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Focusing slabs made of negative index materials based on inhomogeneous dielectric rods

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We show the advantages of using inhomogeneous dielectric rods to design photonic crystals that behave as materials with negative refractive index. We found that the analysis of light scattering properties of inhomogeneous dielectric rods allows one to estimate the interval of frequencies where a photonic crystal exhibits negative refractive index. A triangular-lattice photonic crystal – assembled from multilayer dielectric rods designed to approximate a fish-eye profile – is shown to exhibit negative refractive index and good focusing properties at frequencies where the fish-eye dielectric rods scatter the light like a medium with negative refractive index.

1 Introduction The concept of left-handed electromagnetic media, which are also known as negative-index materials (NIMs), was introduced by Veselago [1]. Interest in these metamaterials was rejuvenated by Pendry [2] and Smith [3] who noted that the growth of evanescent fields within a NIM provides the opportunity for building a "perfect lens" that can focus electromagnetic waves to a spot size much smaller than a wavelength. Negative-index materials have recently been designed on the basis of composite wire and split ring resonator structures, backward-wave transmission lines, and photonic-band-gap crystals.

Usually, NIMs based on dielectrics are designed from homogeneous rods. In such a case, thick slabs consisting of a large number of rods are required for realizing good focusing. In the present work, we propose the design of a NIM lens from dielectric rods with a specific dielectric constant profile. This design leads to comparable or even improved focusing from much thinner slabs thus containing a much smaller number of rods.

2 Details of the approach A highly efficient and accurate multiple-scattering approach [4] is used to calculate propagation of electromagnetic waves through these structures. This procedure is much more efficient than alternative approaches such as high frequency structure simulation (HFSS) and finite difference time domain (FDTD) methods, which require much greater computer resources.

The elementary building blocks of our design are dielectric rods approximating a "fish-eye" refractive index profile [5] given by

$$n(r) = n_0/(1 + (r/r_0)^2)\,, \tag{1}$$

where r is the distance from the center of the rod and n_0, r_0 are constants. In such a material light propagates in circular (or spiral) trajectories with a radius comparable to the quantity r_0, i.e. a medium with the "fish-eye" dielectric constant profile behaves like a NIM from the point of view of light scattering. To prove that a "fish-eye" rod will exhibit properties of NIM, let us first consider the design of dielectric rods from the point of view of light scattering. We use an approach based on the effective medium concept [6] to choose appropriate parameters for the dielectric profile of the rod. This method relies on using a hypothetic background medium with variable index of refraction in which the dielectric rod is immersed. The scattering cross section of the rod is then calculated for each value of the refractive index of the background medium, and the dependence of the scattering cross section upon the refractive index of the medium involved is plotted. It is obvious that this dependence should exhibit a sharp decrease when the refractive index of the background medium approaches that of the rod being investigated.

3 Results and discussion For practical purposes, we approximate the fish-eye medium by use of several discrete layers of different refractive indices. Figure 1a illustrates the result of such a calculation for a rod consisting of three layers with the radii $r_1 = 0.5a$, $r_2 = 0.25a$, and $r_3 = 0.1a$, and refractive indices $n_1 = 1.5$, $n_2 = 3$, and $n_3 = 4$. The calculations were performed for the wavelength $\lambda = a/0.67$.

As one can see from Fig. 1a, a decrease of the scattering cross section occurs around the refractive index values of –0.85. This means that at this wavelength the rod behaves as a NIM with the refractive index equal to –0.85. One should note for completeness that the calculations performed for a homogeneous rod with the refractive index of –0.85 exhibit a sharper decrease of the scattering cross section.

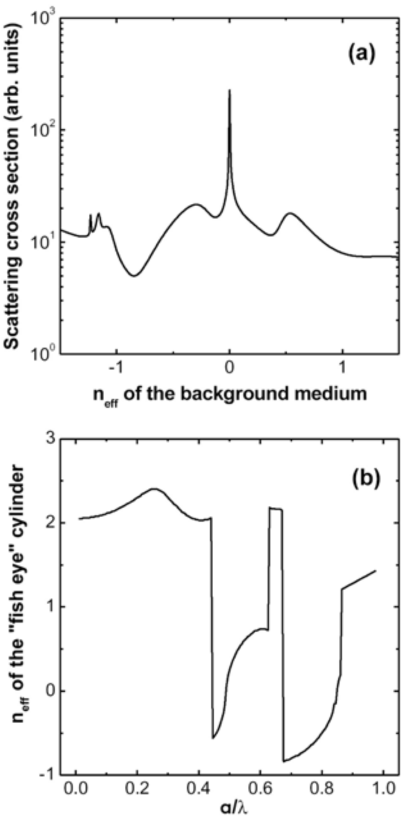

Figure 1 Dependence of the scattering cross section on the refractive index of the background medium (a), and the dependence of the refractive index on the radiation wavelength (b) for a “fish-eye” rod consisting of three layers.

Analogous calculations were performed for other wavelengths, and the effective refractive index of the rod upon the radiation wavelength is illustrated in Fig. 1b. One can see that within the wavelength ranges given by $0.67 < a/\lambda < 0.83$ and $0.44 < a/\lambda < 0.49$ the rod under consideration behaves like a NIM from the point of view of light scattering, exhibiting the best properties at wavelengths around $\lambda = a/0.67$.

The positive refractive index of conventional optical lenses implies that they need curved surfaces to form an image, whereas a negative index of refraction allows a flat slab of a material to behave as a lens that can focus electromagnetic waves to produce a real image. The distance between the source and the image is determined only by the thickness of the material and the value of the refractive index. Conventional optical systems have a single optical axis and limited aperture, and cannot focus light onto an area smaller than a square wavelength [2]. In contrast, flat NIM lenses have no unique optical axis and are not restricted by aperture size. In such a system, the movement of the source in a direction parallel to the slab surface will result in the movement of the image by the same distance and in the same direction [7].

Now let us next consider a two-dimensional photonic crystal assembled from the fish-eye rods with the parameters described above as elementary units. We present results for a photonic crystal possessing a triangular lattice with the lattice constant a. Figure 2 shows the image of a

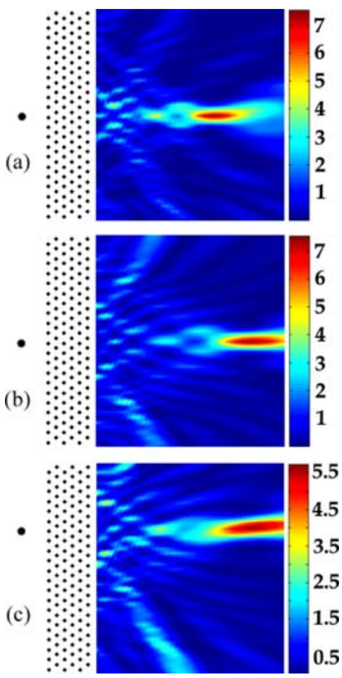

Figure 2 Electric field intensity map of a cross-sectional view of the 2D source-image system when imaging by a triangular-lattice photonic crystal slab consisting of “fish-eye” rods: (a) the source is placed at distance $6a$ from the slab; (b) and (c) the source is placed at the distance $2a$ from the slab. Note that (c) illustrates transverse displacement of the image by the same distance that the source is moved.

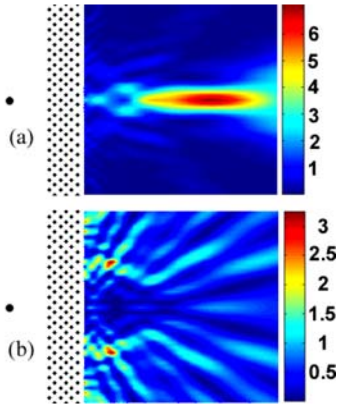

Figure 3 Distribution of the transmitted EM power through a photonic crystal slab constructed from homogeneous rods with refractive index –0.8 (a) and 4.1 (b).

point source with wavelength $\lambda = a/0.67$ placed in front of a triangular-lattice photonic-crystal slab with dimensions $4.3a \times 20.5a$ and consisting of 126 cylindrical rods. The pattern represents the distribution of the transmitted EM power $T = (E/E_0)^2$, where E is the electric field amplitude of the radiation passed through the photonic crystal slab, and E_0 is the electric field amplitude without the photonic crystal slab. The source is placed at the distance $6a$ from the slab in Fig. 3a and at the distance $2a$ in Fig. 3b and c. The calculations are performed for a polarization with the electric field vector $\boldsymbol{E}$ parallel to the cylindrical rods axis. One can see a clear focusing effect in all the illustrated patterns. An especially high-quality image is seen in Fig. 3a.

The movement of the image when the source is moved as illustrated in Fig. 3b and c is a clear indicative of the negative refractive index of the slab, in accordance with the test described above. Note that the focusing effect is observed also for other wavelengths in the range $0.40 < a/\lambda < 0.85$, but the image quality is especially good in the frequency ranges $0.67 < a/\lambda < 0.83$ and $0.44 < a/\lambda < 0.49$, where the individual fish-eye rods scatter light like media with a negative refractive index.

For the purpose of comparison, in Fig. 3 we show the distribution of the transmitted EM power for slabs of dimensions and rod sizes identical to those of Fig. 2, but constructed from *homogeneous* rods with refractive index equal to –0.8 (Fig. 3a) and 4.1 (Fig. 3b). One can see that the imaging by first slab is a slightly better than, but comparable to, the imaging with the lens of Fig. 2a. No focusing effect is realized for the second slab (Fig. 3b).

It is worthwhile to mention that the focusing effect of the lens with the "fish-eye" rods is fairly tolerant to the fabrication imperfections. Simulations showed that a variation of up to 10% both in the cylinders arrangement as well as in their transverse dimensions acts negligibly on the focusing properties of the lens.

It was previously reported (see, for instance, Ref. [8]) that focusing is also achieved by flat photonic-crystal lenses constructed from homogeneous rods with positive refractive index. However, thick slabs consisting of a big number of rods are generally necessary for this purpose. For instance, we have performed calculations for a PC slab of dimensions and rod sizes identical to those of Fig. 2 but assembled from homogeneous rods with positive refractive index from 1 to 4 and have found no focusing in the frequency interval $0 < a/\lambda < 1$. Design of flat lenses on the basis of rods with special dielectric constant profile allows one to achieve better focusing effect with much thinner slabs. In addition, these lenses work in a much broader wavelength range.

4 Conclusion

The results of our calculations show that dielectric rods with a fish-eye refractive index profile are promising for designing and manufacturing negative index materials. The NIM designed in this work on the basis of multilayer dielectric rods is suitable for focusing the EM radiation in a relatively large wavelength range. At present, we are assembling a flat lens made from "fish-eye" dielectric rods for the purpose of testing the calculations in the microwave region. Note that photonic crystal lenses made from homogeneous dielectric rods were found recently to focus microwaves at specific wavelengths [9, 10] as well as to show tolerances to the induced disorder [11, 12].

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