [2-4] for transverse resolution.

What about longitudinal resolution (which is also very important)?

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1. M. Tsang, R. Nair, and X.-M. Lu, Phys. Rev. X 6, 031033 (2016).

2. W.-K. Tham, H. Ferretti, and A. M. Steinberg, Phys. Rev. Lett. 118, 070801 (2017).

3. M. Paúr, B. Stoklasa, Z. Hradil, L. L. Sánchez-Soto, and J. Rehacek, Optica 3, 1144 (2016).

4. F. Yang, A. Tashchilina, E. S. Moiseev, C. Simon, and A. I. Lvovsky, Optica 3, 1148 (2016).

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optica

Quantum-limited estimation of the axial separation of two incoherent point sources

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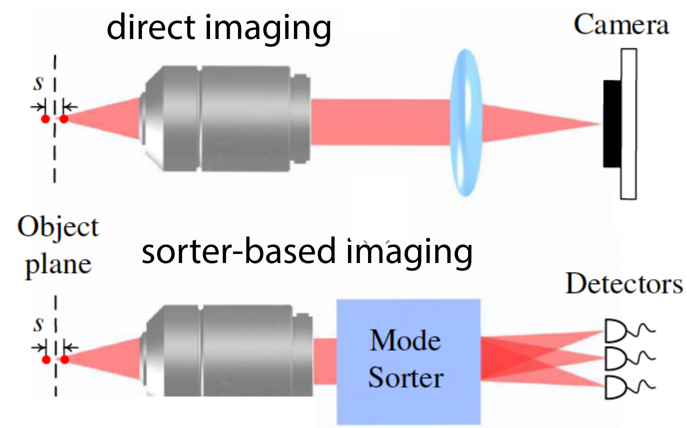
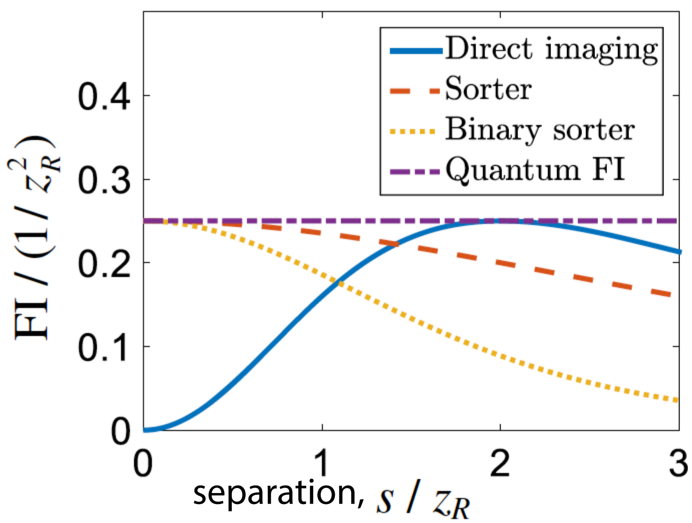
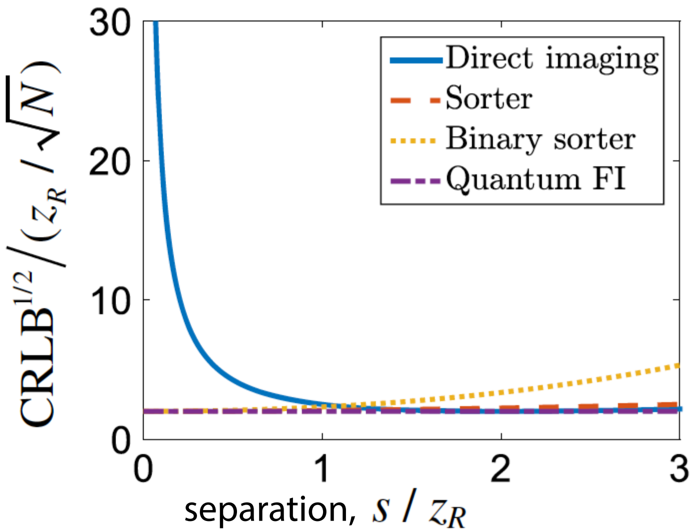
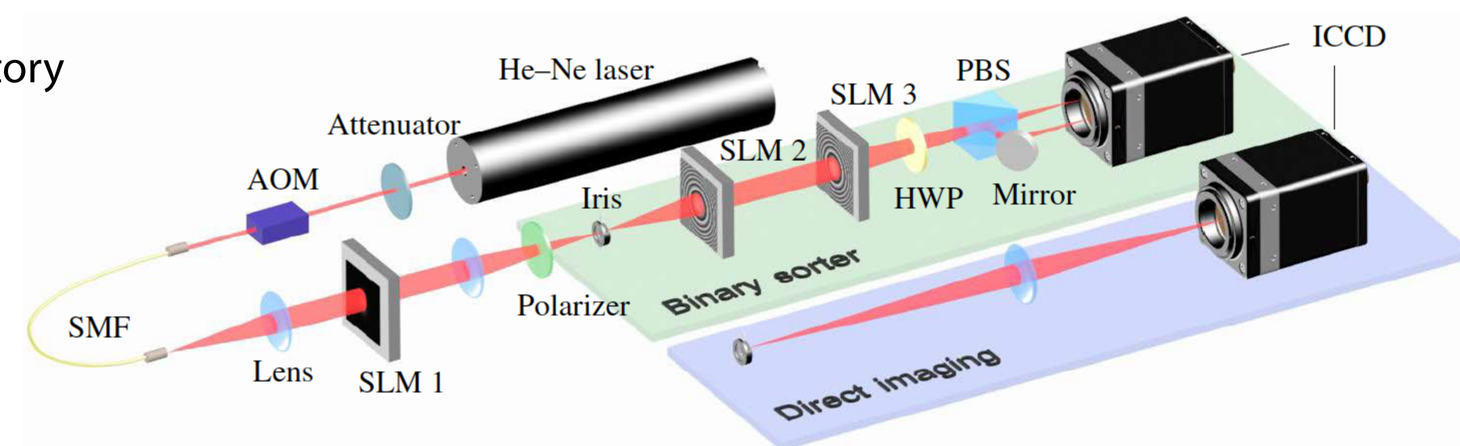


Theory

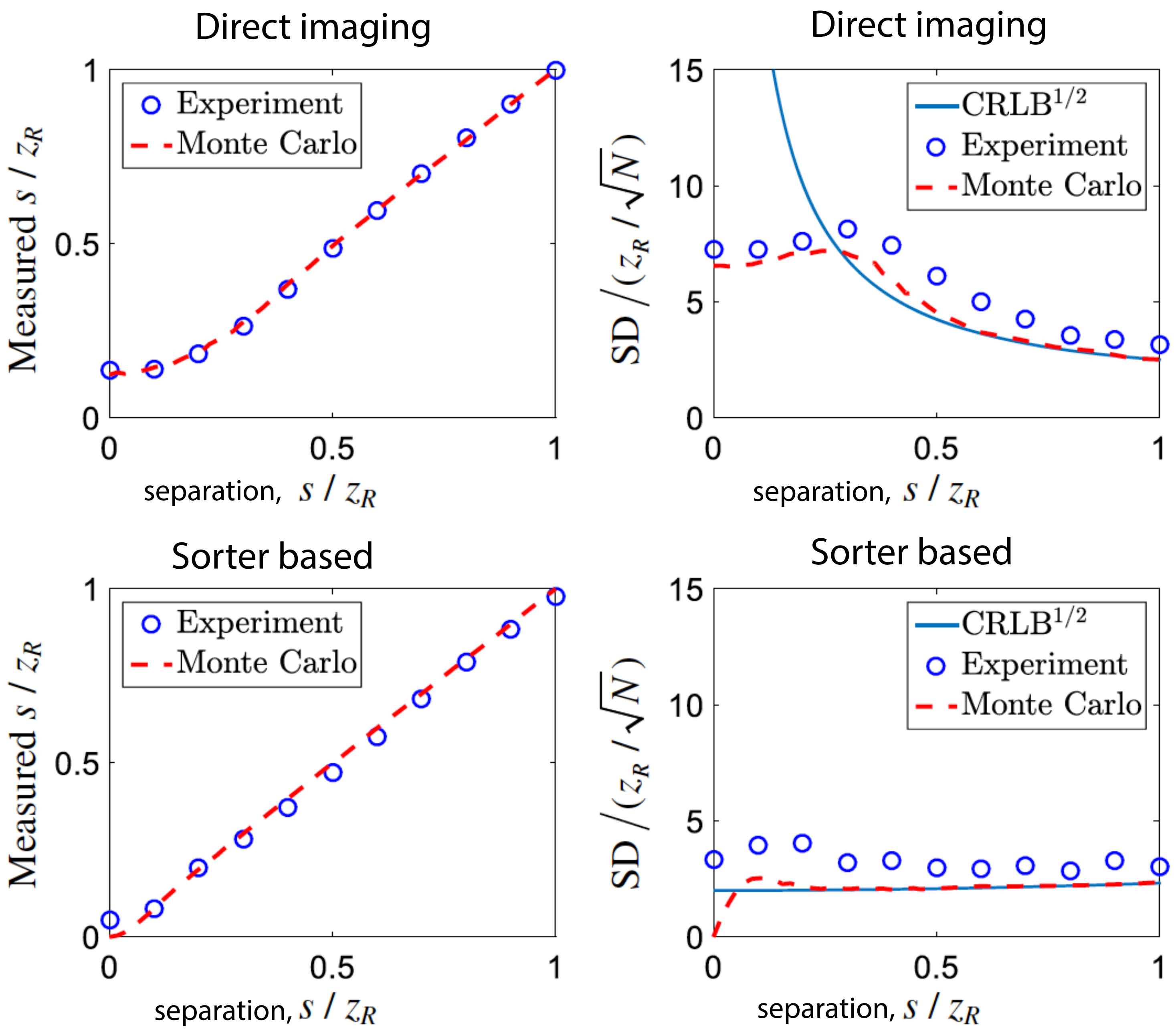




Laboratory



Laboratory Results: Axial Superresolution

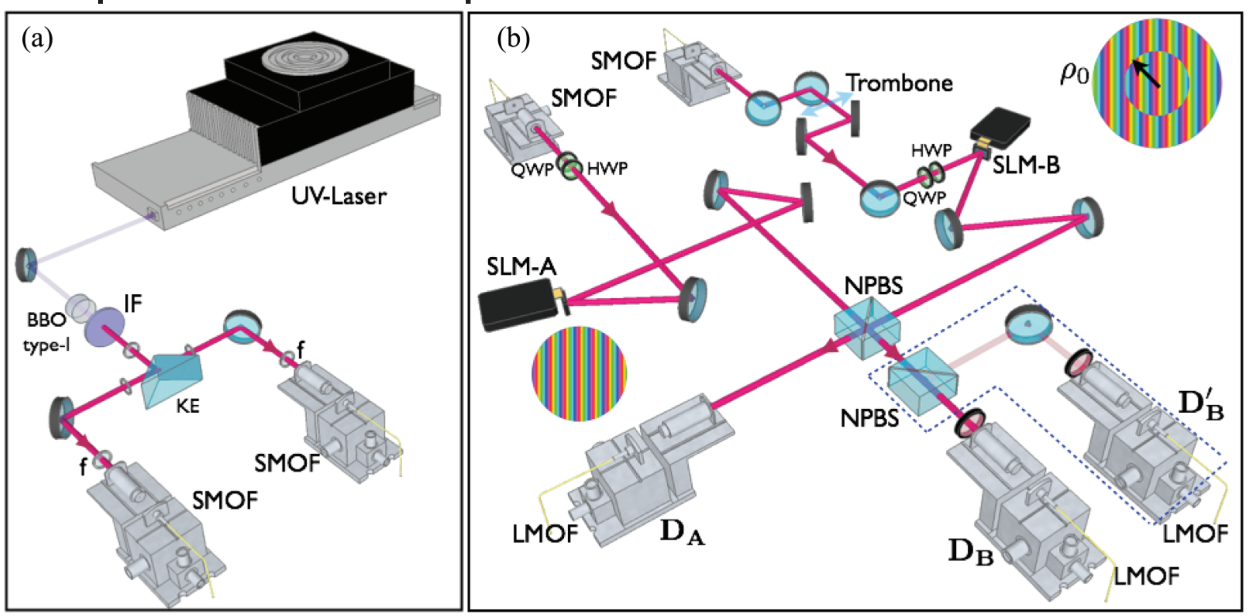


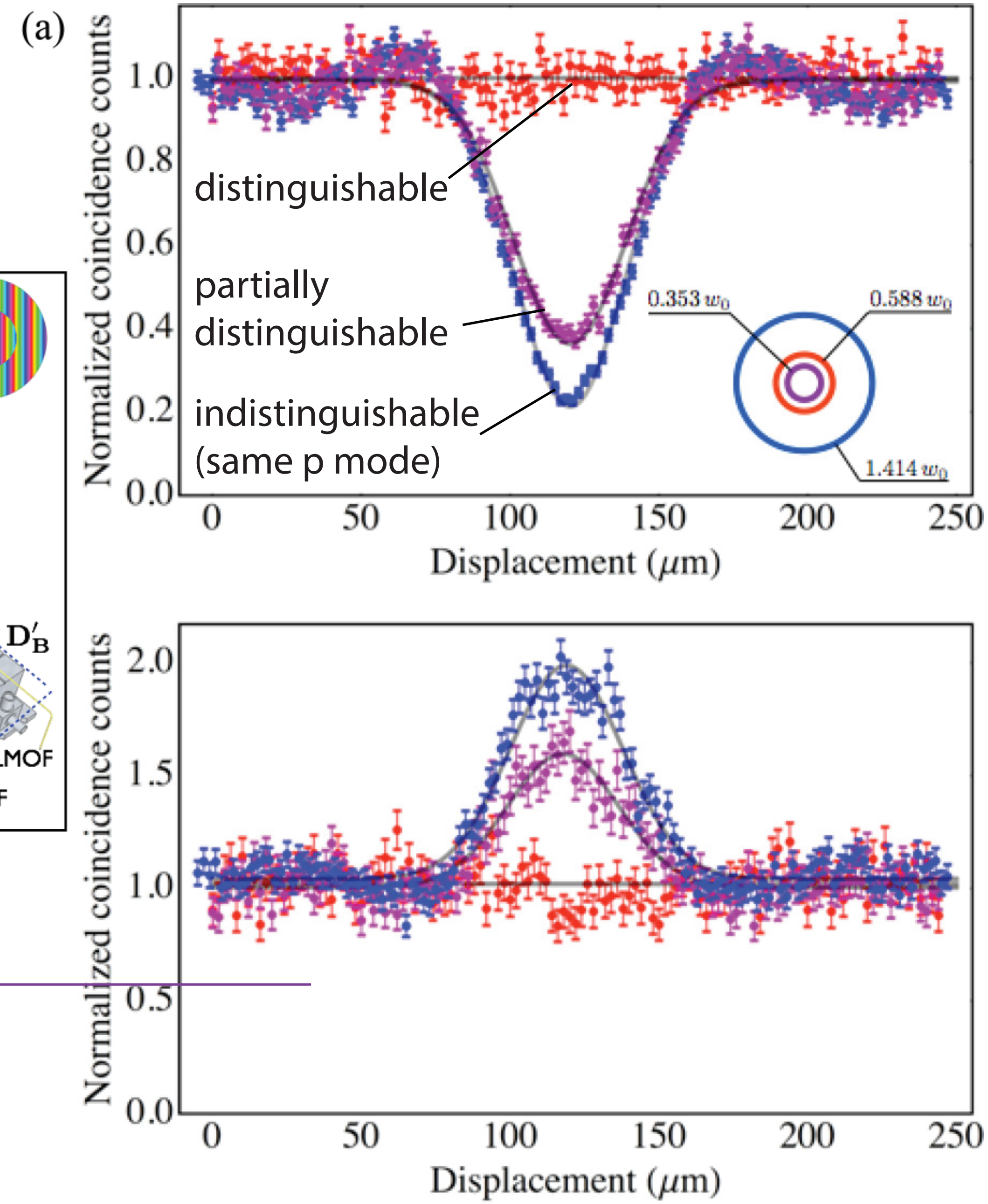
Quantum Properties of the Radial Degree of Freedom

- Hong-Ou-Mandel effect and and LG beams
- We can control the degree of distinguishability through the radial degree of freedom of a LG beam.
- Note that the radius is a continuous variable; azimuthal index is a discrete variable.

coincidences between D_A and D_B

Experimental setup





- coincidences between D_B and $D_{B'}$
- measurement of coalescence

Karimi et al. PHYSICAL REVIEW A 89, 013829 (2014)

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PAPER

Orbital angular momentum modes do not increase the channel capacity in communication links

fidelity = 97%
efficiency = 0.2%

Abstract
The orbital momentum of optical or radio waves can be used as a degree of freedom to transmit information. However, mainly for technical reasons, this degree of freedom has not been widely used in communication channels. The question is if this degree of freedom opens up a new, hitherto unused 'communication window' supporting 'an infinite number of channels in a given, fixed bandwidth' in free space communication as has been claimed? We answer this question in the negative by showing that on the fundamental level, the mode density, and thus room for mode multiplexing, is the same for this degree of freedom as for sets of modes lacking angular momentum. In addition we show that modes with angular momentum are unsuitable for broadcasting applications due to excessive crosstalk or a poor signal-to-noise ratio.

What do we mean by an OAM mode?

Summary: Quantum State Sorters

- New quantum-state sorters for full Laguerre-Gauss characterization enable enhanced performance for free-space quantum communications.

- This technology is useful more generally in advanced imaging by allowing a complex optical field to be decomposed into a complete set of orthogonal modes.

- In particular, these sorters can be used in superresolution imaging.