Interference pattern produced on reflection at a phase-conjugate mirror. I: Theory

E. Wolf and L. Mandel

Department of Physics and Astronomy, University of Rochester, Rochester, New York 14627

R. W. Boyd

The Institute of Optics, University of Rochester, Rochester, New York 14627

T. M. Habashy

Schlumberger-Doll Research, Ridgefield, Connecticut 06877

M. Nieto-Vesperinas

Instituto de Optica, Consejo Superior de Investigaciones Cientificas, Serrano 121, 28006 Madrid, Spain

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The structure of the fringe pattern that results from the interference between a plane monochromatic wave of any state of polarization incident upon a phase-conjugate mirror and the wave reflected from the mirror is analyzed theoretically. It is found that the locations of the fringe maxima and minima depend on the phase of the incident wave, in contrast to the situation involving an ordinary metal mirror. Some of the results are applied to situations that represent the "phase-conjugate analogs" of classic experiments of O. Wiener on standing waves of light. A comparison is made between his results and those that would be obtained in experiments involving a phase-conjugate mirror in place of an ordinary metallic mirror.

1. INTRODUCTION

In classic experiments carried out toward the end of the last century Wiener[1] demonstrated the existence of standing waves of light and also showed that the photochemical action responsible for the blackening of a photographic plate is directly related to the electric rather than to the magnetic field vector. These experiments involved measurements of the positions of fringes formed by interference between a plane electromagnetic wave incident upon a highly reflecting plane mirror and the wave reflected from the mirror.

In the present paper we investigate theoretically the structure of the interference pattern that is formed when a plane electromagnetic wave is incident upon a phase-conjugate mirror rather than upon an ordinary mirror. We then specialize the results to situations that are analogous to those pertaining to Wiener's experiments.

In Section 2 we derive general expressions for the distribution of the time-averaged electric energy density in the interference pattern formed when a plane electromagnetic wave of any state of polarization and any direction of incidence falls upon a phase-conjugate mirror. The phase-conjugate mirror is of arbitrary reflectivity and is assumed to produce a complete reversal of the state of polarization of the incident wave.

In Section 3 we analyze the structure of the pattern. We find, in particular, that unlike in the situation involving an ordinary mirror, the locations of the intensity maxima and minima depend on the phase of the incident wave.[4,5]

In Section 4 we specialize the results to the situation when the absolute value of the reflectivity of the phase-conjugate mirror is unity. This is the situation that, with certain directions of incidence, is the "phase-conjugate analog" of Wiener's experiments. With an ordinary highly reflecting mirror a standing wave is formed only when a plane wave is incident upon it along the normal to the mirror surface. In contrast to this situation we find that with a phase-conjugate mirror a standing wave is formed irrespective of the angle of incidence.

Some of our results can be understood from qualitative considerations based on the well-known action of a phase-conjugate mirror in reflecting the incident wave back upon itself. For this reason the form of the interference pattern does not depend on the angle of incidence. Other results are, however, intuitively less obvious, and their derivation needs a more detailed mathematical analysis such as is presented in this paper. Examples are the dependence of the fringe visibility on the reflectivity of the phase-conjugate mirror and on the state of polarization of the incident field.

Some of our predictions have been recently confirmed by experiment.[6,7]

2. DISTRIBUTION OF THE TIME-AVERAGED ELECTRIC ENERGY DENSITY PRODUCED BY REFLECTION OF A PLANE WAVE AT A PHASE-CONJUGATE MIRROR

Let[8]

$$\mathbf{E}^{(i)}(\mathbf{r}, t) = \epsilon A^{(i)} \exp[i(\mathbf{k}_i \cdot \mathbf{r} - \omega t)] \tag{2.1}$$

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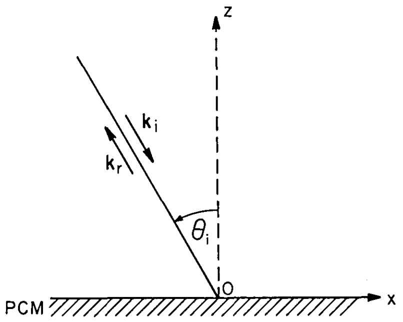

Fig. 1. Reflection of a monochromatic plane wave at a PCM. Plane of incidence. $\mathbf{k}_i$ is the wave vector of the incident wave, and $\mathbf{k}_r = -\mathbf{k}_i$ is the wave vector of the wave that is reflected at the PCM.

be the electric field at a point $\mathbf{r}$ at time t of a plane monochromatic wave with wave vector $\mathbf{k}_i$ and frequency ω incident upon a plane phase-conjugate mirror (PCM) located in the plane $z = 0$. We take the positive z direction to point into the half-space from which the wave is incident upon the PCM (see Fig. 1). In Eq. (2.1) ϵ denotes a unit polarization vector:

$$\epsilon^* \cdot \epsilon = 1. \tag{2.2}$$

For a linearly polarized wave ϵ may be taken to be real, but ϵ will be complex for more general states of polarization. For example, for a circularly polarized wave we may take $\epsilon = (\epsilon_1 + i\epsilon_2)/\sqrt{2}$, where ϵ_1 and ϵ_2 are real, mutually orthogonal unit vectors. The amplitude factor $A^{(i)}$ in Eq. (2.1) is a (generally complex) constant.

We assume that the PCM is ideal in the sense that the electric field of the reflected wave leaving the PCM is given by[9–12]

$$\mathbf{E}^{(r)}(\mathbf{r}, t) = \mu\epsilon^* A^{(i)*} \exp[i(-\mathbf{k}_i \cdot \mathbf{r} - \omega t)]. \tag{2.3}$$

Here

$$\mu = |\mu| e^{i\phi} \tag{2.4}$$

denotes the amplitude reflectivity of the PCM. According to this definition of an ideal PCM, the wave vector $\mathbf{k}_i$ is reversed, and the complex amplitude $A^{(i)}$ is replaced by its complex conjugate in the reflection process. We have also assumed that the PCM produces a reversal of the state of polarization of the incident wave, which implies that ϵ is replaced by ϵ^*. PCM's having this property are sometimes referred to as "vector phase-conjugate mirrors."

From Eqs. (2.1) and (2.3) it follows that the total electric field $\mathbf{E} = \mathbf{E}^{(i)} + \mathbf{E}^{(r)}$ in the half-space $z > 0$ in front of the PCM is given by

$$\mathbf{E}(\mathbf{r}, t) = [\epsilon A^{(i)} \exp(i\mathbf{k}_i \cdot \mathbf{r}) + \mu\epsilon^* A^{(i)*} \exp(-i\mathbf{k}_i \cdot \mathbf{r})] e^{-i\omega t}. \tag{2.5}$$

Let us now consider the time-averaged electric energy density of the total field at a typical point in the half-space $z > 0$. It is given by the expression [cf. Ref. 2, Sec. 1.4, Eq. (54)] $\langle w_e \rangle = (1/16\pi)\mathbf{E} \cdot \mathbf{E}^*$, which is independent of time. On substituting into this expression from Eq. (2.5) and using Eq. (2.2) we readily find that

$$\langle W_e(\mathbf{r}) \rangle = \frac{1}{16\pi} \{(1 + |\mu|^2)|A^{(i)}|^2 + [\epsilon^2 \mu^* A^{(i)2} \exp(2i\mathbf{k}_i \cdot \mathbf{r}) + \text{c.c.}]\}, \tag{2.6}$$

where c.c. denotes the complex conjugate. It will be useful to set

$$A^{(i)} = |A^{(i)}| e^{i\alpha}, \qquad \epsilon^2 = |\epsilon^2| e^{i\delta}. \tag{2.7}$$

If we make use of Eqs. (2.7) and (2.3), the expression (2.6) for the time-averaged electric energy density becomes, if we also use an elementary trigonometric identity,

$$\langle W_e(\mathbf{r}) \rangle = \frac{|A^{(i)}|^2}{16\pi} [1 + |\mu|^2 + 2|\mu|\,|\epsilon^2| \times \cos(2\mathbf{k}_i \cdot \mathbf{r} + 2\alpha - \phi + \delta)]. \tag{2.8}$$

3. STRUCTURE OF THE INTERFERENCE PATTERN

We will now analyze the structure of the interference pattern in front of the PCM. We see from Eq. (2.8) that $\langle W_e(\mathbf{r}) \rangle$ has a constant value when

$$2\mathbf{k}_i \cdot \mathbf{r} + 2\alpha - \phi + \delta = \text{constant}, \tag{3.1}$$

i.e., along planes perpendicular to the direction of propagation $\mathbf{k}_i$ of the incident wave. In directions perpendicular to these planes $\langle W_e \rangle$ varies sinusoidally between the values

$$\langle W_e \rangle_{\max} = \frac{|A^{(i)}|^2}{16\pi} [1 + |\mu|^2 + 2|\mu|\,|\epsilon^2|] \tag{3.2a}$$

and

$$\langle W_e \rangle_{\min} = \frac{|A^{(i)}|^2}{16\pi} [1 + |\mu|^2 - 2|\mu|\,|\epsilon^2|], \tag{3.2b}$$

with the period

$$\Delta = \frac{\pi c}{\omega} = \frac{\lambda}{2}, \tag{3.3}$$

where λ is the wavelength of the incident wave.

As is customary, we will characterize the sharpness of the interference fringes by the visibility $\mathcal{V}$, defined as

$$\mathcal{V} = \frac{\langle W_e \rangle_{\max} - \langle W_e \rangle_{\min}}{\langle W_e \rangle_{\max} + \langle W_e \rangle_{\min}}. \tag{3.4}$$

On substituting from Eqs. (3.2) into Eq. (3.4), we find that in the present case

$$\mathcal{V} = \frac{2|\mu|}{1 + |\mu|^2} |\epsilon^2|. \tag{3.5}$$

Some interesting consequences can readily be derived from the formula (3.5). Since $|\epsilon^2| \leqslant \epsilon^* \cdot \epsilon = 1$, it is clear that (with μ fixed) the visibility will attain its maximum value when $|\epsilon^2| = 1$. In view of Eq. (2.2) this condition can readily be shown to be satisfied only when the incident field is *linearly polarized*, and we then have from Eq. (3.5) that

$$\mathcal{V} = \frac{2|\mu|}{1 + |\mu|^2}. \tag{3.6}$$

In Fig. 2 the visibility $\mathcal{V}$ given by this expression is plotted as a function of the absolute value $|\mu|$ of the reflectivity of the PCM. We see that in the range $0 \leqslant |\mu| \leqslant 1$ the visibility increases monotonically from the value $\mathcal{V} = 0$ when $\mu = 0$ to its maximum value $\mathcal{V} = 1$ when $|\mu| = 1$, i.e., when the reflected wave is generated by the PCM without any losses or gains.

We also see from Eq. (3.5) that irrespective of the value of

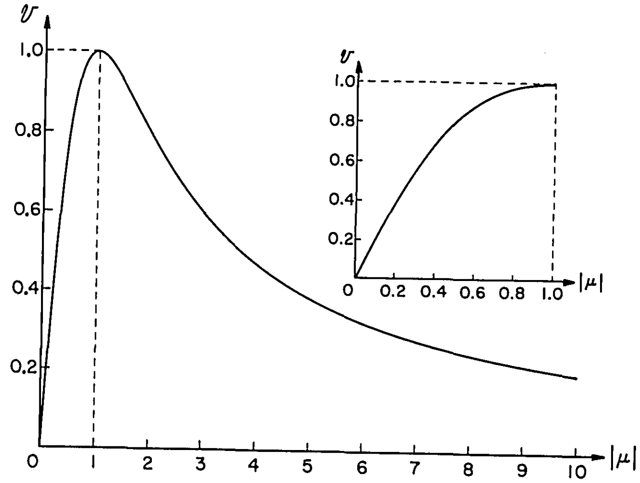

Fig. 2. Visibility $\mathcal{V}$ of the interference fringes as a function of the absolute $|\mu|$ of the reflectivity of the PCM [Eq. (3.6)].

μ the visibility will be zero when $|\epsilon^2| = 0$. It can be readily verified [cf. the remarks under Eq. (2.2)] that this will be the case when the incident wave is *circularly polarized*. Thus we conclude that when a circularly polarized wave is incident upon the PCM no interference fringes are formed in front of it, irrespective of the value of the reflectivity of the PCM, as is also seen at once by setting $|\epsilon^2| = 0$ in Eq. (2.8).

Let us return to the situation when the incident wave is linearly polarized. As we have seen, one then has $|\epsilon^2| = 1$; this implies, according to the second expression in Eqs. (2.7), that $\delta = 2m\pi$, where m is an integer. Under these circumstances the expression (2.8) for the time-averaged electric density may be expressed in the form

$$\langle W_e(\mathbf{r})\rangle = \frac{|A^{(i)}|^2}{16\pi}\,[(1-|\mu|)^2 + 4|\mu|\cos^2(\mathbf{k}_i\cdot\mathbf{r} - \frac{\phi}{2} + \alpha + m\pi)]. \tag{3.7}$$

It follows at once from this formula that the locations of the maxima of the fringe pattern are given by

$$\mathbf{k}_i\cdot\mathbf{r} - (\phi/2) + \alpha = n\pi \qquad (n = 0, \pm1, \pm2, \ldots) \tag{3.8a}$$

and the locations of the minima are given by

$$\mathbf{k}_i\cdot\mathbf{r} - (\phi/2) + \alpha = [n + (1/2)]\pi \qquad (n = 0, \pm1, \pm2, \ldots). \tag{3.8b}$$

We see from these two formulas that the positions of the maxima and the minima in the fringe pattern depend on the phase α of the incident light and on the phase ϕ of the complex reflectivity of the PCM and are independent of the absolute value $|\mu|$ of the reflectivity of the PCM. Thus the measurements of the locations of the maxima and minima would provide information about the phase of the incident light, which appears at first sight to be unphysical because the phase depends on the choice of origin for space and time. However, before concluding that the positions of the interference fringes depend on the origin, we need to recall that the PCM is not a passive device but relies on the presence of a pump electromagnetic field. As a result, both α and ϕ depend on the origin of space and time.

In order to make these considerations a little more explicit, let us consider the simple but important case in which the PCM is produced by degenerate four-wave mixing in a nonlinear medium, with all the four waves being linearly polarized perpendicular to the plane of incidence (TE waves). Let ϵ_0 be a real unit vector that characterizes the (linear) polarization. Let $\mathbf{E}^{(\mathrm{I})}(\mathbf{r}, t)$ and $\mathbf{E}^{(\mathrm{II})}(\mathbf{r}, t)$ represent the two counterpropagating pump waves with wave vectors $\mathbf{k}_0$ and $-\mathbf{k}_0$ (see Fig. 3), for which we write

$$\mathbf{E}^{(\mathrm{I})}(\mathbf{r}, t) = \epsilon_0 A^{(\mathrm{I})}\exp[i(\mathbf{k}_0\cdot\mathbf{r} - \omega t)],$$
$$\mathbf{E}^{(\mathrm{II})}(\mathbf{r}, t) = \epsilon_0 A^{(\mathrm{II})}\exp[i(-\mathbf{k}_0\cdot\mathbf{r} - \omega t)]. \tag{3.9}$$

As is well known,[13] in the weak-field limit $|\mu| \ll 1$ the complex reflectivity μ of the PCM is given by

$$\mu = igLA^{(\mathrm{I})}A^{(\mathrm{II})}, \tag{3.10}$$

where L is the thickness of the optically pumped nonlinear medium and g is a coupling constant proportional to the nonlinear susceptibility. It then follows on substituting the expression (3.10) for μ into the expression (2.3) for $\mathbf{E}^{(r)}(\mathbf{r}, t)$ that the electric field of the reflected wave is given by

$$\mathbf{E}^{(r)}(\mathbf{r}, t) = -igL\epsilon_0 A^{(\mathrm{I})}A^{(\mathrm{II})}A^{(i)*}\exp[i(-\mathbf{k}_i\cdot\mathbf{r} - \omega t)]. \tag{3.11}$$

Suppose now that we displace the origin of space by $\Delta\mathbf{r}$ and the origin of time by Δt. As this is a purely formal notational change, one would not expect this to change the reflected wave in any way, despite the explicit appearance of the phase of the incident wave in Eq. (3.11). Let us check this point by making use of Eq. (3.11). Under the translation of origins, the physical fields $\mathbf{E}^{(\mathrm{I})}$, $\mathbf{E}^{(\mathrm{II})}$, and $\mathbf{E}^{(i)}$ do not change, so the complex amplitudes $A^{(\mathrm{I})}$, $A^{(\mathrm{II})}$, and $A^{(i)}$ must change to $A^{(\mathrm{I})\prime}$, $A^{(\mathrm{II})\prime}$, $A^{(i)\prime}$, such that

$$A^{(\mathrm{I})}\exp[i(\mathbf{k}_0\cdot\mathbf{r} - \omega t)] = A^{(\mathrm{I})\prime}\exp\{i[\mathbf{k}_0\cdot(\mathbf{r} - \Delta\mathbf{r}) - \omega(t - \Delta t)]\},$$
$$A^{(\mathrm{II})}\exp[i(-\mathbf{k}_0\cdot\mathbf{r} - \omega t)] = A^{(\mathrm{II})\prime}\exp\{i[-\mathbf{k}_0\cdot(\mathbf{r} - \Delta\mathbf{r}) - \omega(t - \Delta t)]\},$$
$$A^{(i)}\exp[i(\mathbf{k}_i\cdot\mathbf{r} - \omega t)] = A^{(i)\prime}\exp\{i[\mathbf{k}_i\cdot(\mathbf{r} - \Delta\mathbf{r}) - \omega(t - \Delta t)]\},$$

or

$$A^{(\mathrm{I})} = A^{(\mathrm{I})\prime}\exp[i(-\mathbf{k}_0\cdot\Delta\mathbf{r} + \omega\Delta t)],$$
$$A^{(\mathrm{II})} = A^{(\mathrm{II})\prime}\exp[i(\mathbf{k}_0\cdot\Delta\mathbf{r} + \omega\Delta t)],$$
$$A^{(i)} = A^{(i)\prime}\exp[i(-\mathbf{k}_i\cdot\Delta\mathbf{r} + \omega\Delta t)]. \tag{3.12}$$

If we now substitute for $A^{(\mathrm{I})}$, $A^{(\mathrm{II})}$, and $A^{(i)}$ from Eqs. (3.12) into Eq. (3.11), we obtain

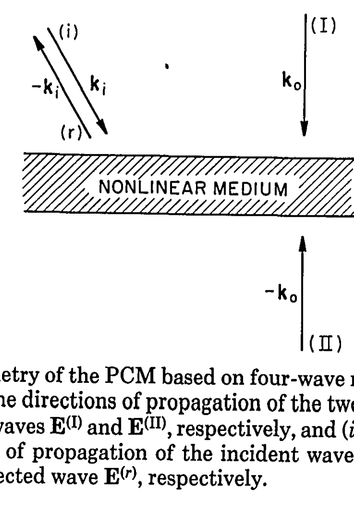

Fig. 3. Geometry of the PCM based on four-wave mixing. (I) and (II) indicate the directions of propagation of the two counterpropagating pump waves $\mathbf{E}^{(\mathrm{I})}$ and $\mathbf{E}^{(\mathrm{II})}$, respectively, and ($i$) and ($r$) denote the directions of propagation of the incident wave $\mathbf{E}^{(i)}$ and of the conjugate reflected wave $\mathbf{E}^{(r)}$, respectively.

$$\mathbf{E}^{(r)}(\mathbf{r}, t) = -igL\epsilon_0 A^{(\mathrm{I})'} A^{(\mathrm{II})'} A^{(i)'*} \times \exp\{i[-\mathbf{k}_i \cdot (\mathbf{r} - \Delta\mathbf{r}) - \omega(t - \Delta t)]\}. \quad (3.13)$$

The right-hand side of Eq. (3.13) represents precisely the electric field of the reflected wave, expressed in terms of the transformed variables, and it is evidently the same wave as in Eq. (3.11). From this it follows that the nature of the interference patterns described by Eq. (2.8) is independent of the origin of space and time and therefore independent of the absolute phase in the strict sense. Nevertheless, through the difference $\alpha - (\phi/2)$ the position of the interference fringes depends on the difference between the phases of the incident wave and of the complex response of the PCM. The fringes will move if the phase of the incident wave is retarded by use of a phase retarder, for example, and this was recently demonstrated experimentally.[6,7] These conclusions are unchanged if $|\mu|$ is not necessarily small compared with unity, although the expression for the reflectivity μ of the PCM then becomes more complicated.

We shall now examine in more detail the case in which $|\mu| = 1$, when the fringe visibility is greatest.

4. STRUCTURE OF THE INTERFERENCE PATTERN WHEN $|\mu| = 1$. COMPARISON WITH WIENER'S EXPERIMENTS

When $|\mu| = 1$, the reflected wave is generated at the PCM without any losses or gains. This situation, which in a sense is analogous to reflection at a perfect conductor, is of particular interest in applications of the technique of phase conjugation. It has been shown[12,14] that under these circumstances the effect of distortion imparted on an incident wave by scattering on a dielectric is completely eliminated by phase conjugation.[15] This situation ($|\mu| = 1$) is also of interest, because for certain directions of incidence it represents the "phase-conjugate analog" of classic experiments of Wiener[1] by which he demonstrated the existence of standing waves of light and by which he showed that the blackening of a photographic plate (i.e., the photochemical action) is caused by the electric and not by the magnetic field. In this section we specialize some of our formulas to the situation when $|\mu| = 1$, and we will discuss their consequences.

When $|\mu| = 1$, Eq. (2.5) for the components of the total electric field in the half-space $z > 0$ in front of the PCM become

$$\mathbf{E}(\mathbf{r}, t) = [\epsilon A^{(i)} \exp(i\mathbf{k}_i \cdot \mathbf{r}) + e^{i\phi}\epsilon^* A^{(i)*} \exp(-i\mathbf{k}_i \cdot \mathbf{r})]e^{-i\omega t}, \quad (4.1)$$

where Eq. (2.3) was used. The expression (4.1) may be rewritten in the form

$$\mathbf{E}(\mathbf{r}, t) = 2\{\mathbf{R}\epsilon A^{(i)} \exp[i(\mathbf{k}_i \cdot \mathbf{r} - \phi/2)]\}\exp\{i[(\phi/2) - \omega t]\}. \quad (4.2)$$

We see that the electric field is now given by a product of a function of position and a function of time. Hence the wave in the half-space $z > 0$ is a *standing wave*, irrespective of the angle of incidence θ_i. This is in contrast to the case of reflection from an ordinary, perfectly reflecting plane mirror, in which a standing wave is formed only when the wave is incident upon it in the direction of the normal ($\theta_i = 0$). The formation of the standing wave for any angle of incidence, with a PCM that generates the conjugate wave without losses and gains (i.e., when $|\mu| = 1$), plays a basic role in the elimination of distortions by the technique of phase conjugation.[12,15]

The time-averaged electric energy density is in this case given by Eq. (2.8) with $|\mu| = 1$, viz.,

$$\langle W_e(\mathbf{r})\rangle = \frac{|A^{(i)}|^2}{8\pi}\{1 - |\epsilon^2| + 2|\epsilon^2| \cos^2[\mathbf{k}_i \cdot \mathbf{r} + \alpha - (\phi/2) + (\delta/2)]\}. \quad (4.3)$$

Let us now specialize some of our results to two cases that are related to Wiener's classic experiments involving an ordinary mirror.

A. Normal Incidence ($\theta_i = 0$, $|\mu| = 1$)

When the incident wave falls normally upon the PCM, the wave vector $\mathbf{k}_i$ has Cartesian components $(0, 0, -k_i)$, where $k_i = 2\pi/\lambda$, λ being the wavelength. For the sake of simplicity we assume that the electric field $\mathbf{E}^{(i)}$ is linearly polarized, in the x direction, say. The unit polarization vector ϵ [cf. Eq. (2.1)] may then be taken to be the real unit vector along that direction. It follows at once from Eq. (3.8) that the locations of the maxima of the time-averaged electric energy density are now given by[16]

$$z = \frac{n\lambda}{2} + \frac{\lambda}{2\pi}[(\phi/2) - \alpha] \qquad (n = 0, 1, 2, \ldots) \quad (4.4a)$$

and the locations of the minima are given by

$$z = \frac{(n + 1/2)\lambda}{2} + \frac{\lambda}{2\pi}[(\phi/2) - \alpha] \qquad (n = 0, 1, 2, \ldots). \quad (4.4b)$$

The planes specified by Eqs. (4.4a) and (4.4b) are, of course, also the locations of the antinodes and of the nodes, respectively, of the electric field.

In the classic experiments of Wiener, mentioned at the beginning of this section, an ordinary, almost perfectly reflecting mirror M was illuminated by a normally incident, linearly polarized electromagnetic wave. A standing wave was formed by interference of the incident and the reflected wave, with the antinodes of the electric field being at distances

$$z = [n + (1/2)]\frac{\lambda}{2} \qquad (n = 0, 1, 2, \ldots) \quad (4.5a)$$

and the nodes at distances

$$z = n\frac{\lambda}{2} \qquad (n = 0, 1, 2, \ldots) \quad (4.5b)$$

in front of the mirror. The antinodes of the magnetic field then coincided with the nodes of the electric field and vice versa. Wiener placed a glass plate G in front of the mirror M. The plate was coated with a film of transparent photographic emulsion of thickness less than $\lambda/20$ and was inclined to the mirror at a small angle [exaggerated in Fig. 4(a)]. On development, the emulsion was found to be blackened along equidistant parallel bands, with transparent regions between them. The maxima of blackening coincided with the intersection of F with the antinodal planes of the electric field. These and related experiments demonstrated that photochemical action is directly caused by the electric field, a result that came to be fully understood later, from electron theory.

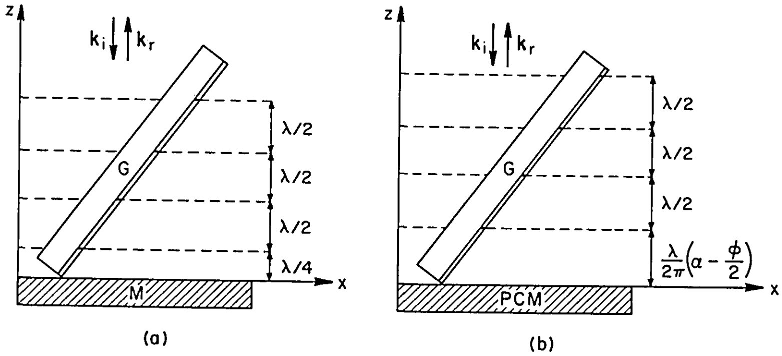

Fig. 4. (a) Wiener's experiments on standing waves produced on reflection from a highly reflecting mirror M. The horizontal lines indicate the position of the maxima of the time-averaged electric energy density. (b) Predicted locations of the maxima in similar experiments performed with a PCM rather than with an ordinary mirror. α denotes the phase of the complex amplitude $A^{(i)}$ of the incident wave, linearly polarized along the x direction, and ϕ denotes the phase of the complex reflectivity μ of the PCM.

Suppose that we would carry out similar experiments to Wiener's but with the ordinary mirror M replaced by a PCM [Fig. 4(b)]. The photographic plate would then again be blackened along equidistant parallel bands, but the maxima of the blackening would now be given by Eq. (4.4a) rather than by Eq. (4.5a). Their locations depend not only on the wavelength λ but also on the phase α of the complex amplitude $A^{(i)}$ of the electric vector of the incident field and on the phase ϕ of the complex reflectivity of the PCM, as was to be expected from the remarks made at the end of Section 3.

B. Incidence at 45° ($\theta_i = 45°$, $|\mu| = 1$)

Next we consider the situation when the incident wave falls upon the PCM at 45°. We again assume that its electric vector is linearly polarized.

Since the wave vector $\mathbf{k}_i$ now has the Cartesian components $(k_i/\sqrt{2}, 0, -k_i/\sqrt{2})$, it readily follows from Eqs. (3.8) that the locations of the maxima of the time-averaged electric energy density are now given by

$$x - z = \frac{n\lambda}{\sqrt{2}} + \frac{\lambda}{\pi\sqrt{2}}[\phi/2 - \alpha] \qquad (n = 0, \pm 1, \pm 2, \ldots), \tag{4.6a}$$

and the locations of the minima are given by

$$x - z = \frac{(n + 1/2)\lambda}{\sqrt{2}} + \frac{\lambda}{\pi\sqrt{2}}[(\phi/2) - \alpha] \qquad (n = 0, \pm 1, \pm 2, \ldots). \tag{4.6b}$$

These two formulas show that the positions of the interference fringes again depend on the phase α of the incident wave. This result, too, is in contrast with the corresponding situation when an ordinary, highly reflecting mirror is employed in place of the PCM, a situation that was also investigated experimentally by Wiener. Moreover, with an ordinary mirror, the nature of the interference pattern formed in front of the mirror depends on the angle that the (linearly polarized) electric vector of the incident field makes with the plane of incidence. In particular, Wiener found in agreement with theory [cf. Ref. 2, Sec. 7.4, Eqs. (12)–(14)] that, when the incident wave is linearly polarized with its electric vector perpendicular to the plane of incidence, the interference fringes are parallel to the plane of the mirror; and that, when it is polarized in the plane of incidence, no fringes are formed, the time-averaged energy density then being constant throughout the half-space $z > 0$. With a PCM, on the other hand, as we see from Eqs. (4.6), interference fringes will be formed irrespective of the angle that the electric vector makes with the plane of incidence, and they are perpendicular to the direction of incidence.

5. DISCUSSION

We have investigated the structure of the fringe pattern produced by interference between a plane electromagnetic wave of any state of polarization falling upon a PCM at any direction of incidence and the wave leaving the mirror. We have found that the position of the fringes depends on the phase of the incident wave. This result is in contrast to that which occurs with an ordinary metal mirror. In that case the positions of the fringes are independent of the phase of the incident wave. We have shown that for the case of a PCM based on degenerate four-wave mixing, the phases of the waves used to pump the mirror provide a reference with respect to which the phase of the incident wave is determined. Hence, while the positions of the fringes can be used to determine the phase of the incident field, the phase is determined only with respect to the phases of the pump waves, and in this respect the situation is somewhat similar to that encountered in traditional interferometers.

We have also shown that with a PCM the maximum fringe visibility is obtained when the incident field is linearly polarized and that no fringes are formed when it is circularly polarized.

We also considered the special case when the absolute value of the reflectivity of the PCM is unity, i.e., when the conjugate wave is generated without any loss or gain of energy. For certain directions of incidence and states of polarization these situations are analogous to those pertaining to Wiener's classic experiments involving an ordinary, highly reflecting mirror. We have compared Wiener's results with those that would be obtained when a PCM rather than an ordinary mirror is used.

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16. When $(\phi/2) - \alpha$ $(0 \leqslant \phi/2 - \alpha < 2\pi)$ exceeds π, the value $n = -1$ also gives the location of a maximum, and when it exceeds $\pi/2$, the value $n = -1$ also gives a location of a minimum.