Nature of the interference pattern produced on reflection at a phase-conjugate mirror

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Received June 30, 1986; accepted October 5, 1986

We show theoretically and experimentally that the positions of the fringes produced by interference between a wave incident upon a phase-conjugate mirror and the wave leaving the mirror depend on the phase associated with the incident wave. This result is in contrast to that obtained when an ordinary metal mirror is used, in which case the interference pattern is found to be independent of the phase of the incident wave.

The virtues of a phase-conjugate mirror (PCM) in correcting for optical aberrations are by now well known.[1] Because of the phase reversal suffered by the incident wave on reflection, any phase distortion experienced by the forward wave is canceled out when the reflected wave propagates back through the same (nonabsorbing) medium. PCM's have also been used to produce interferometers with esoteric properties, such as the ability to be self-referencing.[2]

In this Letter we present the results of a theoretical and experimental study of the pattern resulting from the interference between the wave incident upon a PCM and that reflected from it. The primary motivation of this study was to determine how the results of the classic experiments of Wiener[3–5] would be modified if the ordinary metal mirror used in his work were replaced by a PCM. Wiener's experiment involved measurements of the positions of the fringes produced by interference between a plane electromagnetic wave and the wave reflected from the mirror. The experiment is of considerable historical interest because it demonstrated the existence of standing light waves and also showed that the photochemical action responsible for the blackening of a photographic plate is directly related to the electric rather than to the magnetic field vector. The latter inference follows from the boundary conditions for a metal mirror, which require the existence of a node of the electric field at the mirror surface. In this Letter we show both theoretically and experimentally that with a PCM the positions of the fringes are not fixed with respect to the PCM but depend on the phase of the incident optical field (see also Feinberg[6] and Nieto-Vesperinas[7]).

Let us consider the situation illustrated in Fig. 1. $\mathbf{E}^{(\mathrm{I})}(\mathbf{r}, t)$ and $\mathbf{E}^{(\mathrm{II})}(\mathbf{r}, t)$, given by

$$\mathbf{E}^{(\mathrm{I})}(\mathbf{r}, t) = \epsilon_0 A^{(\mathrm{I})}\exp[i(\mathbf{k}_0 \cdot \mathbf{r} - \omega t)],$$
$$\mathbf{E}^{(\mathrm{II})}(\mathbf{r}, t) = \epsilon_0 A^{(\mathrm{II})}\exp[i(-\mathbf{k}_0 \cdot \mathbf{r} - \omega t)], \quad (1)$$

represent the complex electric fields of the counterpropagating monochromatic pump waves with wave vectors $\mathbf{k}_0$ and $-\mathbf{k}_0$; and

$$\mathbf{E}^{(i)}(\mathbf{r}, t) = \epsilon A^{(i)}\exp[i(\mathbf{k} \cdot \mathbf{r} - \omega t)] \quad (2)$$

represents the wave with wave vector $\mathbf{k}$ incident upon the PCM. In practice all three waves are usually derived from a single laser beam in order to achieve identical frequencies ω. ϵ_0 and ϵ are complex unit polarization vectors satisfying the condition $\epsilon_0^* \cdot \epsilon_0 = 1 = \epsilon^* \cdot \epsilon$. A real unit vector represents a linearly polar-

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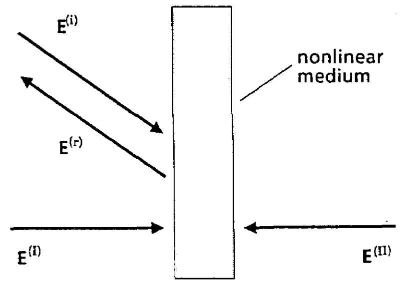

Fig. 1. The geometry for a PCM based on four-wave mixing.

ized wave, but more general states of elliptic polarization require complex polarization vectors. We assume that the reflected wave $\mathbf{E}^{(r)}(\mathbf{r}, t)$ leaving the PCM is in general given by

$$\mathbf{E}^{(r)}(\mathbf{r}, t) = \mu\epsilon^* A^{(i)*}\exp[i(-\mathbf{k}\cdot\mathbf{r} - \omega t)], \tag{3}$$

where μ is the complex reflectivity of the mirror. As is well known, in the weak-field limit ($|\mu| \ll 1$) μ is expressible in the form[8]

$$\mu = -igLA^{(\mathrm{I})}A^{(\mathrm{II})}, \tag{4}$$

where g represents the strength of the coupling between the waves through the nonlinear susceptibility of the phase-conjugating medium and is real for a purely dispersive nonlinearity. L is the optical path length through the medium. The superposition of the incident and reflected waves results in a total field

$$\begin{aligned}\mathbf{E}(\mathbf{r}, t) &= \mathbf{E}^{(i)}(\mathbf{r}, t) + \mathbf{E}^{(r)}(\mathbf{r}, t)\\ &= (\epsilon A^{(i)}e^{i\mathbf{k}\cdot\mathbf{r}} + \mu\epsilon^* A^{(i)*}e^{-i\mathbf{k}\cdot\mathbf{r}})e^{-i\omega t}\end{aligned} \tag{5}$$

and a total light intensity

$$\begin{aligned}I(\mathbf{r}, t) &= |\mathbf{E}(\mathbf{r}, t)|^2\\ &= |A^{(i)}|^2 + |\mu|^2|A^{(i)}|^2 + \mu^*\epsilon^2 A^{(i)2}e^{2i\mathbf{k}\cdot\mathbf{r}} + \mathrm{c.c}\\ &= |A^{(i)}|^2[(1+|\mu|^2) + 2|\mu||\epsilon^2|\cos(2\mathbf{k}\cdot\mathbf{r} - \phi + 2\alpha + \delta)],\end{aligned} \tag{6}$$

where we have written $\mu = |\mu|\exp(i\phi)$, $A^{(i)} = |A^{(i)}|\exp(i\alpha)$, and $\epsilon^2 = |\epsilon^2|\exp(i\delta)$. Equation (6) represents a standing interference pattern with fringes perpendicular to $\mathbf{k}$ and with fringe spacing π/k. The visibility $\mathcal{V}$ of the interference fringes[9] is given by

$$\mathcal{V} = \frac{2|\mu||\epsilon^2|}{1 + |\mu|^2} \tag{7}$$

and has a maximum value of unity when $|\mu| = 1$ and $|\epsilon^2| = 1$. This maximum visibility is achievable only when ϵ is real; the incident wave is then linearly polarized. Indeed, for a circularly polarized incident wave with $\epsilon = (\epsilon_1 + i\epsilon_2)\sqrt{2}$, where ϵ_1, ϵ_2 are real, mutually orthogonal unit vectors, we have $\epsilon^2 = 0$ and consequently $V = 0$.

From Eq. (6) the positions of the interference fringes are seen to depend on the phase α associated with the incident wave and also on the phase shift ϕ associated with reflection from the PCM. The latter quantity is given by

$$\phi = \phi^{(\mathrm{I})} + \phi^{(\mathrm{II})} - \pi/2, \tag{8}$$

where we have written $A^{(\mathrm{I})} = |A^{(\mathrm{I})}|\exp(i\phi^{(\mathrm{I})})$ and $A^{(\mathrm{II})} = |A^{(\mathrm{II})}|\exp(i\phi^{(\mathrm{II})})$. Although the positions of the interference fringes depend on α, the phase of the incident wave, this does not mean that a PCM can be used to measure the phase of a light beam in any absolute sense. This remark can be made more explicit by noting that the concept of the absolute phase of an electromagnetic wave depends on the choice of the origin of space and time but that the position of the interference pattern is independent of this choice. To see this, we translate the origin of space by $\Delta\mathbf{r}$ and the origin of time by Δt. As this is a purely formal, mathematical transformation, it leaves all the fields unaffected, and therefore $A^{(\mathrm{I})}$, $A^{(\mathrm{II})}$, and $A^{(i)}$ must change so that

$$\begin{aligned}A^{(\mathrm{I})} &\to A^{(\mathrm{I})}\exp[-i(\mathbf{k}_0\cdot\Delta\mathbf{r} - \omega\Delta t)],\\ A^{(\mathrm{II})} &\to A^{(\mathrm{II})}\exp[-i(-\mathbf{k}_0\cdot\Delta\mathbf{r} - \omega\Delta t)],\\ A^{(i)} &\to A^{(i)}\exp[-i(-\mathbf{k}\cdot\Delta\mathbf{r} - \omega\Delta t)].\end{aligned} \tag{9}$$

As a result the phase α changes:

$$\alpha \to \alpha - \mathbf{k}\cdot\Delta\mathbf{r} + \omega\Delta t, \tag{10}$$

and it follows from Eq. (4) and formulas (9) that the phase angle ϕ changes also:

$$\phi \to \phi + 2\omega\Delta t. \tag{11}$$

The position of the interference pattern is determined by the argument $2\mathbf{k}\cdot\mathbf{r} - \phi + 2\alpha + \delta$ of the cosine factor in Eq. (6), and, under the translation that we have been considering, by virtue of formulas (10) and (11), this argument remains unchanged.

We have confirmed these predictions by an experiment in which the PCM is formed by degenerate four-wave mixing in a nonlinear medium consisting of fluorescein-doped boric acid glass. This material has a relatively large nonlinear susceptibility of 4×10^{-3} e.s.u. and negligible absorption at the laser wavelength.[10] Figure 2 shows an outline of the setup. The three waves $\mathbf{E}^{(\mathrm{I})}$, $\mathbf{E}^{(\mathrm{II})}$, $\mathbf{E}^{(i)}$, which are linearly polarized in the plane perpendicular to the figure, are derived from an argon-ion laser operating in a single longitudinal mode at wavelength 488 nm. Phase shifts are introduced by varying the air pressure between 0 and 1 atm in a glass cell placed at one of five different positions A–E shown in Fig. 2. The position of the interference pattern relative to some arbitrary but fixed reference point was then determined from photoelec-

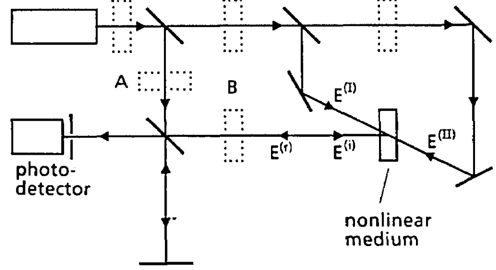

Fig. 2. Outline of the experimental setup.

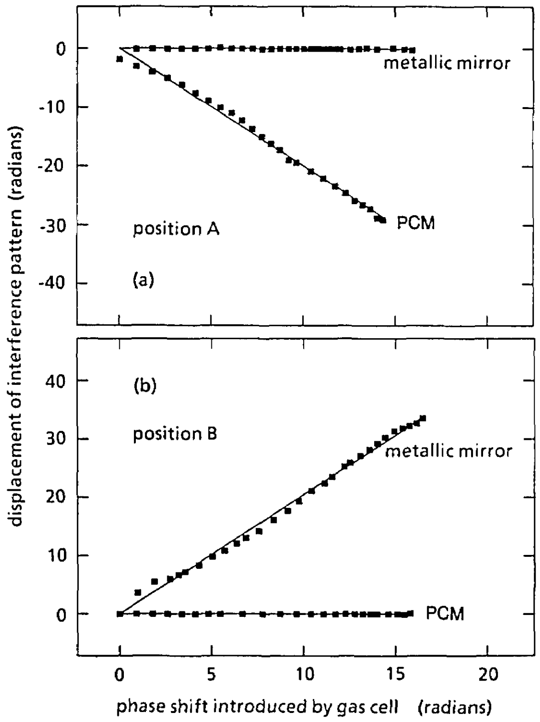

Fig. 3. Measured displacement of the interference pattern as a function of phase shift introduced by the air cell (a) at position A, (b) at position B.

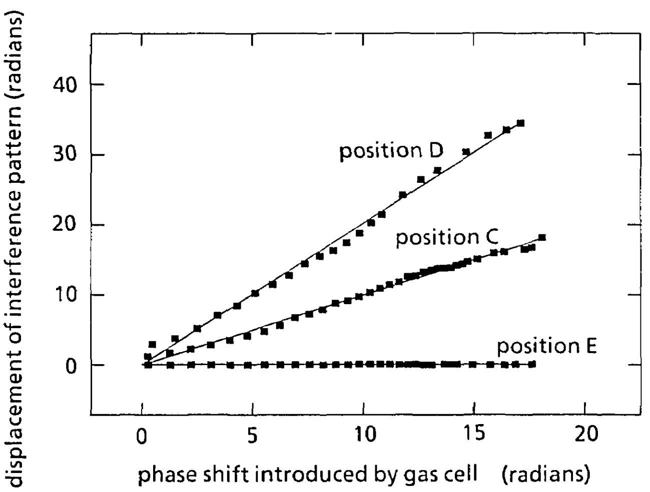

Fig. 4. Measured displacement of the interference pattern formed by the PCM as a function of phase shift introduced by the air cell at positions C, D, and E.

tric measurements of the light intensity. For comparison, the PCM was replaced by an ordinary metallic mirror in some measurements.

Figures 3(a) and 3(b) show the results of measurements made with PCM's and metallic mirrors at positions A and B, respectively. With the PCM in use and the cell at position A the fringes move linearly with the phase α associated with the incident wave, as predicted by Eq. (6), whereas no change is observed with a metallic mirror. With the cell at position B the phase cancellation introduced by the PCM makes the fringe pattern independent of path difference. However, the pattern shifts linearly with path difference when a metallic mirror replaces the PCM, because changing the gas pressure is then equivalent to moving the mirror. Figure 4 shows the effect, with the PCM in position, of placing the air cell at the locations C, D, and E and thereby introducing various relative phase shifts between the pump and incident waves. At location E there is no relative phase shift and the fringes do not move. At position C one pump wave is shifted in phase, and at position D both pump waves are phase shifted. The displacement of the fringe pattern is therefore twice as great in the third case, as expected from Eq. (4).

In conclusion, we have repeated the classic experiment of Wiener with the ordinary metal mirror replaced by a PCM. We find that, unlike in the Wiener experiments, the position of the interference pattern depends on the phase of the incident wave, as predicted by the theory outlined above.

A fuller discussion of the theory[11] and the experiment[12] described in this Letter will be published elsewhere.

This research was supported by the National Science Foundation and by the U.S. Office of Naval Research.

E. Wolf is also with the Institute of Optics, University of Rochester.

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