Quantum Radiometry:
Vacuum fluctuations as a primary standard for radiometry

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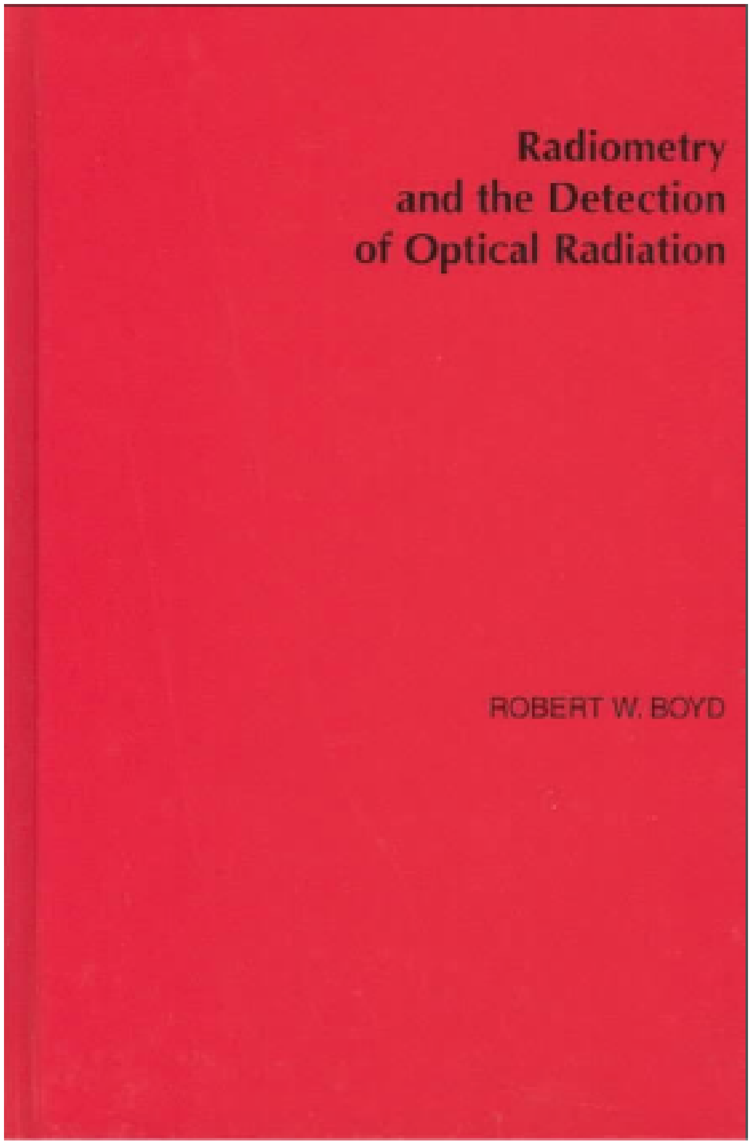
The visuals of this talk will be posted at boydnlo.ca/presentations

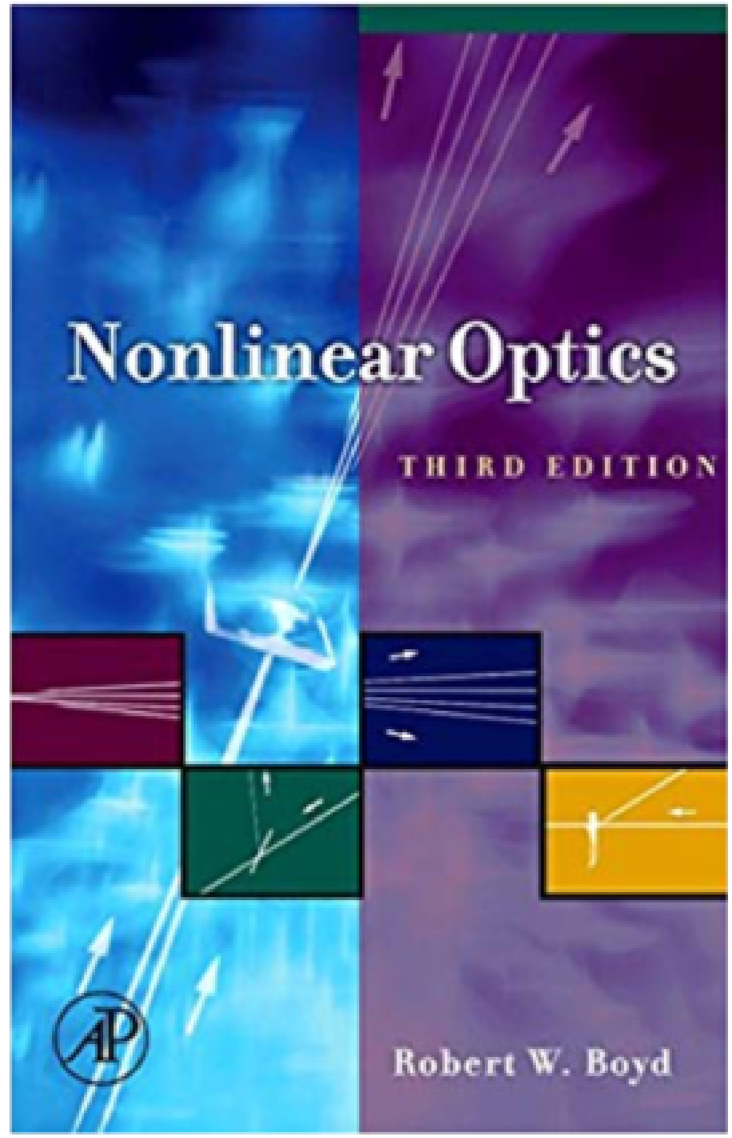
Presented at the *49th Winter Colloquium on the Physics of Quantum Electronics*, Snowbird Utah, January 6–11, 2019.

Quantum Radiometry

Two of my research interests are radiometry and nonlinear optics.
Why not combine them both into a single project?



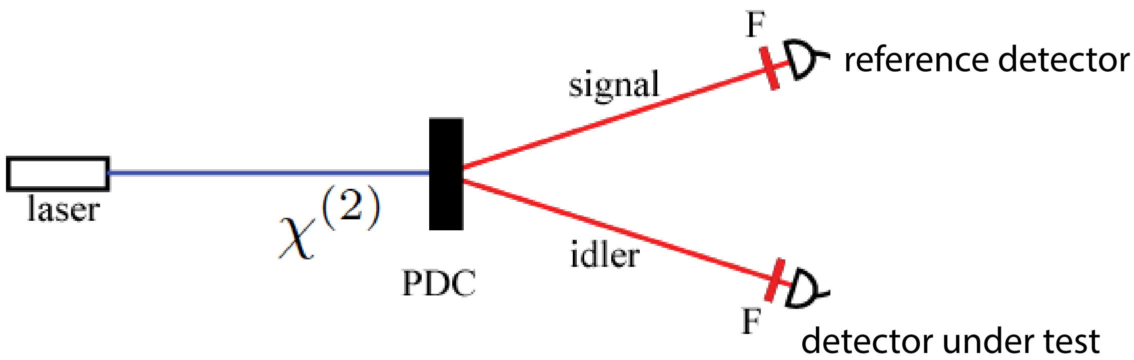
John Wiley, 1983

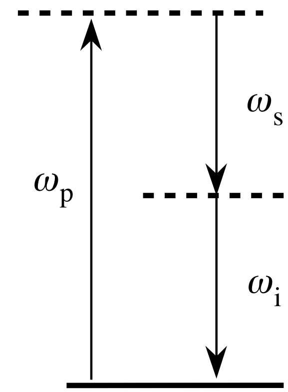


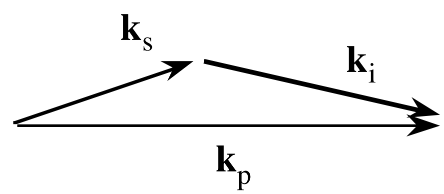
Academic Press, 2008

Earlier Work on Quantum Radiometry

- Absolute measurement of detector quantum efficiency (Klyshko, Sergienko, Migdall, Polyakov, etc.)



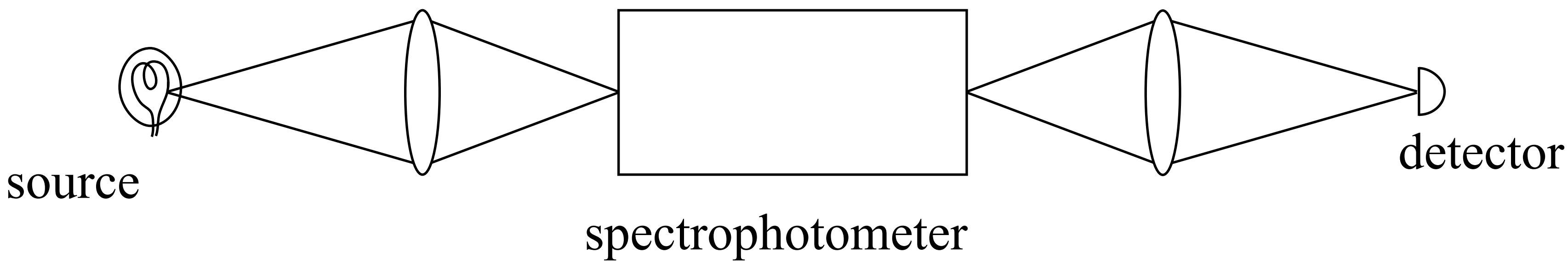


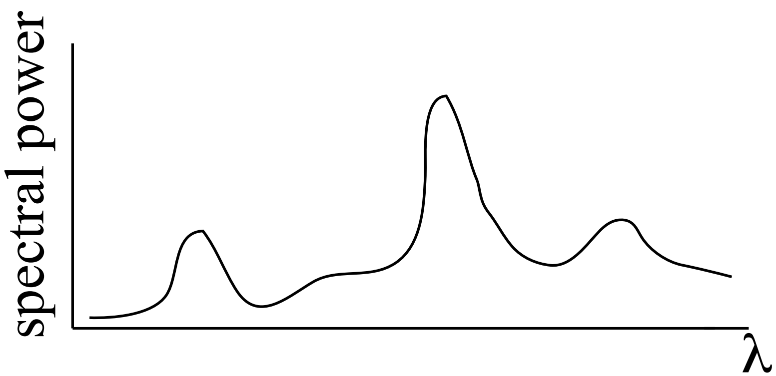


- Earlier work (Klyshko) established that the light produced by spontaneous parametric downconversion (SPDC) can be characterized in terms of the radiometric property known as brightness (or radiance).

Goal of Our Research

- How to perform absolute calibration of a spectrophotometer?





How do we perform an absolute measurement of the vertical axis.

Units of Watts per nm of spectral bandwidth.

Traditional Approach to Calibration

- Use a black body source, or a lamp calibrated to a black body source
- Theory of black body radiation is very well understood

(1) Density of field modes (number of modes per unit volume per unit frequency interval) is given by

$$\rho_\nu = \frac{8\pi\nu^2}{(c/n)^3}$$

(2) Energy per field mode is $h\nu\bar{n}$ where $\bar{n}$ is the mean number of photons per mode:

$$\bar{n} = \frac{1}{e^{(h\nu/k_B T)} - 1}$$ Planck distribution

(3) Energy density of black body radiation (energy per unit volume) give by

$$u_\nu = 2\rho_\nu h\nu\,\bar{n} = \frac{8\pi h\nu^3}{(c/n)^3(e^{h\nu/k_B T} - 1)}$$ Planck radiation law

(4) Brightness (radiance) of black body radiation (power per unit area per unit solid angle) is given by

$$B_\nu = \frac{(c/n)}{4\pi}u_\nu = \frac{2h\nu^3}{(c/n)^2(e^{h\nu/k_B T} - 1)}$$ Planck radiation law

Problem with Traditional Approach to Calibration

Need to use a non-optical means to determine the temperature of a black body source

This step is not easy and is prone to error.

Our New Approach to Absolute Calibration

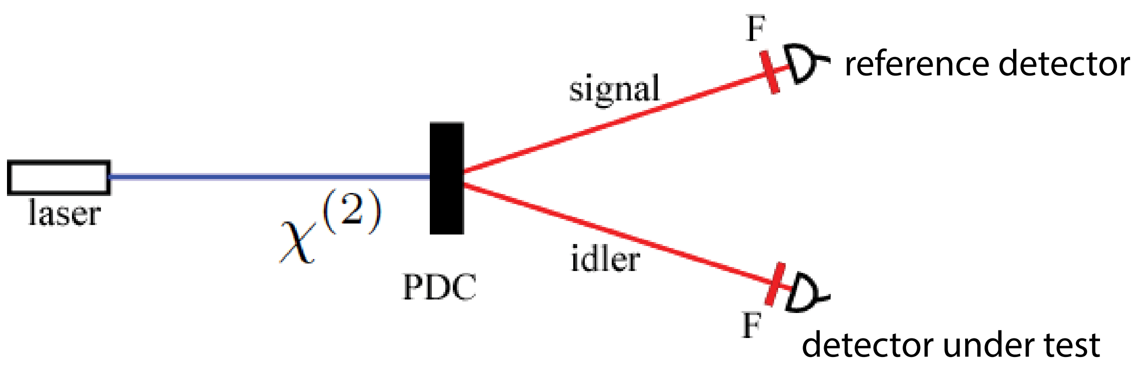
We make use of black body radiation, but at very low temperature.

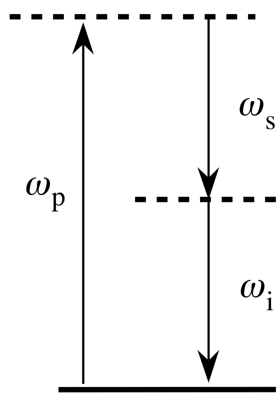
At low temperature the emission vanishes, but the field fluctuations remain.

We use these fluctuations to seed the process of spontaneous parametric down conversion (SPDC).

We calibrate our spectrophotometer with this radiation, whose strength can be traced back to Planck’s constant *h*.

Our approach builds upon the work of Klyshko, Sergienko, Migdall, Polyakov, etc., but is distinct from it





Theory of Spontaneous Parametric Down Conversion (SPDC)

The theory of SPDC is very well developed (see, for instance, D. N. Klyshko, *Photons and Nonlinear Optics,* Gordon and Breach, 1989). Here we convey only a few key elements.

- Macroscopic Hamiltonian

$$\hat{H} \sim E_p \chi^{(2)} \hat{E}_s^\dagger \hat{E}_i^\dagger + \text{h.c.}$$

- Quantization

$$\hat{E}_j \sim \int \mathrm{d}^3 k_j \underbrace{\sqrt{\frac{\omega_j}{n_j}}}_{\text{Vacuum amplitude}} \hat{a}_j \, \mathrm{e}^{i(\boldsymbol{k}_j \boldsymbol{r} - \omega_j t)}$$

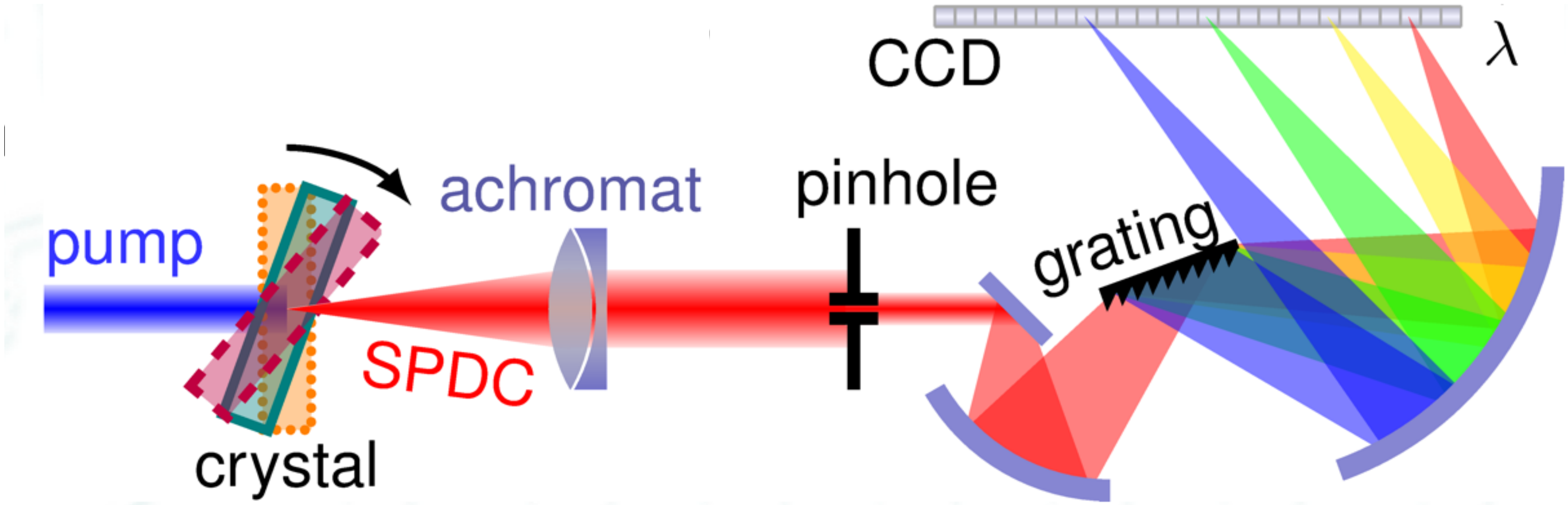
with

$$\boldsymbol{k}_j = (\boldsymbol{q}_j, \kappa_j)$$

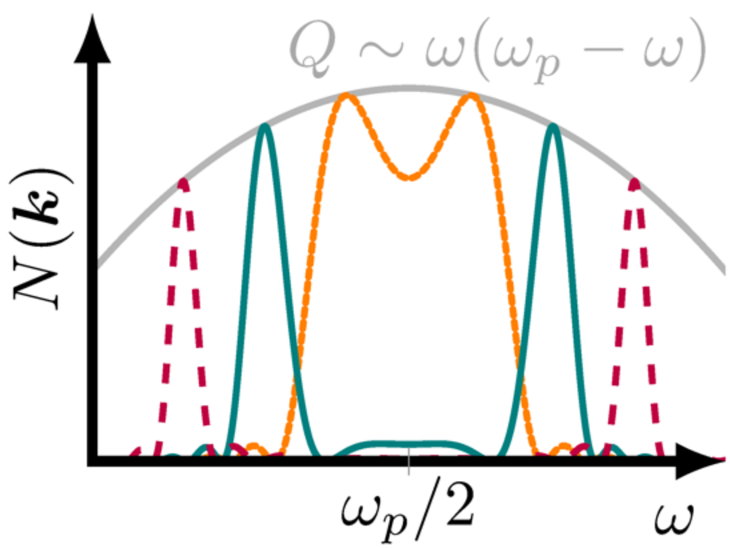
We then determine the number of downconverted signal photons per mode

$$N(\boldsymbol{k}_s) \sim \underbrace{\left(\frac{\mathcal{E}_p \chi^{(2)} L}{\sqrt{n_s n_i}} \right)^2}_{\mathcal{G}(\lambda_s)} \underbrace{\omega_s \omega_i}_{Q(\lambda_s)} \underbrace{\mathrm{sinc}^2 \frac{\Delta\kappa L}{2}}_{S(\lambda_s)}$$

Our Experimental Setup



We rotate the crystal to vary the phase matched wavelength. We end up with data that looks like this.



Note that

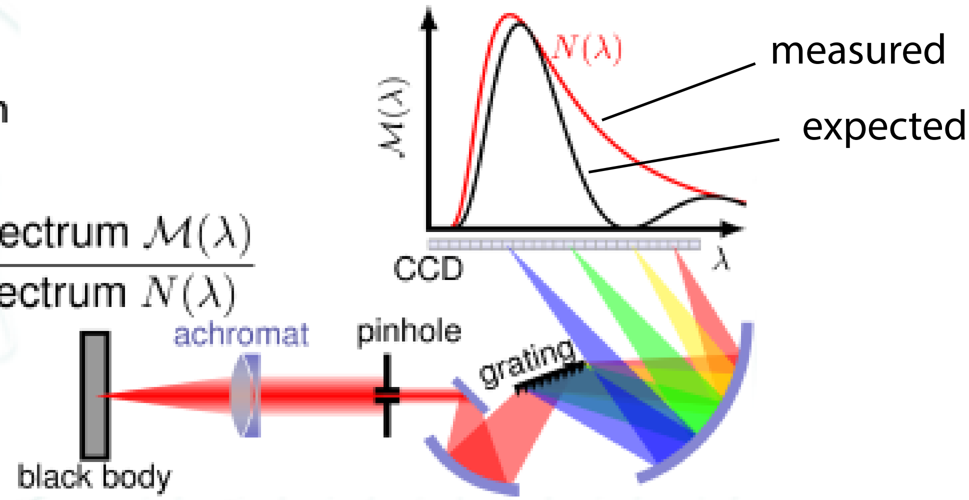
$$\omega_s \omega_i = \omega_s(\omega_p - \omega_s)$$

which give the form of the parabola shown in the figure.

Determining the Response Function

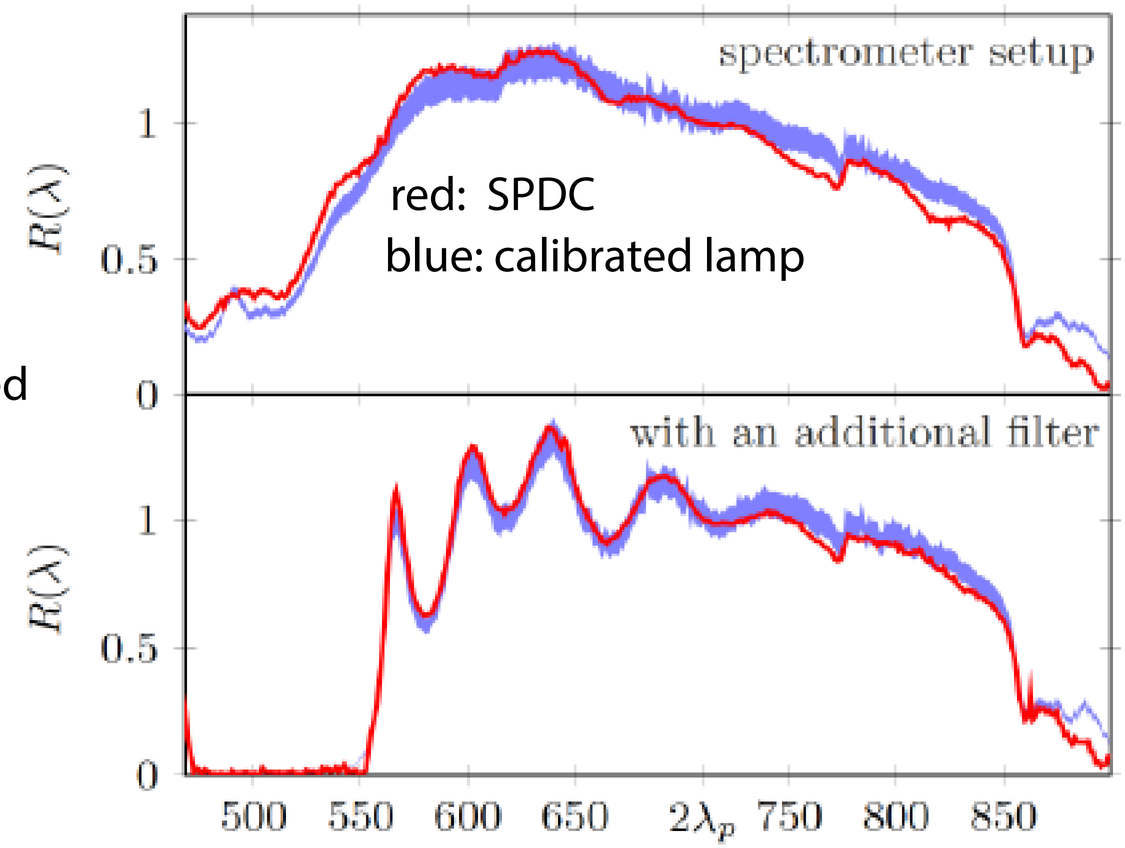
- Spectrometer might not show expected spectrum
- Response function:

$$\mathcal{R}(\lambda) = \frac{\text{measured spectrum } \mathcal{M}(\lambda)}{\text{expected spectrum } N(\lambda)}$$



Laboratory measurements of the response function

There is no discernable difference between the results obtained with a calibrated lamp and those obtained with the new methods based on SPDC



Relative and Absolute Calibration

The results from the last slide demonstrated that SPDC can provide good ***relative*** calibration.

Specifically, we presented a protocol for determining the relative response $R(\lambda)$. Through knowledge of $R(\lambda)$ one can correct for any wavelength dependent inefficiencies of a spectrophometer.

A more challenging task in to determine the absolute calibration, that is to know exactly how many watts (or watts per spectral bandwidth) are leaving the spectrophotometer.

The theory and data presented up till now have been taken in the limit of a weak pump wave, that is for ***spontaneous*** parametric downconversion. As we will show next, when this procedure is repeated for a strong pump wave, the process is one of ***stimulated*** parametric down conversion. For this situation, additional information becomes available and it is possible to perform an absolute calibration.

Absolute Calibration

- The theory for absolute calibration is formulated as as follows:
- We define the efficiency η(λ) of the downconversion process through

$$M(\lambda) = \eta(\lambda)\, N(\lambda)$$

where *M*(λ) is the measured signal and *N*(λ) is the expected signal.

- We express the efficiency a $\eta(\lambda) = \alpha R(\lambda)$ where α is wavelength independent.
- The predicted signal under phase-matched conditions is give by

$$\mathcal{N}_{\rm PM} = \sinh^2\left(\mathcal{G}\sqrt{\omega(\omega_p - \omega)}\right)$$

where we have introduced the gain parameter $\mathcal{G} = c^{-1}\, L\, \chi^{(2)} E_p / \sqrt{n\, n_i},$

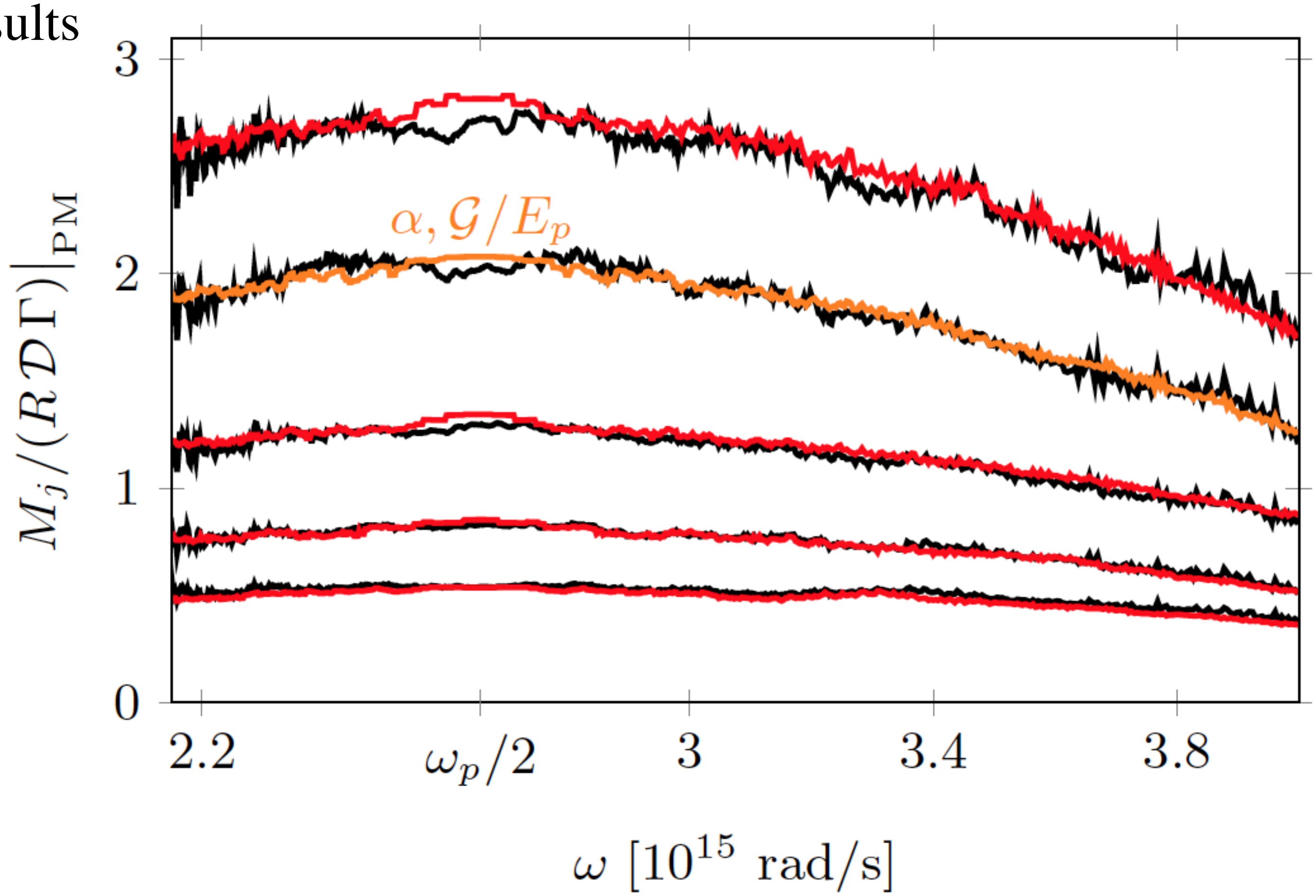
- Finally, we deduce the result

$$\alpha \sinh^2\left(\mathcal{G}\sqrt{\omega(\omega_p - \omega)}\right) = \left.\frac{M(\lambda)}{R(\lambda)\mathcal{D}(\lambda)\,\Gamma}\right|_{\rm PM} \quad \text{where} \quad \Gamma = \Delta\Omega\Delta\lambda\, A_s c \tau_s$$

In this equation, everything is known or directly measurbable except for α, which can thereby be determine.

Absolute Calibration

Laboratory results



Black curve: right-hand side of displayed equation

Red and orange curves: fit to the left-hand side of the equation. The deduced value of α is 0.38.

$$\alpha \sinh^2\left(\mathcal{G}\sqrt{\omega(\omega_p - \omega)}\right) = \left.\frac{M(\lambda)}{R(\lambda)\mathcal{D}(\lambda)\,\Gamma}\right|_{\mathrm{PM}}$$

Conclusions

We have proposed and demonstrated a new protocol for calibrating a spectrophometer.

The method is based on parametric downconversion (PDC).

For a weak pump beam, the emission is induced only by vacuum fluctuation and is known as spontaneous PDC. The emission can provide a relative calibration of a spectrophometer

For a strong pump beam, the emission is initiated by vacuum fluctuation, although the emission is then amplified by parametric gain processes. The process is then stimulated PDC, and when combined with spontaneous PDC can provide an absolute calibration of a spectrophometer.

The advantage of this method is that it relies only on basic quantum physics to provide the calibration

Max Planck Centre for Extreme
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Research Interests:
Nonlinear optics, quantum optics
integrated photonics, metamaterials, etc.

!

Special Thanks To My Students and Postdocs!

Ottawa Group

Rochester Group