Analysis of the pressure, density, and velocity distributions in a spectroscopic heat-pipe oven and the resulting limitations on device performance

Robert W. Boyd

Institute of Optics, University of Rochester, Rochester, New York 14627

Jack G. Dodd

Physics Department, Colgate University, Hamilton, New York 13346

The pressure, density, and velocity distributions within a cylindrical heat-pipe oven are analyzed for inviscid, compressible flow in either the radial or axial direction. For sufficiently large but readily attainable heat flows, density variations within the heat pipe as large as 50% which can lead to inaccuracies in quantitative spectroscopic investigations are predicted.

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I. INTRODUCTION

The heat-pipe oven has recently become an important tool for use in spectroscopic investigations of atomic vapors.[1] The heat-pipe oven produces a well-defined region of nearly pure isothermal atomic vapor, which is commonly believed to be also at nearly constant pressure and density. This paper analyzes the pressure, density, and velocity distributions within a heat-pipe oven, and predicts that under certain attainable operating conditions the variations in density within a heat pipe can be sufficiently large to produce inaccuracies in quantitative spectroscopic measurements.

The heat pipe[2] itself has been the subject of extensive theoretical study[3] because of its use as a device for heat transfer. Most such studies have emphasized the heat-transport properties of the device, however, and have not treated its performance as a spectroscopic oven, as we do here. Vidal,[4] however, has presented a theoretical analysis of the spectroscopic heat pipe, and his results will be compared to ours in Sec. V.

In the normal form of the spectroscopic heat-pipe oven, shown in Fig. 1, an atomic vapor is confined to the center of a cylindrical pipe by a buffer gas of pressure p_0. Heat is applied, usually electrically, to the center of the heat pipe, causing the atomic species to evaporate and stream outward. On encountering the region near the cooling coils, atomic vapor condenses onto a stainless-steel mesh and is pulled back to the center of the heat pipe as a liquid by capillary action. The molecular pumping action of the streaming atomic vapor expels the buffer gas from the central, or active region. The transition region between the central active region of nearly pure atomic vapor and the outer region of nearly pure buffer gas is quite short in practice, being on the order of a mean free path of the working species in the buffer gas. The condition that the transition region be in mechanical equilibrium requires that the pressure of the active fluid at the boundary be equal to that of the buffer gas less the momentum flux density of the working fluid due to its streaming.

Similar considerations apply to the case of a disk-shaped heat-pipe oven,[5] illustrated in Fig. 2. In this design the walls of the cell are flat plates separated by a short, cylindrical Pyrex window, thus allowing unrestricted 360° viewing of the active medium. Heat is again applied to the center of the active region which occupies the disk-shaped central portion of the heat pipe and is surrounded by an annular region of buffer gas.

For either geometry, the active region has the shape of a truncated cylinder, but in the conventional design of Fig. 1 the flow is axial, whereas in the disk-shaped design of Fig. 2 the flow is radial. The terms axial-flow heat pipe and radial-flow heat pipe will thus be applied to these two cases in the remainder of the discussion.

The active region in either case is very nearly isothermal, as can be seen from the following argument. At each point on the wetted mesh, the liquid will be in equilibrium with its vapor at the local temperature. Since the vapor pressure of common materials increases very rapidly with temperature, the temperature of the vapor must be nearly constant unless the pressure variations are extremely large, but large pressure variations are usually inconsistent with the condition of steady-state mechanical equilibrium. By the Clausius-Clapeyron relation, the rapid increase of vapor pressure with temperature is related to the large latent heat of vaporization displayed by most materials. The nearly isothermal conditions within the active region can thus equivalently be understood as resulting from the very large heat flows that would accompany a thermal gradient.

While the temperature of the active region is thus very nearly isothermal, the pressure and density will not necessarily be constant within the active region. Since the vapor pressure varies rapidly with temperature, no inconsistency is thereby implied. In fact, the inevitable variation of the flow velocity with position requires, by Bernoulli's law, a variation of pressure with position.

It is the purpose of this paper to present an analysis of these pressure and density variations within a heat-pipe oven, and to determine how these variations depend on the operating parameters. In Sec. II, the equations which govern the pressure, density, and velocity fields within a heat-pipe oven are formulated. In Sec. III, these equations are solved for the case of an axial-flow heat pipe, and in Sec. IV they are solved for a special case of a radial-flow heat pipe. The experimental implications of these results, including possible
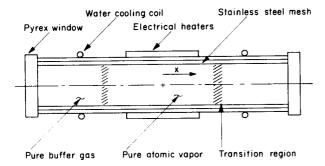
FIG. 1. Axial-flow heat-pipe oven.

inaccuracies in spectroscopic measurements, are discussed in Sec. V.

II. MATHEMATICAL FORMULATION

The equations that describe the steady-state operation of a heat-pipe oven are formulated in this section. The equation of continuity is given by

$$\nabla \cdot \rho \mathbf{v} = \sigma, \tag{1}$$

where ρ is the density of the working fluid, $\mathbf{v}$ is its flow velocity, and σ is a source term giving the rate per unit volume at which mass is added to the flow by evaporation.

The equation of motion for inviscid flow is given in component form by[6]

$$\frac{\partial}{\partial x_k} \rho v_i v_k = -\frac{\partial p}{\partial x_i}. \tag{2a}$$

It can be expressed in vector notation as

$$\rho(\mathbf{v} \cdot \nabla)\mathbf{v} + \mathbf{v}\nabla \cdot \rho \mathbf{v} = -\nabla p, \tag{2b}$$

where, in either form, p is the pressure of the working fluid. We shall show later that neglecting viscous forces is justifiable for many laboratory heat pipes. For the special case in which the source term is equal to zero, the equation of motion reduces to Euler's equation

$$\rho(\mathbf{v} \cdot \nabla)\mathbf{v} = -\nabla p. \tag{3}$$

Equation (2) is a generalization of Euler's equation in that it includes the contribution to the pressure gradient from the acceleration required to bring mass added to the flow by evaporation to the local flow velocity. If it is assumed that the flow is irrotational, or that $\nabla \times \mathbf{v} = 0$, Euler's equation (3) can be expressed through use of the vector identity

$$(\mathbf{A} \cdot \nabla)\mathbf{A} = (\nabla \times \mathbf{A}) \times \mathbf{A} + \tfrac{1}{2}\nabla(\mathbf{A} \cdot \mathbf{A})$$
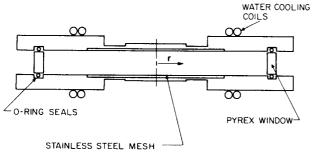

FIG. 2. Radial-flow or disk-shaped heat-pipe oven.

as

$$\tfrac{1}{2}\nabla(\mathbf{v} \cdot \mathbf{v}) = -(1/\rho)\nabla p. \tag{4}$$

Since the vapor can be assumed to be isothermal, the pressure and density are related by

$$p/p_0 = \rho/\rho_0, \tag{5}$$

where p_0 and ρ_0 are the pressure and density of the working fluid at some reference pressure which is taken to be that of the buffer gas.

III. AXIAL-FLOW HEAT PIPE

For flow in one dimension, the continuity and momentum equations (1) and (2) become

$$\frac{d}{dx}\rho v = \sigma(x), \tag{6}$$

$$\frac{d}{dx}\rho v^2 = -\frac{dp}{dx}, \tag{7}$$

where the coordinate x is measured from the center of the heat pipe. It is assumed that $\sigma(x)$ is a known function of position, $\sigma(x)$ being proportional to the spatial distribution of the heat input to the heat pipe. Equation (6) can then be integrated to give

$$\rho v = s_1(x), \tag{8}$$

where

$$s_1(x) = \int_0^x \sigma(x')dx' \tag{9}$$

is the flow rate at position x in units of mass per unit time per unit cross-sectional area. Equation (7) can likewise be integrated to give

$$\rho v^2 = p_0 - p, \tag{10}$$

where the integration constant is set equal to the external buffer-gas pressure to ensure mechanical equilibrium. The result (10) can be considered to be a form of Bernoulli's law. Equations (5), (8), and (10) can then be combined to give

$$S(x)^2 \equiv 4s_1(x)^2/p_0\rho_0 = (4p/\rho_0)(1 - \rho/\rho_0), \tag{11}$$

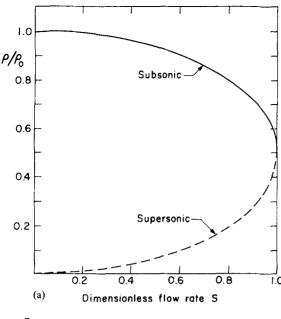where $S(x)$ is the dimensionless flow rate at position x. We can express the velocity in terms of $S(x)$ using (8) as

$$v = \frac{S(x)(p_0/\rho_0)^{1/2}}{2\rho/\rho_0} = \frac{S(x)v_{th}}{2\rho/\rho_0}, \tag{12}$$

where $v_{th} = (kT/m)^{1/2}$ is the rms thermal atomic velocity. Equations (11) and (12) completely determine the density and velocity fields in terms of the dimensionless flow rate $S(x)$, and are illustrated graphically in Fig. 3. Two solutions exist for each value of $S(x)$, one corresponding to a subsonic and one to a supersonic velocity. Under normal operating conditions the subsonic solution applies.

IV. RADIAL-FLOW HEAT PIPE

For axially symmetric radial flow, the continuity equation (1) becomes

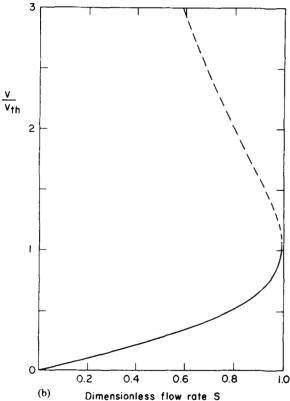$$\frac{1}{r}\frac{\partial}{\partial r}\rho v r = \sigma(r), \tag{13}$$

FIG. 3. Density (a) and flow velocity (b) of the active fluid in an axial-flow heat-pipe oven as a function of the dimensionless flow rate S defined by Eq. (11).

which can be integrated to give

$$\rho v r = s_2(r),\tag{14}$$

where

$$s_2(r)\equiv\int_0^r \sigma(r')r'\,dr'\tag{15}$$

can be interpreted as the total outward mass flow per unit time at radius r divided by $2\pi d$.

The general form (2) of the momentum equation cannot

be integrated in cylindrical coordinates. However, the specialized form (4) can be integrated in these coordinates. Equation (4) is valid outside of the evaporation region where $\sigma(r) = 0$, since it has been assumed that the flow is radial and thus that $\nabla \times v = 0$. Since in practical heat pipes the evaporation region is often a small part of the total working volume, even a solution which is valid only outside the evaporation region is quite useful. For axially symmetric flow, Eq. (4) takes the form

$$\frac{1}{2}\frac{\partial v^2}{\partial r}=-\frac{1}{\rho}\frac{\partial p}{\partial r}.\tag{16}$$

Using the relation (5) between p and ρ, this equation can be integrated to give

$$\tfrac{1}{2}v^2 = -(p_0\rho_0)\ln(\rho/\rho_0),\tag{17}$$

where the integration constant was chosen to ensure mechanical equilibrium. The velocity v can be eliminated from Eqs. (14), and (17), giving the result

$$1/R^2 \equiv s_2(r)^2/p_0\rho_0 r^2 = -(2\rho^2/\rho_0^2)/\ln(\rho/\rho_0).\tag{18}$$

This equation for ρ/ρ_0 as a function of the dimensionless radial coordinate R is displayed graphically in Fig. 4(a). The velocity can be expressed in terms of R and the ratio ρ/ρ_0 as

$$v = \frac{(p_0/\rho_0)^{1/2}}{(\rho/\rho_0)R} = \frac{v_{th}}{(\rho/\rho_0)R},\tag{19}$$

where again the rms thermal velocity is given by $v_{th} = (kT/m)^{1/2}$. Two possible solutions exist for each value of R, one corresponding to supersonic flow and one corresponding to subsonic flow. Again, subsonic flow corresponds to the physical solution for parameters typical of spectroscopic heat pipes, and is shown in Fig. 4(b).

V. DISCUSSION

We consider here those conditions under which the density variations exhibited by a heat-pipe oven give rise to measurable effects. As the input power to the axial-flow heat pipe shown in Fig. 1 increases, the size of the active region initially grows as the increased input power is dissipated as thermal losses on the walls of the heat pipe. Once the active region has expanded to the position of the water-cooling coils, a further increase in input power must be accompanied only by an increase in the heat conducted to the cooling coils by the condensation of the working fluid. The flow rate $s_1(x)$ will thus increase nearly linearly with further increases in the input power. The flow velocity will thus increase as shown in Fig. 3, causing a decrease in the density and pressure of the working fluid, this decrease being understandable as a consequence of Bernoulli's law.

We can estimate the magnitude of the heat transfer rate for which this effect becomes important. The density within the heat pipe will drop to 90% of its low-heat-flow value [corresponding to $S(x) = 0$] for the value $S(x) = 0.6$ of the dimensionless flow rate. For the case of a sodium heat pipe operating at a buffer-gas pressure of 5 Torr, the sodium temperature will be approximately 475 °C and its density will be approximately 2×10^{-6} g/cm^3. The case $S(x) = 0.6$ thus corresponds to a flow rate of 0.03 g/cm^2 sec. The power transferred by this flow will be due mainly to the latent heat of
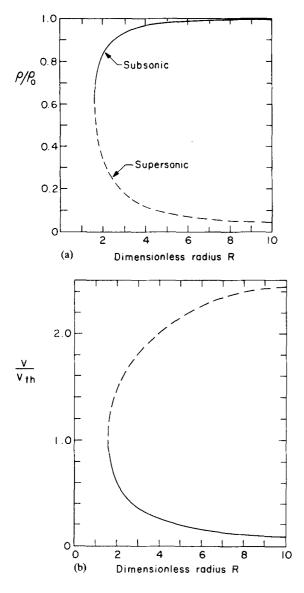
FIG. 4. Density (a) and flow velocity (b) of the active fluid in a radial-flow heat-pipe oven as a function of the demensionless radius R defined by Eq. (18).

vaporization of sodium, which is approximately 4000 J/g, giving a power transfer of 130 W/cm². Such a power transfer is achievable by typical laboratory heat-pipe ovens, and thus great care must be taken to avoid the measurement inaccuracies that could accompany the resulting decrease in density.

The mathematical solution presented here assumes inviscid flow, and thus should well describe the physical situation as long as the Reynold's number of the flow is much greater than unity. For the sodium heat pipe considered above, the viscosity η is approximately $1000\,\mu$P, and if the characteristic heat-pipe demension is $l = 1$cm, the Reynold's number $\mathscr{R} = \rho v l /\eta$ of the flow is equal to 100 for a flow velocity v equal to the thermal velocity, at which point the theory predicts large density variations.

velocity, at which point the theory predicts large density variations.

For a Reynold's number less than or comparable to unity, the theory presented here is not valid. However, the qualitative features of the solution can be expected to hold even in the presence of viscosity. A flow of mass must accompany the flow of heat through a heat pipe, regardless of the possible effects of viscosity. The requirement of mechanical equilibrium at the transition zone will thus still require that the pressure of the working fluid be less than that of the buffer gas by the momentum flux density ρv^2 of the streaming, active fluid, and this will lead to a decreased number density in the working volume.

Vidal's analysis[4] reaches conclusions that are qualitatively similar to those presented here, although the two methods of analysis differ. His analysis is based on energy and momentum conservation. It thus assumes that the flow is adiabatic and hence that evaporation and condensation regions are excluded from consideration. Our treatment, however, is based on solving the equations of fluid mechanics, including source terms, and makes what we feel is the more realistic assumption that the vapor temperature is everywhere equal to the local temperature of the wall of the heat pipe. In addition, Vidal presents some experimental evidence that the flow velocity can approach that of sound, but he does not give measured values of the velocity or density distributions, and thus we cannot compare his results to our theory.

In conclusion, we have analyzed the pressure, density, and velocity distributions within a heat-pipe oven, and we find that for sufficiently large, but readily attainable heat flows, the vapor density will significantly differ from its low-heat-flow value. We thus suggest that for quantitative spectroscopic investigations, the power input to a heat-pipe oven be kept as low as possible, consitent with heat-pipe operation.

ACKNOWLEDGMENT

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