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Canada Excellence Research Chairs

Chaires d'excellence en recherche du Canada

Generation of Caustics and Rogue Waves from Nonlinearity

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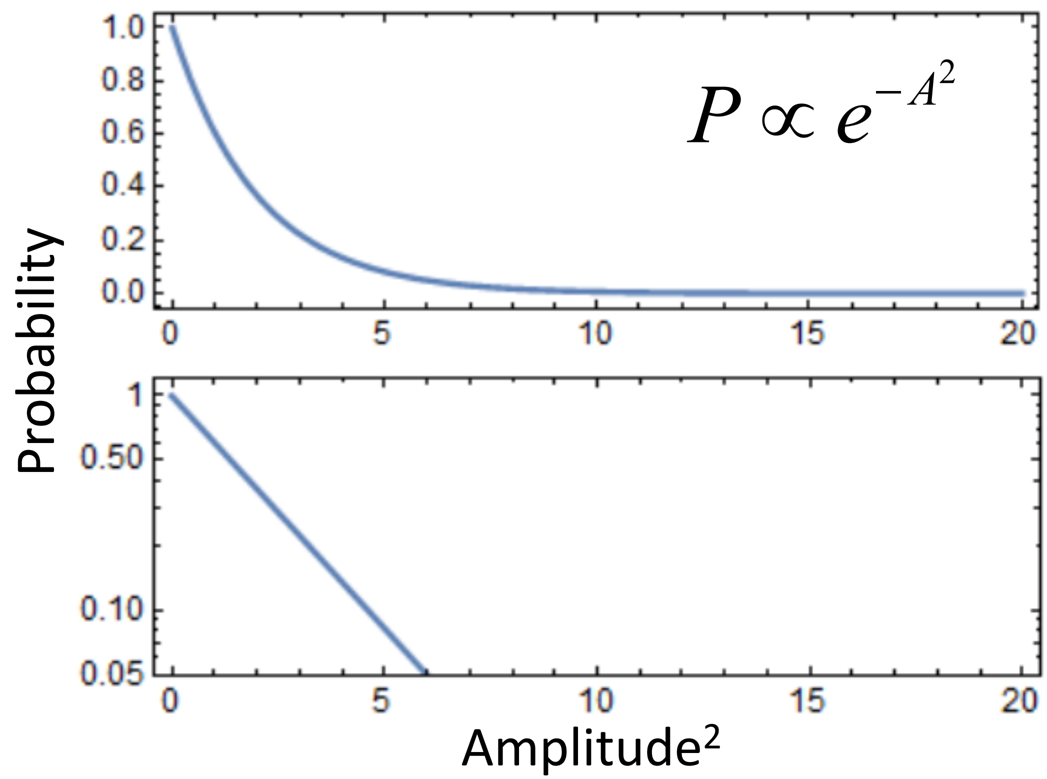
Canada Excellence Research
Chair (CERC) in Nonlinear Quantum Optics

Research interest:
Nonlinear optics, quantum optics,
integrated photonics, meta-materials, etc.

Oceanic rogue waves

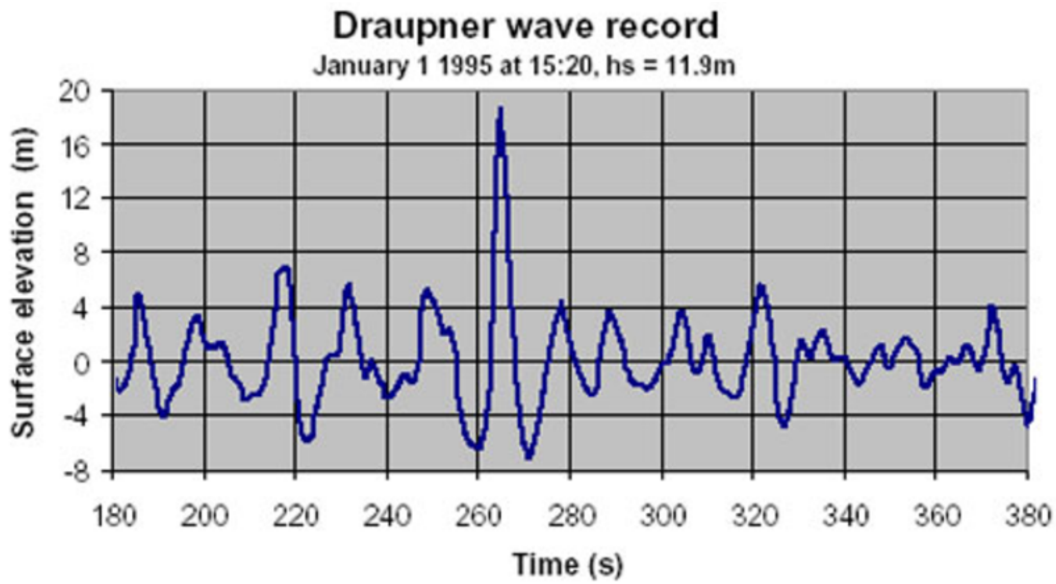
Before 1995

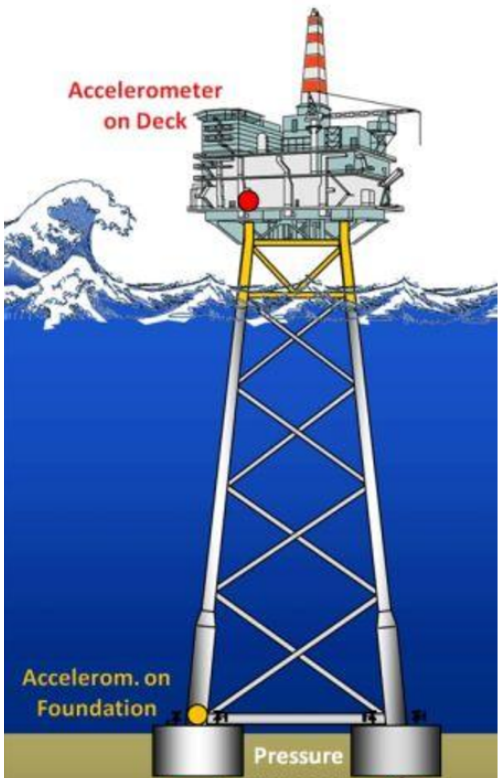
Sailors: we see gigantic waves.
Scientists: it is a fairy tail!
Ocean waves follow
Gaussian distribution.



Oceanic rogue waves

First scientific observation of rogue waves in Draupner oil platform (1995):





Characteristics of rogue waves

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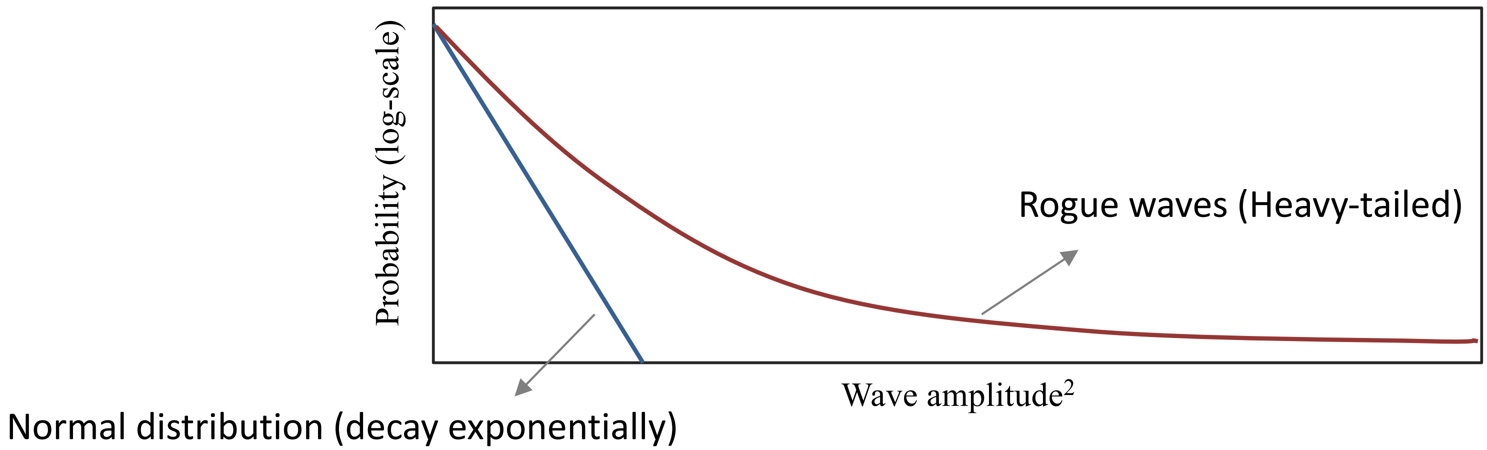
- Rogue waves appear from nowhere and disappear without a trace.
- Rogue waves ≠ accidental constructive interference
- They occur much more frequently than expected in ordinary wave statistics.

Probability distribution in rogue systems:



- Not limited to ocean: Observed in many other wave systems including **optics**.

1D vs 2D systems

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Nonlinear Schrödinger equation explains the wave dynamics in ocean as well as optics.

In 1D systems, such as optical fibers are studied extensively.

J.M. Dudley et al, Nat. Photon, 8, 755 (2014)

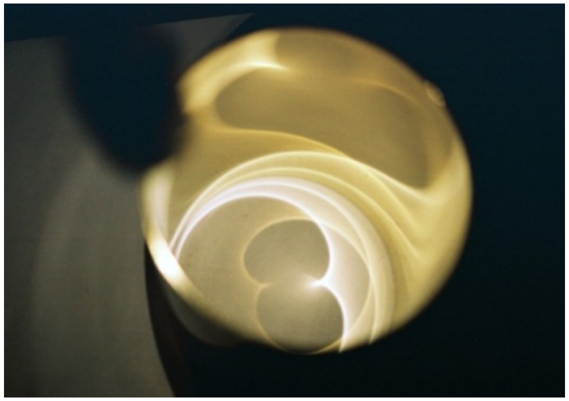
Water waves are not 1D.

Two focusing effects in 2D systems:

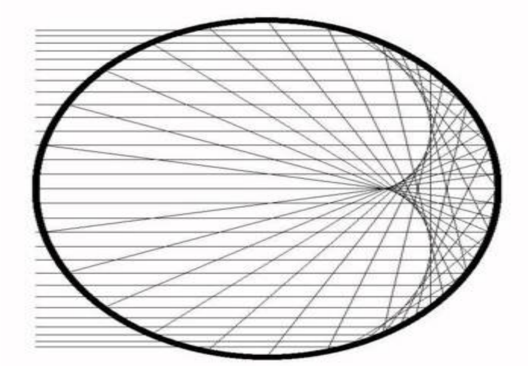
- **Linear**: Spatial (geometrical) focusing
- **Nonlinear**: Self focusing

Optical caustics

Swimming pool



Coffee cup

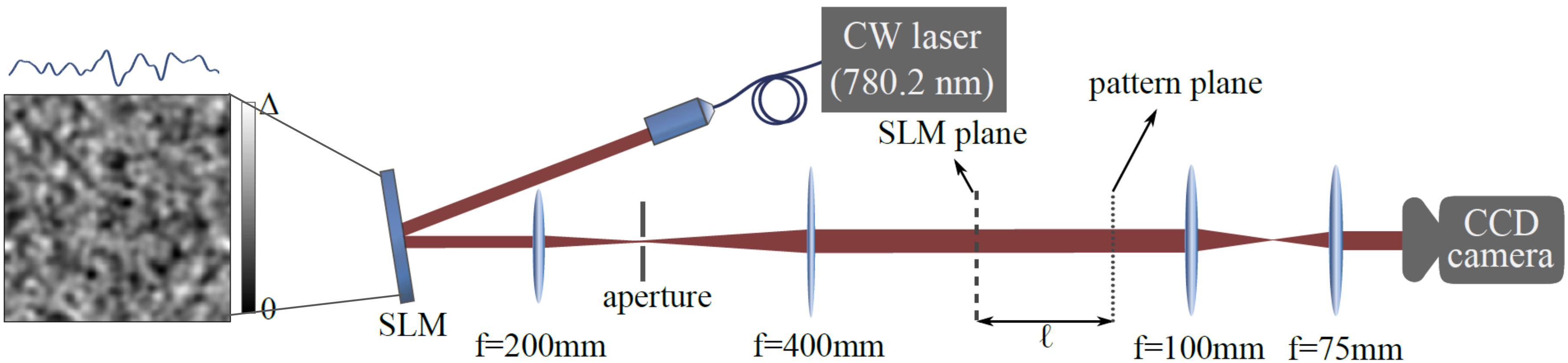


Ray picture

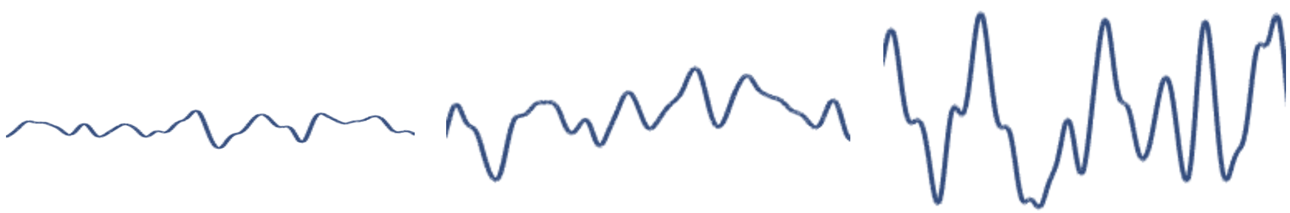
- Caustics are defined as envelope of a family of rays
- Singularities in ray optics
- Catastrophe theory is required to remove singularity

Books:
J.F. Nye, *Natural Focusing and Fine Structure of Light.*
Y.A. Kravtsov, *Caustics, Catastrophes and Wave Fields.*
O.N. Stavroudis, *The Optics of Rays, Wavefronts, and Caustics*.

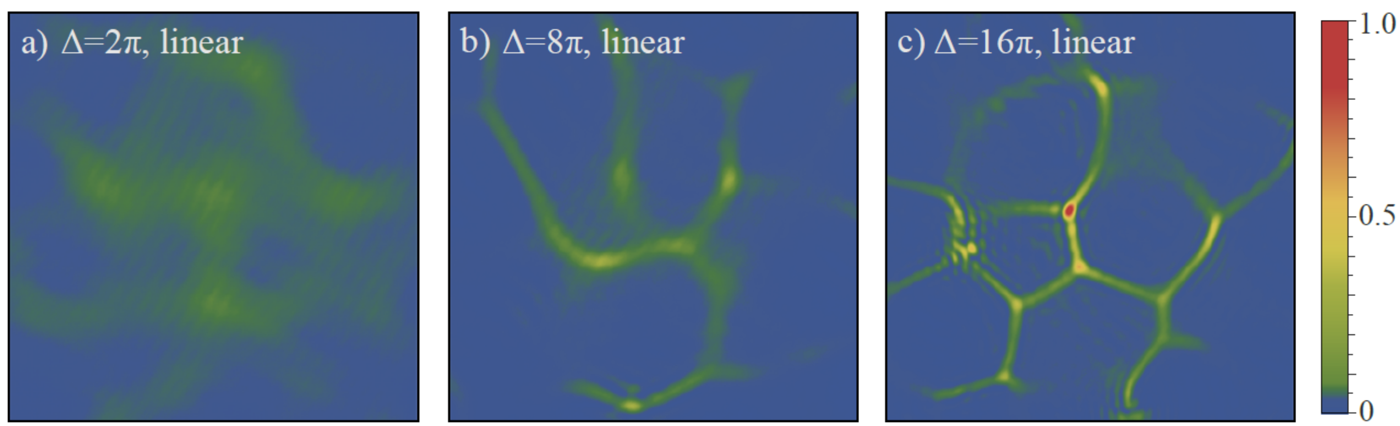
Generation of optical caustics



Phase variations:



Corresponding intensity variations:



A sharp caustic is formed only if the phase variations are large

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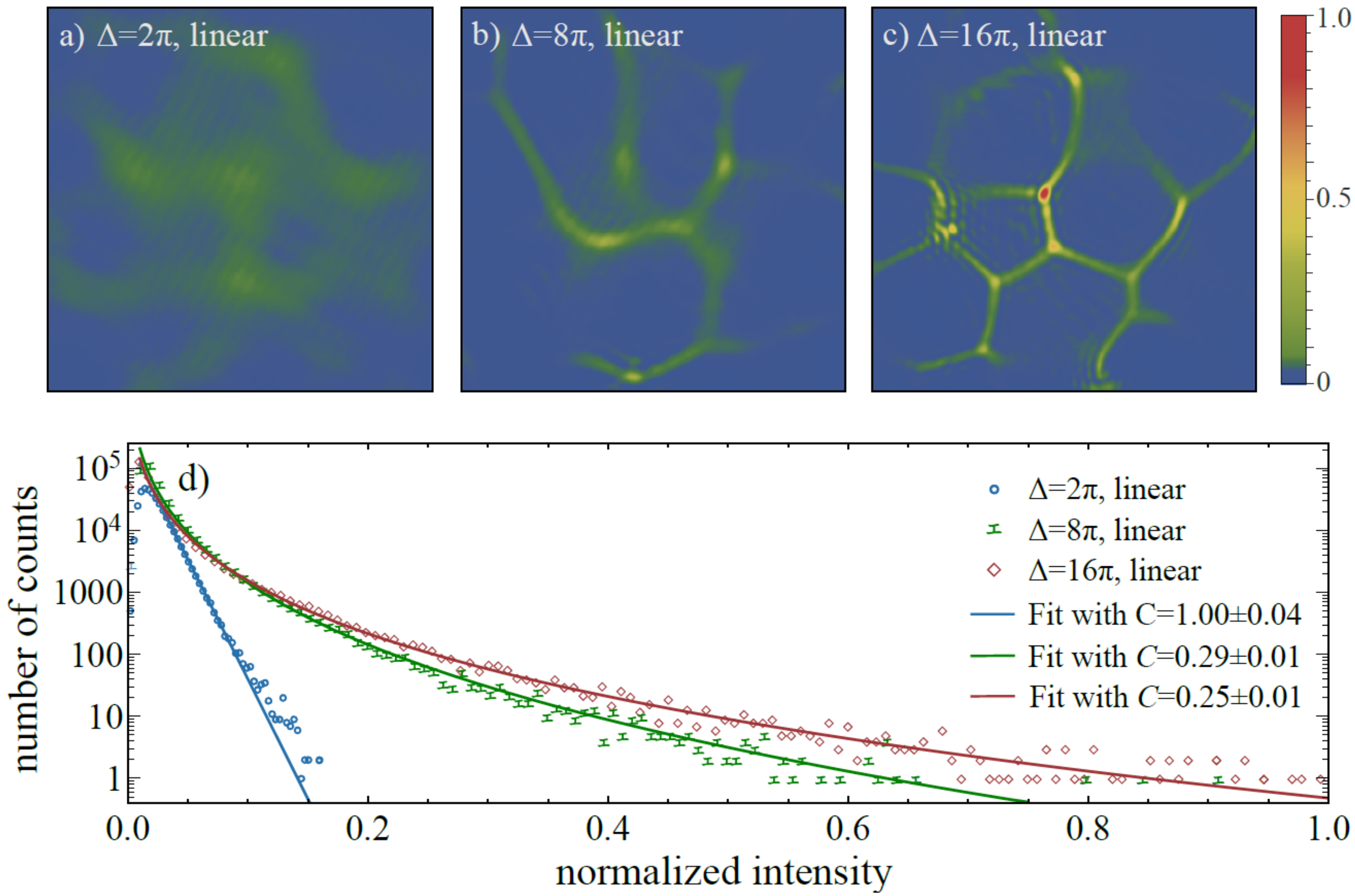
Caustics exhibit long-tailed probability distribution

1000 different patterns for each Δ

Intensity distributions with fit to $A\,Exp\left(-B\,I^{C}\right)$



A. Mathis, L. Froehly, S. Toenger, F. Dias, G. Genty & J. Dudley. *Scientific Reports* **5**, 1 (2015).

Caustics in ocean waves

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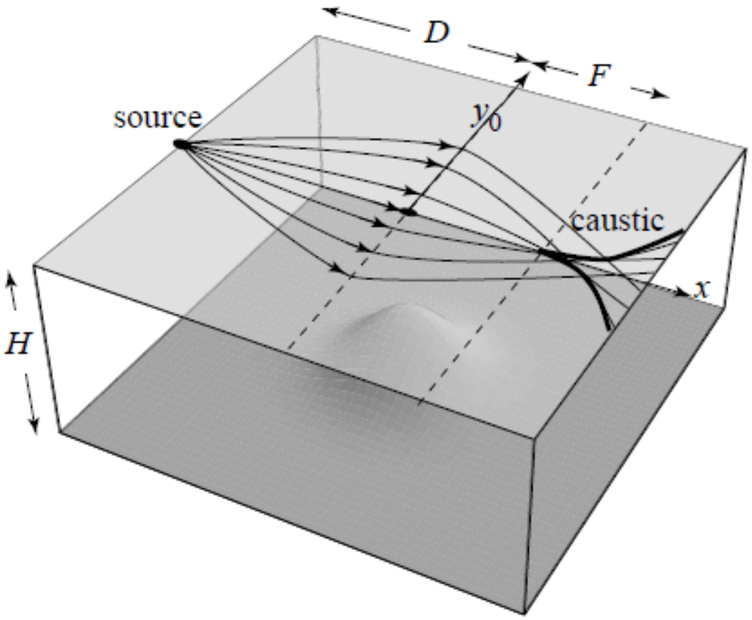
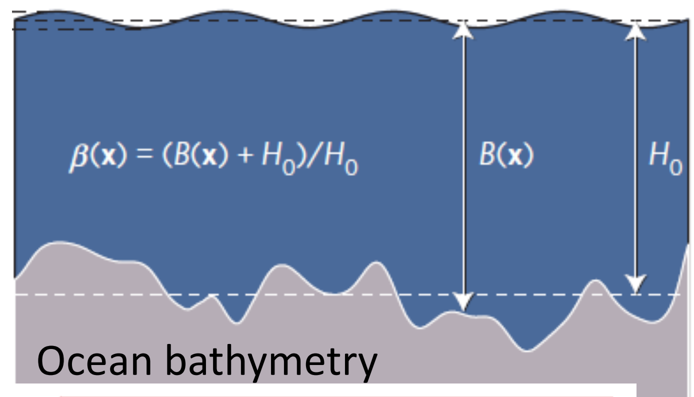
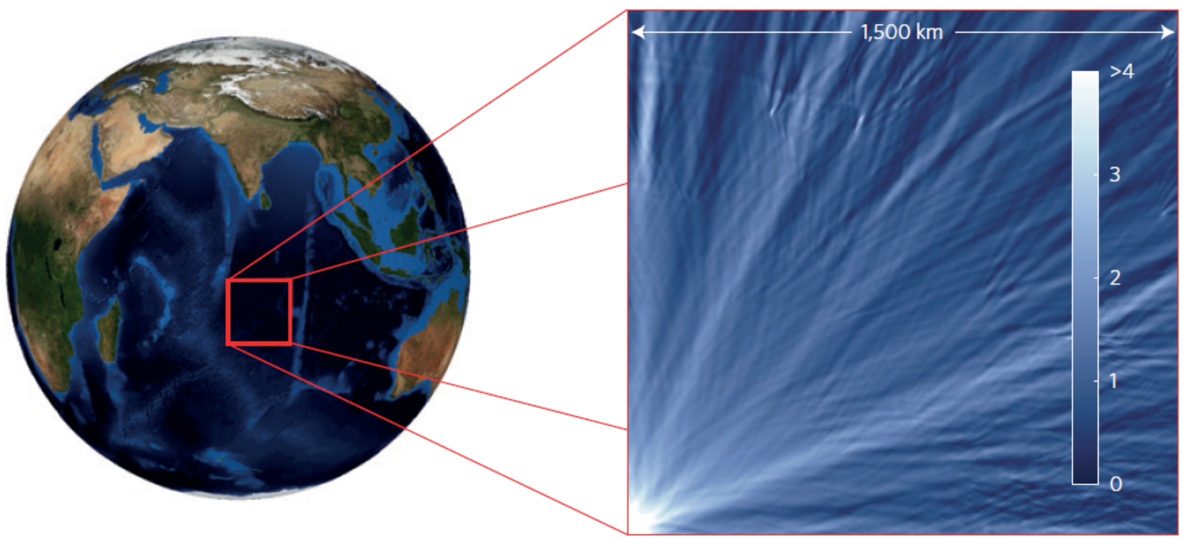


Caustic of tsunami focused by an underwater island lens

M. V. Berry, Focused tsunami waves, *Proc. R. Soc. A* (2007)



Simulated linear propagation of tsunami waves, using real ocean floor bathymetry:



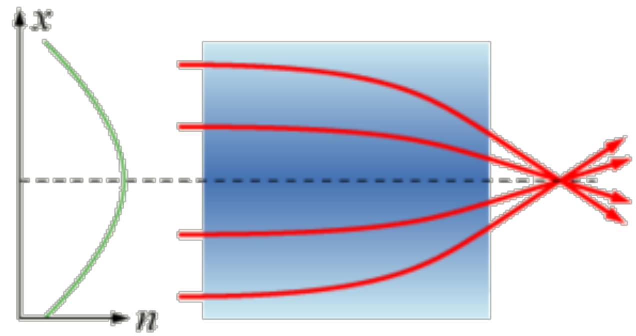
H. Degueldre, J. Metzger, T. Geisel and R. Fleischmann, *Nature Physics* **12, 259** (2016).

Nonlinear focusing

Self focusing:

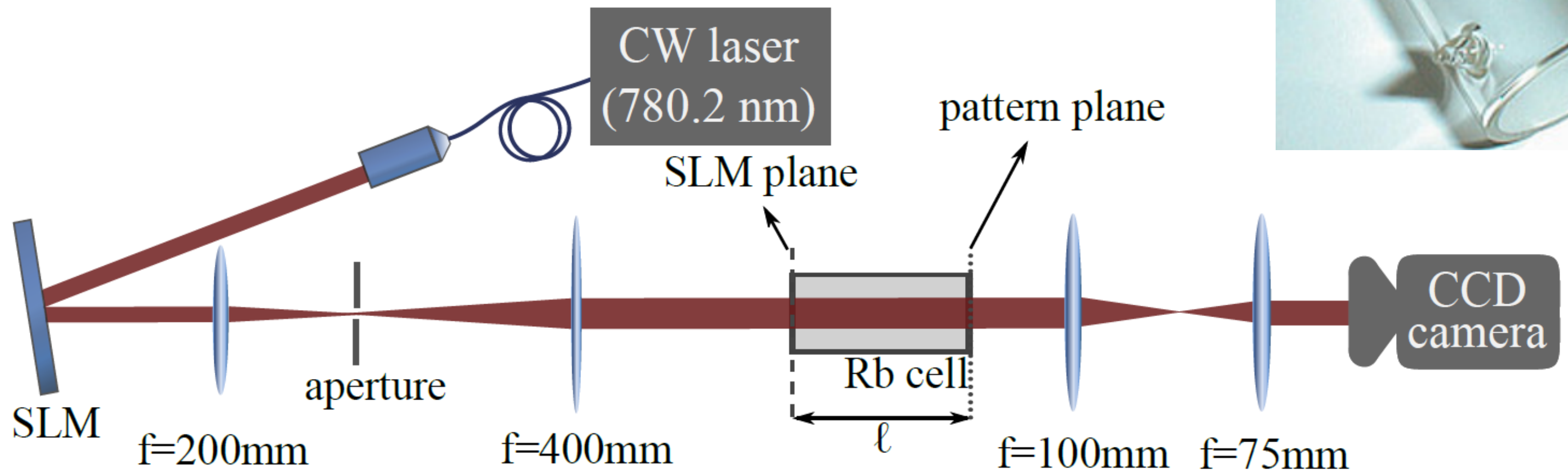
Refractive index depends on intensity:

$$n = n_0 + n_2 I$$



Rubidium cell

Rubidium vapors show large nonlinear effects

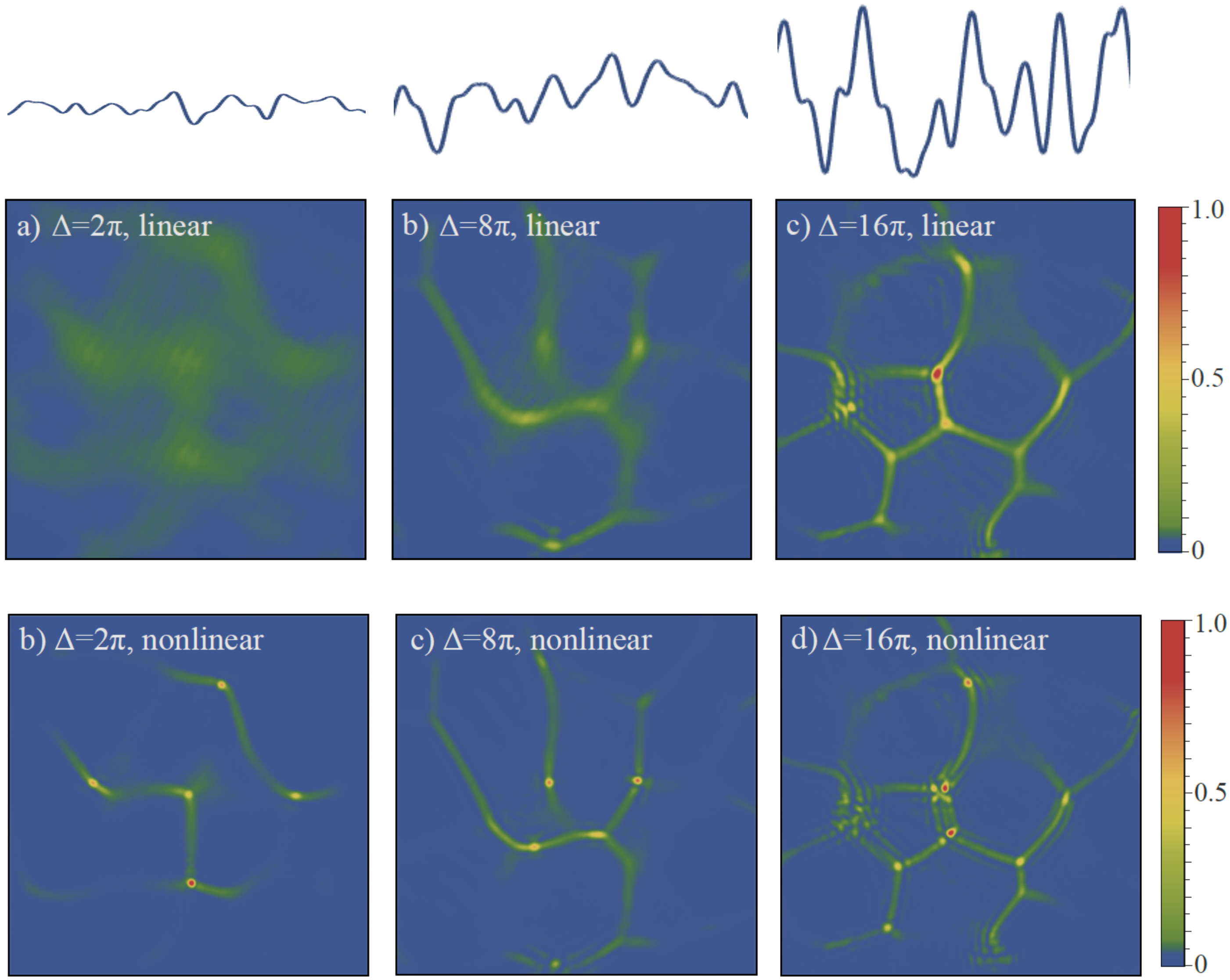


Effect of nonlinearity on caustics

Phase variations:

After linear propagation:

After nonlinear propagation:

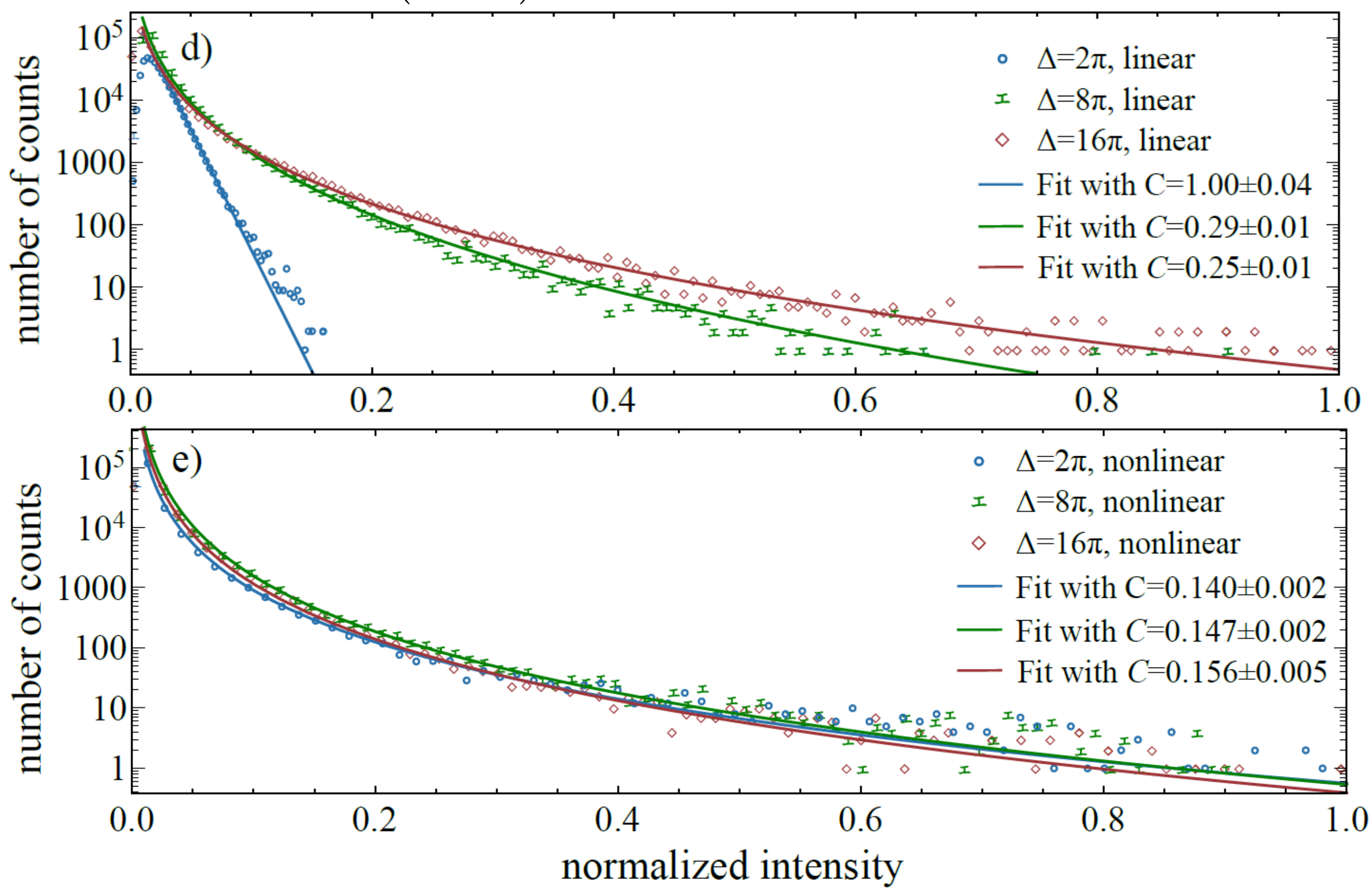


Statistics of caustics

Intensity distributions with fit to $A\,Exp\left(-B\,I^{C}\right)$

Linear propagation:

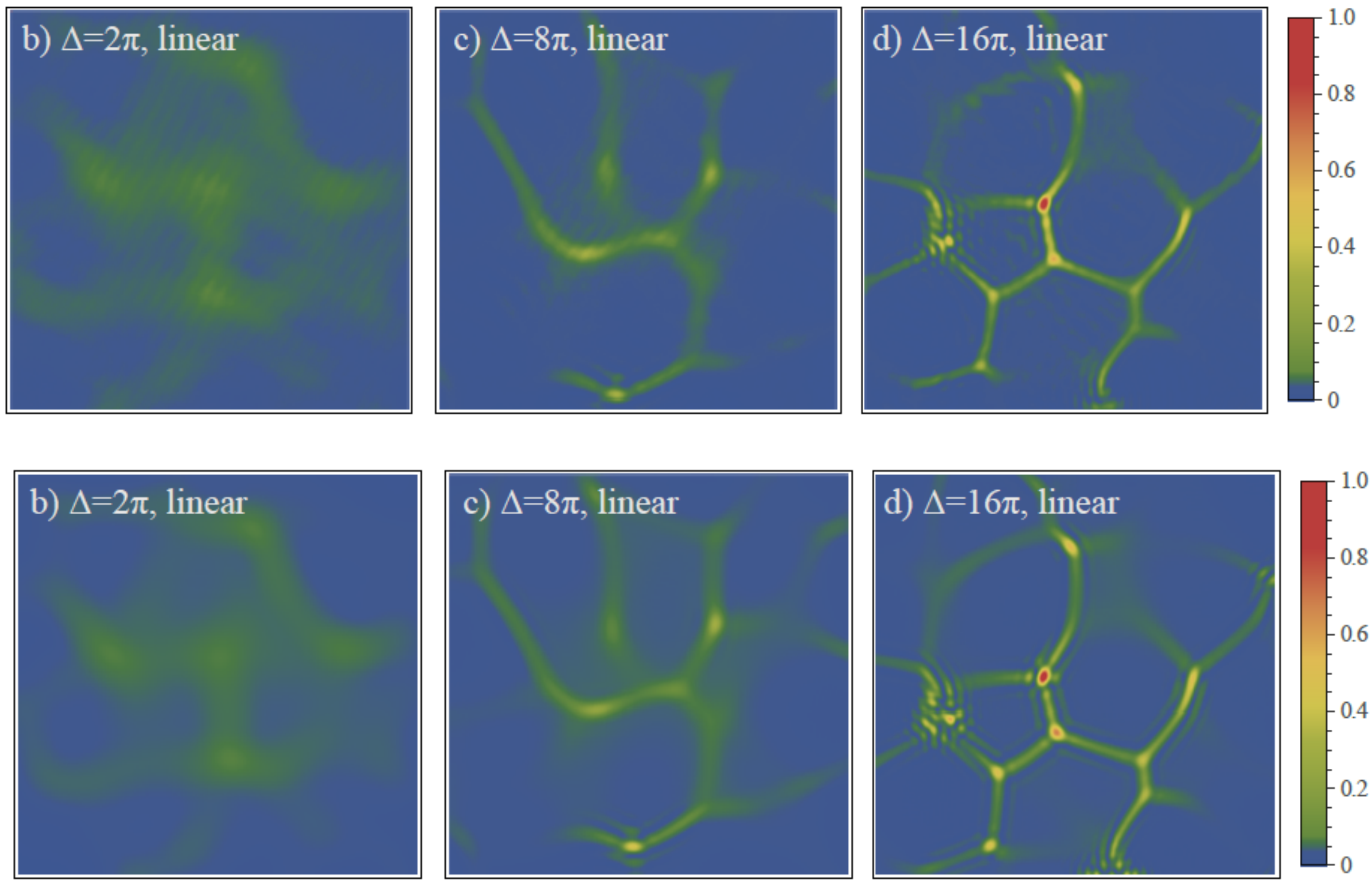
After nonlinear propagation:



Linear propagation was simulated by FFT beam propagation

Experiment:

Simulation:



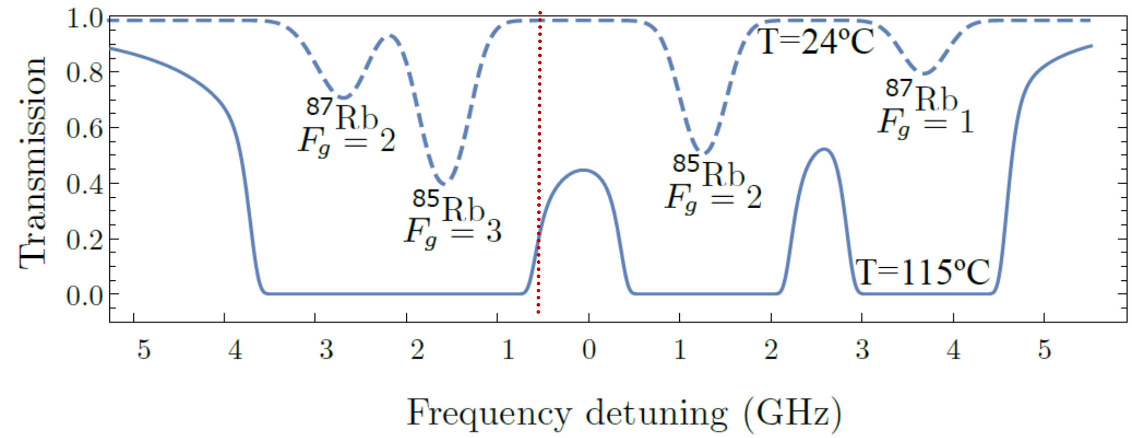
Simulation – Rb model

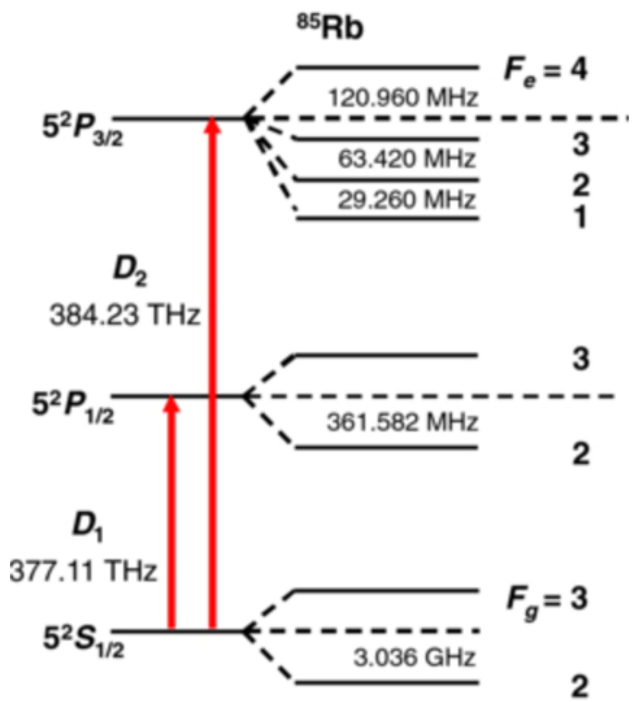
NLSE: $$\frac{\partial \mathcal{E}}{\partial z} - \frac{i}{2k}\nabla_\perp^2 \mathcal{E} = \frac{ik}{2\epsilon_0} P$$

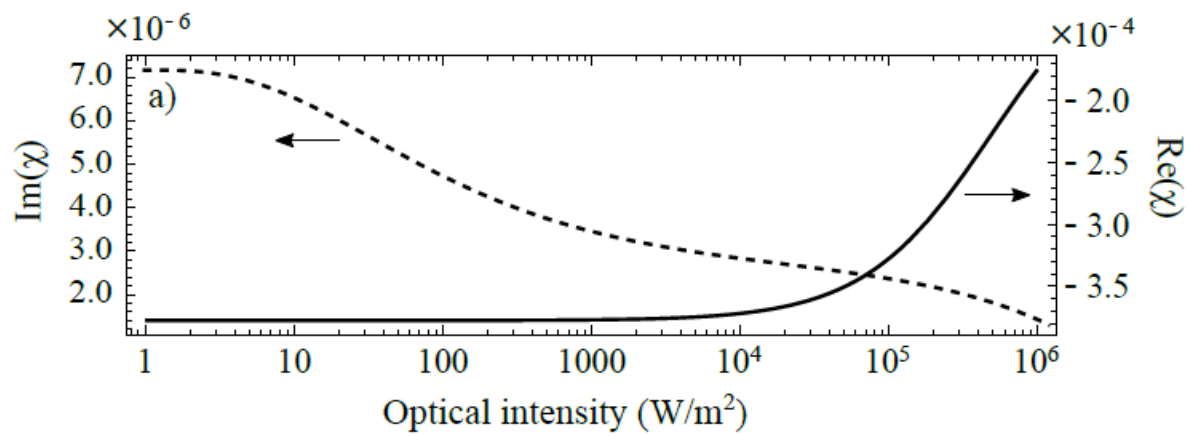
Atomic polarization: $P = \epsilon_0 \chi \mathcal{E}$

Our Rb model includes:

- All hyperfine transitions
- Doppler broadening
- Power broadening
- Collisional broadening
- Optical pumping





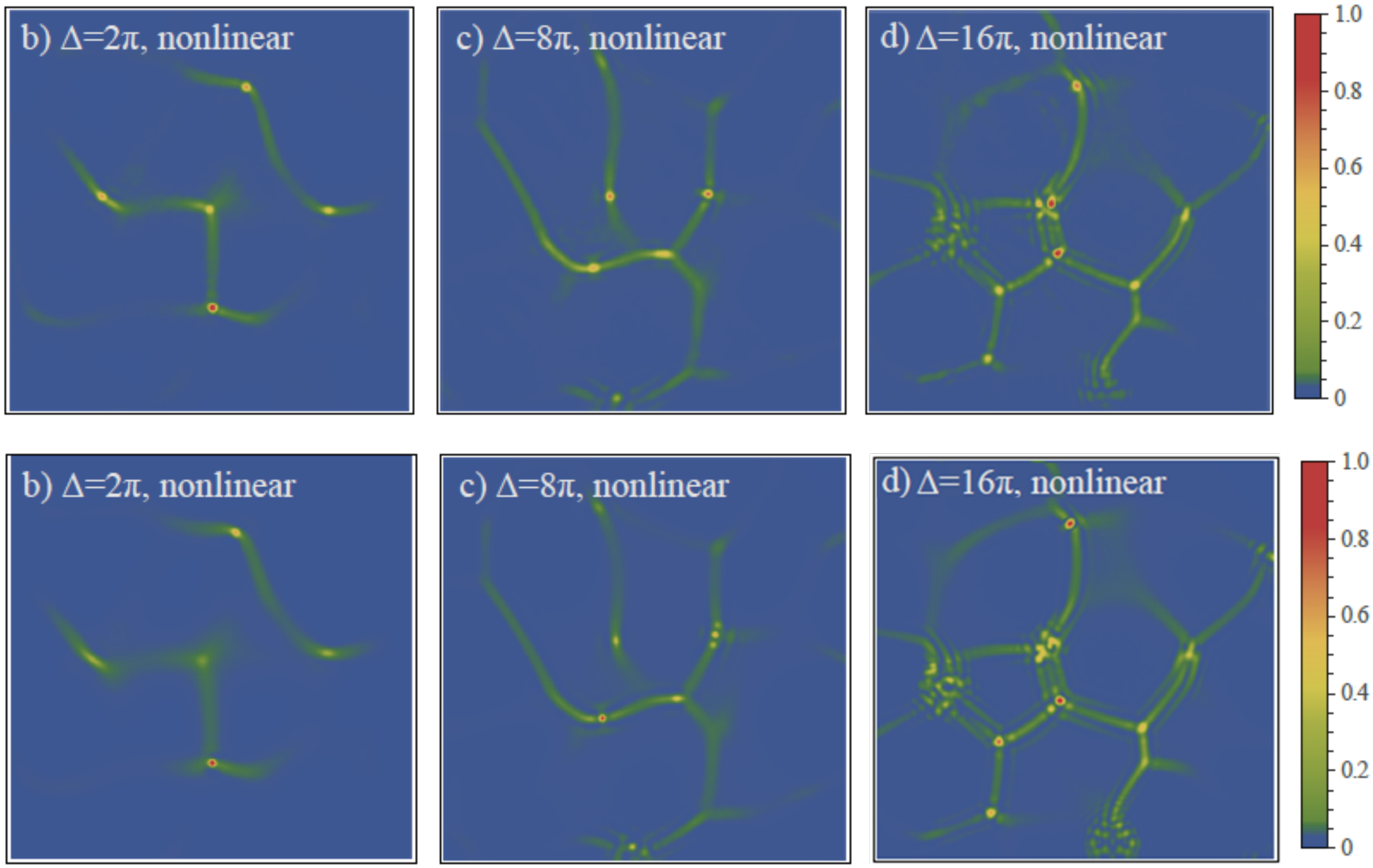


Simulation – Nonlinear propagation

Nonlinear propagation was simulated by FFT beam propagation and split-step

Experiment:

Simulation:

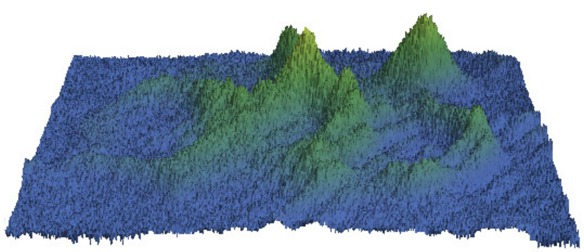


A. Safari, R. Fickler, M. Padgett, R. Boyd ,
Physical Review Letters 119, 203901 (2017)

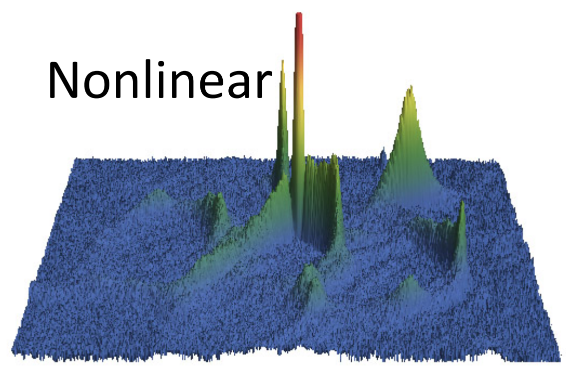
Conclusion

- Caustics are important:
  - Natural way of focusing energy of a wave
  - Can generate large amplitude waves
- Generation of caustics in linear space requires large fluctuation
- Nonlinear instability can generate caustics from small fluctuations

Linear



Nonlinear



A. Safari, R. Fickler, M. Padgett, R. Boyd , Physical Review Letters 119, 203901 (2017)

Thanks to:

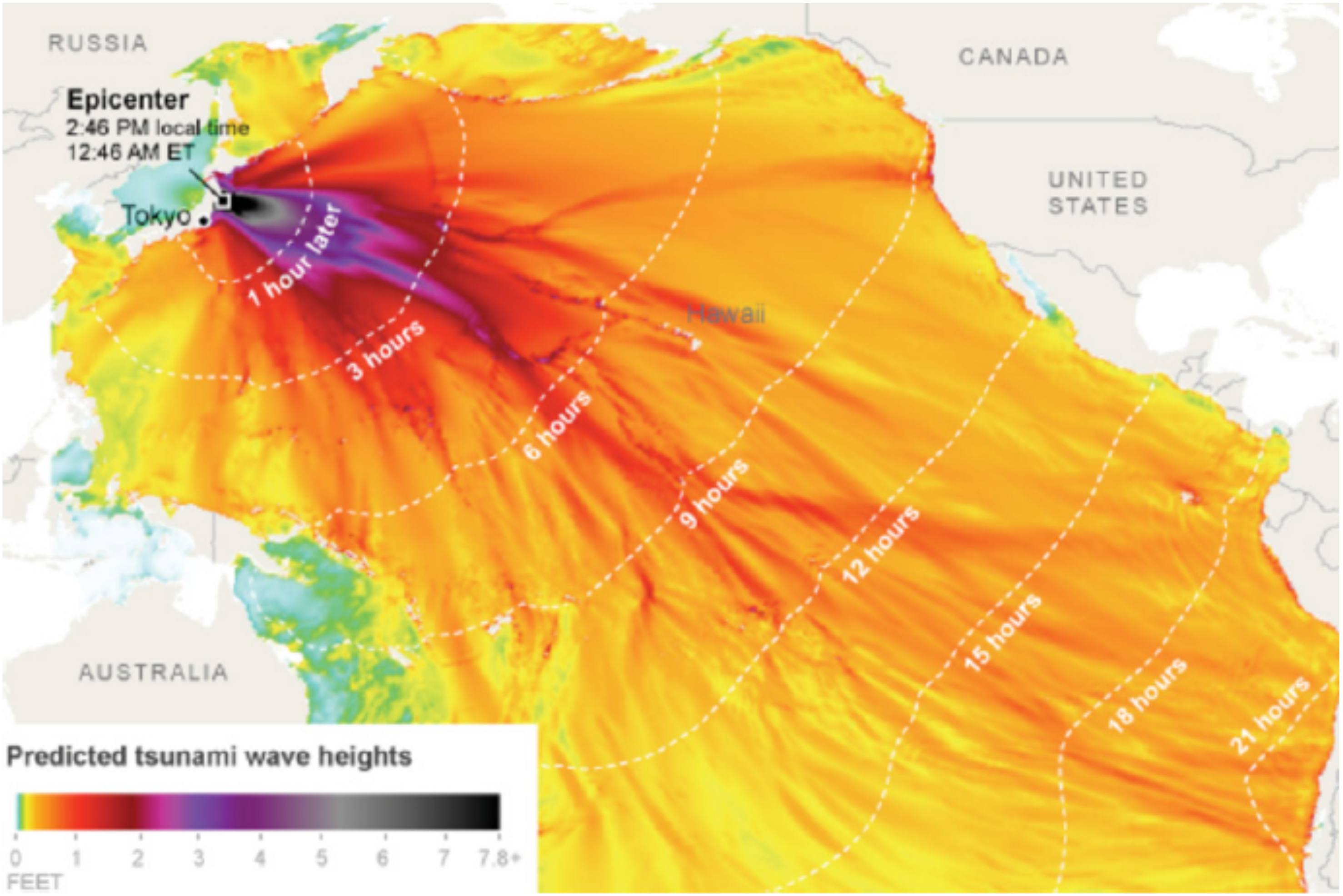
Akbar Safari

Robert Fickler

Miles Padgett

Thank you for your attention

Tsunami wave



Simulation – Nonlinear propagation

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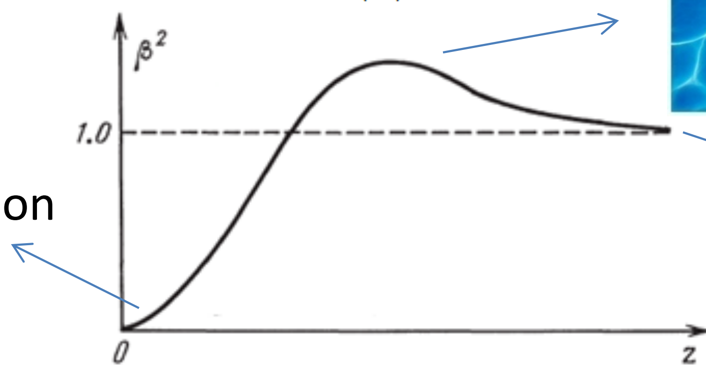
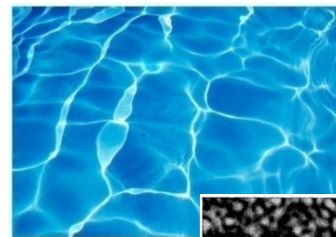
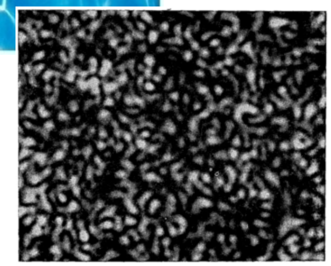
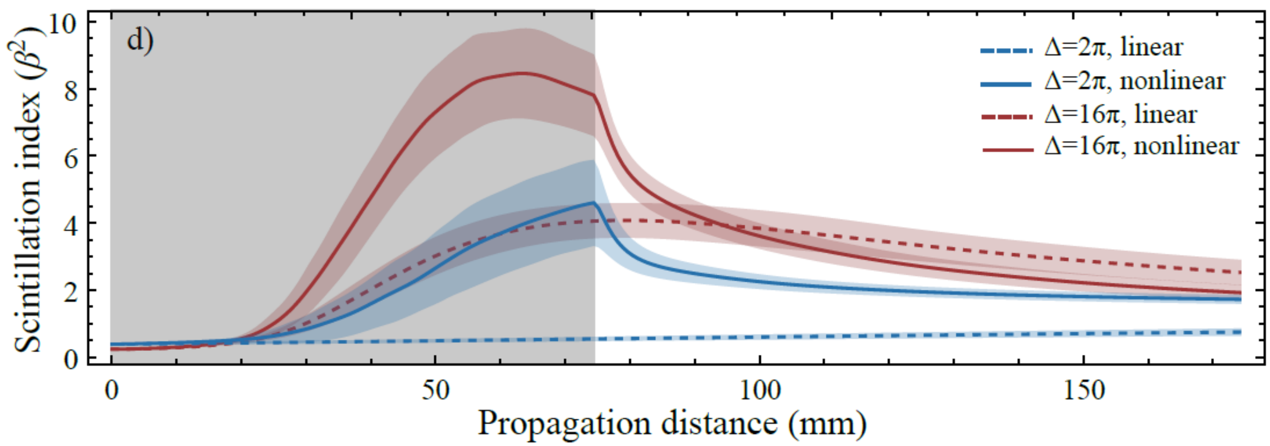
Scintillation index: $\beta^2 = \dfrac{\langle I^2 \rangle - \langle I \rangle^2}{\langle I \rangle^2}$

Uniform distribution









"*Caustics, Catastrophes and Wave Fields*", Kravtsov, Orlov (1993)

1D NLSE

Ocean:

$$i\left(\frac{\partial A}{\partial t}+v_g\frac{\partial A}{\partial x}\right)=\frac{\omega}{8k^2}\frac{\partial^2 A}{\partial x^2}+\frac{\omega k^2}{2}|A|^2 A$$

Optics:

$$\frac{\partial A}{\partial x}+k_1\frac{\partial A}{\partial t}+\frac{1}{2}ik_2\frac{\partial^2 A}{\partial t^2}=i\gamma|A|^2 A$$

Transform the frame and replace time with space in ocean wave based on $x = v_g t$

Ocean:

$$i\frac{\partial A}{\partial x}-\frac{k}{\omega^2}\frac{\partial^2 A}{\partial t^2}=k^3|A|^2 A$$

Optics:

$$\frac{\partial A}{\partial x}+\frac{1}{2}ik_2\frac{\partial^2 A}{\partial t^2}=i\gamma|A|^2 A$$

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2D NLSE

Ocean:

$$i\frac{\partial A}{\partial t}-\frac{\omega}{8k^2}\frac{\partial^2 A}{\partial x^2}+\frac{\omega}{4k^2}\frac{\partial^2 A}{\partial y^2}=\frac{\omega k^2}{2}\left|A\right|^2 A$$

Optics:

$$2ik\frac{\partial A}{\partial x}+\nabla_\perp^2 A=i\gamma\left|A\right|^2 A$$

1D vs 2D systems

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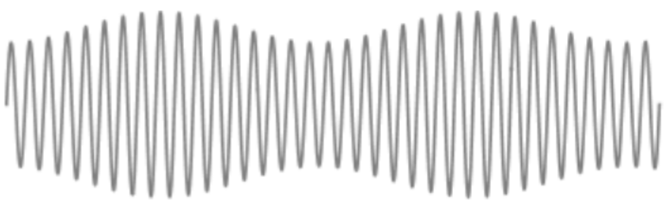
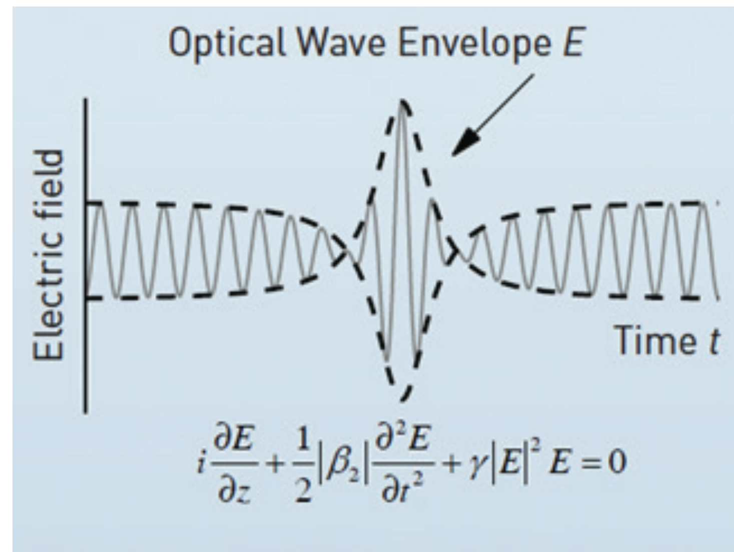
Nonlinear Schrödinger equation explains the wave dynamics in ocean as well as optics.

In 1D systems (studied extensively):

- Nonlinear modulational instability



Modulational instability



J.M. Dudley et al, Nat. Photon, 8, 755 (2014)

In 2D systems:

- Spatial (geometrical) focusing
- Nonlinear focusing