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Noise properties of a beam propagating through an atomic vapor

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Abstract

Saturated absorption is examined theoretically to determine the extent to which this process can lead to an increase in the intensity noise of an intense laser beam as the beam propagates through an atomic vapor. Numerical results for the modification of the intensity fluctuations and the noise figure are presented for both incoherent and coherent incident light. The results show that although saturated absorption is a real process for the generation of excess noise, it is not sufficient to account for the noise observed in laboratory experiments.

1. Introduction

Recent experiments [1] on the noise properties of an intense beam propagating through an atomic vapor have found that the transmitted beam acquires excess noise. A quantum mechanical theory of forward four wave mixing [2] in atomic vapors has been applied to this interaction and it has been found that this model predicts the qualitative features of the observed noise properties of the transmitted beam. According to this model, the excess noise is attributed to the amplification of the vacuum side modes of the laser beam by the forward four wave mixing process. This noise mechanism is well understood in the context of quantum noise properties of optical amplifiers [3]. In particular, optical amplifiers utilizing atomic vapors have been well investigated [4] and their noise properties have been studied [5].

In this paper we consider whether the noise properties of a beam propagating through an atomic vapor can be described by a much simpler model, namely by modelling the atomic vapor as a saturable absorber which modifies the fluctuations initially present on the laser beam. The effect of linear absorption on the statistical properties of the incident beam are trivial and well known. The effect of two-photon absorption on the intensity fluctuations of a beam have been investigated [6,10], mainly with the motivation of generating amplitude squeezed light [7]. In contrast to two-photon absorption, saturated absorption can lead to an enhancement of the intensity fluctuations, due to the fact that higher intensity components of the fluctuating beam are absorbed less than the lower intensity components. Although enhanced intensity fluctuations for chaotic light incident on a saturable absorber have been theoretically studied [8] and experimentally verified [9], the modification of the intensity fluctuations for coherent light incident on a saturable absorber have not previously been investigated.

The intent of the present paper is to examine the effects of saturation on the intensity fluctuations of an intense laser beam propagating through an atomic vapor. We analyze the effect of saturated absorption by solving the nonlinear propagation equation (Eq. (3) below) for the intensity of the beam, where the optical Bloch equations for a two level atom have been used to obtain the form of the intensity-dependent absorption coefficient. We present results for the noise character-

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istics for two distinct cases – one when the incident beam is assumed to be thermal light and the other when the incident beam is assumed to be coherent light. The intensity fluctuations for both these cases are different and saturated absorption is expected to modify the intensity fluctuations of the transmitted beam differently in each case. Our results show that in both cases the beams acquire excess noise on propagation.

2. Theory

We consider the experimental setup shown in Fig. 1 where an intense laser beam of intensity I passes through an atomic vapor cell of length z. To simplify, we model this experimental setup as a system of homogeneously broadened two-level atoms of frequency ω_0, interacting with the intense monochromatic field

$$E = E_0 \exp(-\mathrm{i}\omega t) + \text{c.c.}\,, \tag{1}$$

where ω is the frequency of the laser beam and the intensity of the beam is given by

$$I = \frac{c}{2\pi} |E_0|^2 . \tag{2}$$

The beam intensity satisfies the nonlinear propagation equation

$$\frac{\mathrm{d}I}{\mathrm{d}z} = -\frac{\alpha_0 I}{1 + I/I_s}, \tag{3}$$

where I_s is the standard saturation intensity which can be expressed in terms of atomic parameters as

$$I_s = \frac{ch^2(1 + \Delta^2 T_2^2)}{8\pi |\mu|^2 T_1 T_2} . \tag{4}$$

Here μ is the dipole matrix element of the atom and T_1 and T_2 are the population relaxation time and the dipole

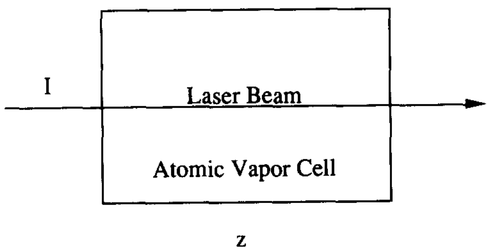

Fig. 1. Experimental setup consisting of a beam of light of intensity I propagating through an atomic vapor cell of length z.

dephasing time, respectively, of the two level atom. The weak field absorption coefficient α_0 is given by

$$\alpha_0 = \frac{4\pi\omega_0 N |\mu|^2 T_2}{hc(1 + \Delta^2 T_2^2)}, \tag{5}$$

where $\Delta = \omega - \omega_0$ is the field–atom detuning and N is the number density of the atoms. Direct integration of Eq. (3) yields

$$\ln\left(\frac{I_0}{I_i}\right) + \frac{I_0 - I_i}{I_s} = -\alpha_0 z, \tag{6}$$

where I_i is the incident beam intensity at $z = 0$, and I_0 is the intensity after the beam has propagated a distance z through the medium. Eq. (6) describes the nonlinear attenuation of the intensity of the beam propagating through a saturable absorber. This equation is valid for the case of a beam of constant input intensity. For a fluctuating beam incident on the medium, the instantaneous intensities are characterized by the probability distribution function $P(I_i)$ defined such that the probability of finding an intensity I_i in the interval I_i to $I_i + \mathrm{d}I_i$ is $P(I_i)\,\mathrm{d}I_i$. Each value of I_i of the incident intensity distribution function is attenuated according to Eq. (6). In order to calculate the statistical properties of the transmitted beam it is necessary to know the intensity probability distribution function $P(I_0)$ of a beam that has propagated a distance z in the medium. This function is determined from Eq. (6) by the relation

$$P(I_0) = (P(I_i) \left|\frac{\mathrm{d}I_i}{\mathrm{d}I_0}\right| = P(I_i)\,\frac{I_0 + I_i I_i}{I_i + I_s I_0} . \tag{7}$$

Eq. (7) can be used to calculate the noise properties of the transmitted beam. A quantitative description of the intensity fluctuations of the transmitted beam is obtained from a knowledge of the noise figure F defined as

$$F = \left[\frac{(S/N)_i}{(S/N)_0}\right]^2, \tag{8}$$

where $(S/N)_i$ is the signal to noise ratio of the incident beam and $(S/N)_0$ is the signal to noise ratio of the transmitted beam. To determine these ratios, the signal parameter S is defined by

$$S = \langle I \rangle , \tag{9}$$

and the rms fluctuation parameter N is defined by

$$N = \sqrt{\langle I^2 \rangle - \langle I \rangle^2} . \tag{10}$$

The mean intensity $\langle I_0 \rangle$ of the transmitted beam is given by

$$\langle I_0 \rangle = \int I_0 P(I_0)\, dI_0\,, \tag{11}$$

and the variance $\langle I_0^2 \rangle$ of the intensity fluctuations of the transmitted beam is given by

$$\langle I_0^2 \rangle = \int I_0^2 P(I_0)\, dI_0\,. \tag{12}$$

Eqs. (7)–(12) can be evaluated for a given form of the input intensity fluctuations.

3. Noise characteristics

We next consider two models for the fluctuations of the incident light beam. First we consider the intensity fluctuations of light produced by a thermal source. Next we consider the intensity fluctuations of laser light, where we describe the statistical properties of a laser oscillator through use of the phenomenological model of Van der Pol.

3.1. Thermal light

The intensity fluctuations of incoherent light [10] are described by the probability distribution function

$$P(I_i)\, dI_i = \frac{1}{\langle I_i \rangle} \exp\left\{-\frac{I_i}{\langle I_i \rangle}\right\} dI_i\,, \tag{13}$$

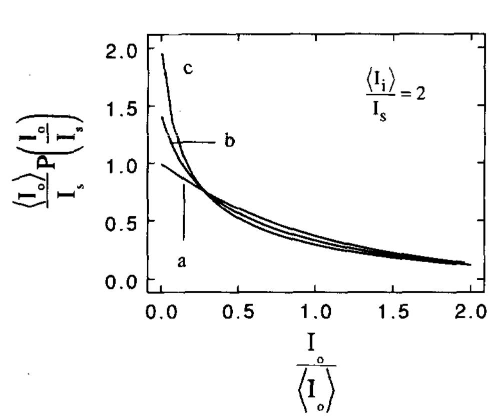

Fig. 2. Probability distribution function of the transmitted intensity for chaotic light of mean intensity $\langle I_i \rangle = 2I_s$ incident on the saturable absorber. Different curves correspond to different values of the normalized propagation distance, (a) $\alpha_0 z = 0$ (incident light), (b) $\alpha_0 z = 0.5$, (c) $\alpha_0 z = 1$.

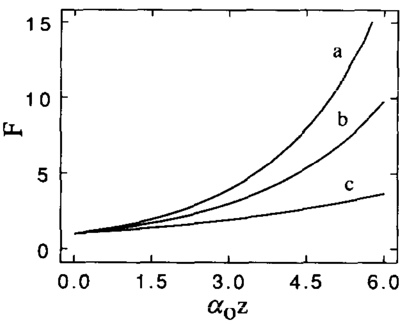

Fig. 3. The noise figure F of the transmitted beam plotted as a function of the normalized cell length $\alpha_0 z$, for chaotic light incident on the medium. Different curves correspond to different values of the mean incident intensity, (a) $\langle I_i \rangle / I_s = 0.5$, (b) $\langle I_i \rangle / I_s = 2$, (c) $\langle I_i \rangle / I_s = 5$, respectively.

where $\langle I_i \rangle$ is the mean value of the incident intensity. To determine $P(I_0)$ we evaluate Eq. (7) numerically for this distribution. In Fig. 2, we show the intensity probability distribution function $\langle I_0 \rangle / I_s\, P(I_0/I_s)$ for the transmitted light for several different values of the normalized propagation length $\alpha_0 z$. We normalize all intensities to the saturation intensity I_s. For $\alpha_0 z = 0$, the probability distribution function is an exponential function (Eq. (13)) of the incident light corresponding to a mean intensity of $\langle I_i \rangle / I_s = 2$. As the light propagates through the medium, the probability distribution function gets modified, as shown in Fig. 2 for $\alpha_0 z = 0.5$ and $\alpha_0 z = 1$. As a consequence, the noise figure (Eq. (8)) also changes from its value of 1 at $\alpha_0 z = 0$. Fig. 3 shows the noise figure of the transmitted beam plotted as a function of the propagation length $\alpha_0 z$ for various values of the mean input intensity $\langle I_i \rangle / I_s$. We see that the beam becomes noisier with increased interaction distance (i.e., F is a monotonically increasing function of z), but that the tendency of a beam to become noisier can be minimized through use of an intensity large enough to saturate the atomic response.

3.2. Van der Pol oscillator model for laser light

Ideal laser light is often characterized as being a shot noise limited beam having a constant value of the classical intensity. However, our present model can describe only how the fluctuations present in the incident laser beam are modified by the interaction of the beam with the atomic medium. We therefore model the laser as a Van der Pol oscillator [11], which produces a fluctuating output intensity. The output is described by the nonlinear stochastic differential equation (with $\beta > 0$)

$$db/dt - \beta(d - |b|^2)b = \sqrt{q}\Gamma(t) , \tag{14}$$

for the complex amplitude $b(t)$ of the single mode laser beam. The parameter d corresponds to the steady state intensity $I = b^* b$ when the noise force $\Gamma(t)$ is neglected. The quantity βd describes the temporal growth rate for small values of $b(t)$. The parameter q is the strength of the δ-correlated noise force $\Gamma(t)$ which is described by the correlation function

$$\langle \Gamma(t) \Gamma^*(t') \rangle = \delta(t - t') . \tag{15}$$

To characterize the regime of operation of the laser it is convenient to introduce the pump parameter

$$a = \sqrt{\beta/q}\, d , \tag{16}$$

which can be interpreted as the ratio of the steady state intensity to the fluctuation intensity of the laser. A value of $a = 0$ corresponds to the laser operating at threshold, and that of $a > 0$ ($a < 0$) corresponds to laser operating above (below) threshold. Eq. (14) leads to a truncated Gaussian distribution function for the intensity fluctuations of the incident laser light which is given by

$$P(I_i) = \frac{C}{\pi} \exp(a^2/4) \exp\{-\tfrac{1}{4}(I_i - a)^2\} , \tag{17}$$

here, I_i is the scaled input intensity variable

$$I_i = \sqrt{\beta/q}\, I , \tag{18}$$

and the normalization constant C is given by

$$C = \frac{1}{\sqrt{\pi} \exp(a^2/4)(1 + \mathrm{erf}(a/2))} . \tag{19}$$

For $a < 0$ the distribution function $P(I_i)$ is the tail of the Gaussian distribution which looks like the exponential distribution characterizing thermal light (Eq. (13)). In Fig. 4 we show the intensity probability distribution function $P(I_0)$ for the above threshold laser ($a = 4$) for different propagation lengths $\alpha_0 z$. We normalize all intensities to the saturation intensity I_s of the medium. We choose to study the case in which the quantity $\sqrt{\beta/q}\, I_s$ which can be interpreted as the ratio of the saturation intensity of the medium to the fluctuation intensity of the laser has the value unity. For $\alpha_0 z = 0$, the distribution is that of the incident laser. The curves corresponding to $\alpha_0 z = 2$, 4 and 5, respectively, show the modification of the probability distribution function as the laser beam travels through the medium. The modified intensity fluctuations result in the laser beam acquiring excess noise as it propagates. This is depicted in Fig. 5 by the increase in the noise figure of the transmitted beam as a function of the length $\alpha_0 z$. Fig. 5 also shows that the amount of noise acquired by the laser depends on how far above threshold it is operated. At $\alpha_0 z = 10$, a laser operating near threshold ($a = 0.5$) is approximately 4 times noisier than a laser operating above threshold ($a = 4$). The noise parameter $\sqrt{\beta/q}$ of a laser can be obtained from experimental measurement of the threshold number of photons [11],

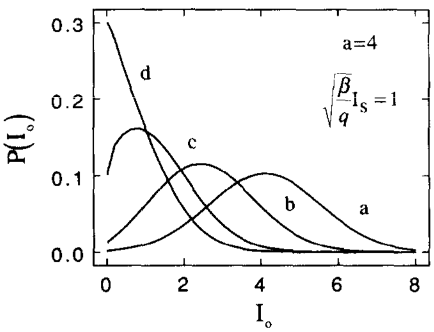

Fig. 4. Probability distribution function $P(I_0)$ of the transmitted intensity for a laser beam incident on the saturable absorber. The laser parameters are $a = 4$ and $\sqrt{\beta/q} = 1$. Different curves correspond to different values of the normalized propagation distance, (a) $\alpha_0 z = 0$ (incident laser light), (b) $\alpha_0 z = 2$, (c) $\alpha_0 z = 4$, and (d) $\alpha_0 z = 5$.

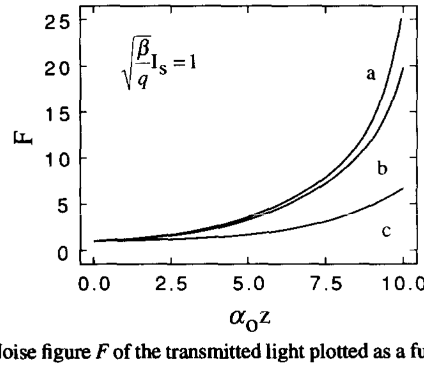

Fig. 5. Noise figure F of the transmitted light plotted as a function of the normalized cell length for laser light incident on the medium. Different curves correspond to different values of the pumping parameter: (a) $a = 0.5$, (b) $a = 1$, (c) $a = 4$, with $\sqrt{\beta/q}\, I_s = 1$ in each case.

and therefore the pump parameter a and the ratio $\sqrt{\beta/q}\,I_s$ are very large numbers, approximately 10^9 for even low power lasers used in typical experiments. Our calculations show that already for $a=4$ and $\sqrt{\beta/q}\,I_s=100$ the noise figure maintains a constant value of unity on propagation. This result shows that the additional noise acquired by an intense laser beam due to saturated absorption would be negligible.

4. Conclusions

We have examined the noise properties of an intense beam propagating through a saturable absorber. Our results show that, for both incoherent and coherent input light, the noise figure increases as the beam travels through the medium. Thus a beam of light travelling through an atomic vapor will acquire excess noise due to saturated absorption. However, as a consequence of the nature of saturation effects, a strongly saturating beam will acquire less excess noise than a weaker beam for the same distance of propagation. Our studies show that although generation of excess noise by saturated absorption is a real process, it is not sufficient to account for the noise observed in the experiment of Davis et al. [1]. We therefore conclude that the full quantum mechanical theory is required for a quantitative understanding of the additional noise.

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