Design of negative-refractive-index materials on the basis of rods with a gradient of the dielectric constant

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Design of negative-refractive-index materials on the basis of rods with a gradient of the dielectric constant

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The authors propose an approach to the design of negative-refractive-index materials based on the use of dielectric rods with a gradient of the dielectric constant. A triangular-lattice photonic crystal assembled from multilayer dielectric rods with a refractive index approximating a fish-eye profile is shown to exhibit a negative refractive index in the wavelength range defined by the inequality $0.67 < a/\lambda < 0.83$, where a is the lattice constant of the photonic crystal. A lens consisting of a plane-parallel slab of such a photonic crystal slab is shown to be able to form an image of a point source in this wavelength range. According to the calculations, particularly high-quality images can be obtained at the wavelength $\lambda = (3/2)a$, where the fish-eye dielectric rods scatter the light like a medium with the refractive index equal to −0.85. © *2007 American Institute of Physics*.

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The concept of left-handed electromagnetic media, which are also known as negative-index materials (NIMs), was introduced by Veselago[1] as a theoretical curiosity. Interest in these metamaterials was rejuvenated by Pendry[2] and Smith *et al.*,[3] who noted that the growth of evanescent fields within a NIM provides the opportunity for building a "perfect lens" that can focus electromagnetic waves to a spot size much smaller than a wavelength. Although various aspects of Pendry's treatment of NIMs have been questioned,[4,5] negative refraction has nonetheless been confirmed in recent experiments,[6–8] and its theoretical background has been further explored.[9–11] Negative-index materials have recently been designed on the basis of composite wire and split ring resonator structures,[3,7,12] backward-wave transmission lines,[13] and photonic-band-gap crystals.[10,14,15] In the work of Leonhardt and Philbin, the use of inhomogeneous spheres is proposed for NIM at wavelengths much smaller than the sphere radius.[16]

Usually, NIMs based on dielectrics are designed from periodically arranged homogenous materials. In such a case, for instance, thick slabs consisting of a large number of rods are required for realizing good focusing. In the present work, we propose the design of a NIM lens from dielectric rods with a specific dielectric constant profile. This design leads to comparable or even improved focusing from much thinner slabs thus containing a much smaller number of rods.

The proposed approach is based on using dielectric rods which themselves possess a negative refractive index at definite wavelengths. The approach consists of the following steps: (i) the design of elementary units (rods) exhibiting properties of NIM; and (ii) different periodic or quasiperiodic structures assembled from these elementary units are considered, and their properties are calculated numerically with the goal of producing negative refraction and optimizing the focusing effect. A highly efficient and accurate multiple-scattering approach[17] is used to calculate propagation of electromagnetic waves through these structures.

The elementary building blocks of our design are dielectric rods with a changeable refractive index. Its gradient resembles a "fish-eye" profile[18] given by

$$n(r) = \frac{n_0}{1 + (r/r_0)^2}, \tag{1}$$

where r is the distance from the center of the rod and n_0, r_0 are constants. In such a material light propagates in circular (or spiral) trajectories with a radius comparable to the quantity r_0, i.e., a medium with the fish-eye dielectric constant profile behaves like a NIM from the point of view of light scattering. Consider the design of dielectric rods from the point of view of light scattering. We use an approach based on the effective medium concept[19] to choose appropriate parameters for the dielectric profile of the rod. This method relies on using a hypothetic background medium with variable index of refraction in which the dielectric rod is immersed. The scattering cross section of the rod is calculated as a function of the refractive index of the background medium. It is obvious that the scattering cross section should exhibit a minimum when the refractive index of the background medium approaches that of the rod under investigation. For practical purposes, we approximate the fish-eye medium by use of several discrete layers of different refractive indices. We analyzed the scattering cross section for a rod which consists of three layers with the radii $r_1 = 0.5a$, $r_2 = 0.25a$, and $r_3 = 0.1a$, and refractive indices $n_1 = 1.5$, $n_2 = 3$, and $n_3 = 4$.

Analogous calculations were performed for a specific wavelength interval. The effective refractive index of the rod as a function of the radiation wavelength is illustrated in Fig.

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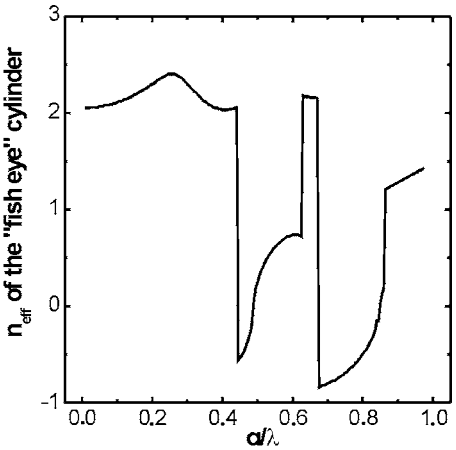

FIG. 1. Dependence of the refractive index on the radiation wavelength for a "fish-eye" rod consisting of three layers.

1. One can see that within the wavelength ranges given by $0.67 < a/\lambda < 0.83$ and $0.44 < a/\lambda < 0.49$, the rod involved behaves like a NIM from the point of view of light scattering, exhibiting the best properties at wavelengths around $\lambda = a/0.67$. The calculations performed for the wavelength $\lambda = a/0.67$ showed a pronounced minimum of the scattering cross section around the refractive index value of −0.85, meaning that at this wavelength the rod behaves as a NIM with the refractive index equal to −0.85.

Now let us next consider a two-dimensional photonic crystal assembled from the fish-eye rods with the parameters described above as elementary units. We present results for a photonic crystal possessing a triangular lattice with the lattice constant a, i.e., the rods are arranged in a manner resulting in minimum voids between the rods. We hypothesize that minimizing the void volume, i.e., the volume characterized by positive refractive index, is important from the point of view of designing a NIM. We have calculated the band structure and have simulated the focusing properties of a triangular-lattice photonic crystal slab having dimensions of $4.3a \times 16.0a$ and consisting of 90 cylindrical rods. The band structure calculations (Fig. 2) suggest that the considered slab may exhibit negative refractive index at wavelengths around $\lambda = a/0.5$ and $\lambda = a/0.7$, i.e., good focusing properties of the slab are expected at frequencies a/λ slightly below 0.5 and 0.7. At the same time the calculations do not predict focusing at frequencies $0.50 < a/\lambda < 0.65$ and $a/\lambda > 0.7$.

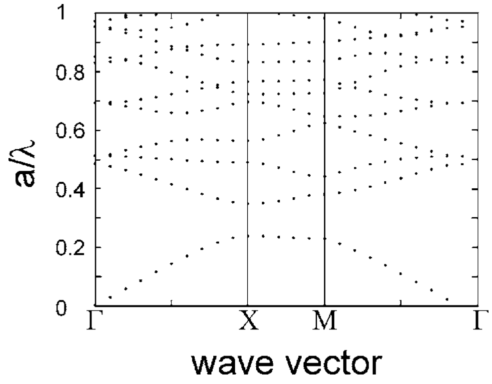

FIG. 2. Photonic band structure of two-dimensional (2D) PC consisting of triangular lattice of fish-eye rods in vacuum.

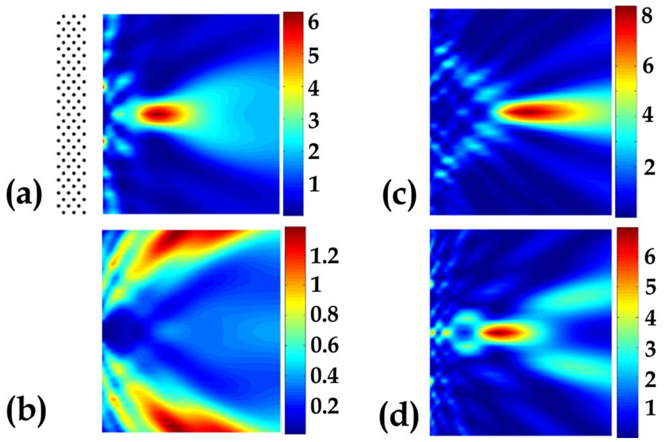

FIG. 3. (Color online) Electric field intensity map of a cross-sectional view of the 2D source-image system when imaging by a triangular-lattice photonic crystal slab consisting of fish-eye rods at the radiation wavelengths $\lambda = a/0.45$ (a), $\lambda = a/0.55$ (b), $\lambda = a/0.67$ (c), and $\lambda = a/0.75$ (d).

Figure 3 shows the image of a point source placed at the distance $3a$ in front of the considered photonic crystal slab at different radiation wavelengths. The pattern represents the distribution of the transmitted electromagnetic (EM) power $T = (E/E_0)^2$, where E is the electric field amplitude of the radiation passed through the photonic crystal slab and E_0 is the electric field amplitude without the photonic crystal slab. The calculations are performed for a polarization with the electric field vector E parallel to the axis of the cylindrical rods. One can see that, as predicted by both the band structure calculations (Fig. 2) and the analysis of the rod properties (Fig. 1), good focusing occurs at wavelengths $\lambda = a/0.45$ and $\lambda = a/0.67$, and no focusing is observed at $\lambda = a/0.55$. At the same time, according to band structure calculations, the slab should not demonstrate focusing at the wavelength $\lambda = a/0.75$, while the rods still scatter light like media with a negative refractive index at this wavelength (Fig. 1). One can see from Fig. 3(d) that the PC slab shows good focusing properties at the wavelength $\lambda = a/0.75$, in accordance with data of Fig. 1. Apart from that, the investigation of imaging properties of the slab demonstrates its inability to focus radiation at frequencies $a/\lambda < 0.44$ and $a/\lambda > 0.85$.

We believe that the properties of individual rods strongly influence the photonic properties of the designed slabs. The design of PC on the basis of rods which behave like a NIM from the point of view of light scattering offers a number of advantages. The analysis of the scattering cross section of rods suggests the frequency ranges where good focusing properties of the assembled PC are expected. Nevertheless, one should mention that the frequency range where the rods behave like a NIM should not necessarily totally coincide with the frequency ranges where the assembled PC exhibits good focusing properties, since the focusing effect of the PC slab is also determined by the arrangement of the rods. It is well known (see, for instance, Ref. 8) that focusing is also achieved by flat photonic crystal lenses constructed from homogenous rods with positive refractive index. However, thick slabs consisting of a big number of rods are generally necessary. For instance, we have performed calculations for a PC slab of dimensions and rod sizes identical to those of Fig. 3 but assembled from homogenous rods with positive refractive index from 1 to 4 and have found no focusing in the

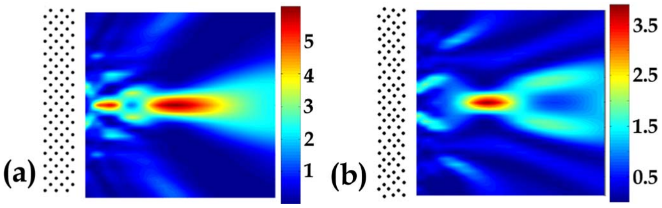

FIG. 4. (Color online) Distribution of the transmitted EM power through a photonic crystal slab constructed from homogenous rods with negative refractive index $n=-0.8$ with perfect triangular lattice (a) and with a degree of disorder of 10% in both the arrangement of rods and their size (b).

frequency interval $0<a/\lambda<1$. Design of flat lenses on the basis of rods with special dielectric constant profile allows one to achieve better focusing effect with much thinner slabs. In addition, these lenses work in a much broader wavelength range.

Another advantage of PC constructed from rods which behave like a NIM is the enhanced tolerance to the disorder and the cylinder size irregularity. Note that for the purpose of comparison we will consider a PC with rods of homogenous NIM in order to reduce the computational complexity. However, this is all done under the strong assumption that the PCs from fish-eye and homogenous NIM cylinders do have the similar optical properties (provided that the refractive indices are the same). In Fig. 4(a) we show the distribution of the transmitted EM power for slabs of dimensions and rod sizes identical to those of Fig. 3, but constructed from homogenous rods with negative refractive index equal to −0.8. One can see that the imaging by this slab is comparable to the imaging with the lens of Fig. 3(c). The introduction of a degree of disorder of 10% in both the arrangement of rods and their size [Fig. 4(b)] slightly diminishes the focusing power of the slab, but the focusing properties still remain good enough. One should mention that the introduction of such a degree of disorder in a PC constructed from homogenous rods with positive refractive index would destroy the focusing properties. We suppose that the decrease of the focusing power of the slab in Fig. 4(b) is caused by PC effects as well as by the increase of the volume of voids (i.e., the volume characterized by positive refractive index), which is the result of the deviation from the closely packed lattice.

According to our calculations, a photonic crystal slab with a square lattice designed from the same fish-eye rods also shows a clear focusing effect. However, the quality of the image is not as good as that of the photonic crystal with a triangular lattice. We attribute this phenomenon to the larger volume of voids inherent to the square lattice. The void volumes are 22% and 9% for the square and triangular lattices, respectively.

In conclusion, the results of our calculations show that dielectric rods with a fish-eye refractive index profile are promising for designing and manufacturing negative-index materials. The NIM designed in this work on the basis of multilayer dielectric rods is suitable for focusing the EM radiation in a relatively large wavelength range. At present, we are assembling a flat lens made from fish-eye dielectric rods for the purpose of testing the calculations in the microwave region. Note that photonic crystal lenses made from homogeneous dielectric rods were demonstrated recently to focus microwaves at specific wavelengths.[20]

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