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Optics Communications 281 (2008) 3732–3738

OPTICS COMMUNICATIONS

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# Propagation of quantum states of light through absorbing and amplifying media

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Received 17 November 2007; received in revised form 12 March 2008; accepted 14 March 2008

**Abstract**

We describe how quantum features of light fields become modified upon propagation through absorbing and amplifying media. Both absorption and amplification add noise to a beam of light. We examine the extent to which quantum features remain after this noise is added. We also examine the question of whether certain quantum states are more robust than others against degradation due to loss. Quantum states of this sort would constitute an important resource for use in quantum information processing. We quantify this thought by determining how the integration time required to achieve a specified signal-to-noise ratio increases in the presence of transmission losses. We find that under certain circumstances the required integration time increases more rapidly with transmission loss for measurement strategies based on coincidence detection of entangled photons than for strategies based on the properties of squeezed light.


## 1. Rationale for this study

Are measurement strategies based on coincidence detection of entangled photons more robust against degradation due to loss than those based on the use of squeezed light? Or is it the converse that is true? The arguments supporting either conjecture are sometimes heard. The argument in support of the increased robustness when using entangled photons is that one can use post selection and simply reject events in which a photon is lost. However, the integration time required to acquire a requisite number of coincidence events increases rapidly with system loss. In contrast, when performing measurements based on the properties of squeezed light, attenuation of the light beam reduces both the intensity of the light and the amount of squeezing that is present. In this sense, squeezed light is more susceptible to degradation due to attenuation than is coincidence detection. Also, there is no way to recover the loss of squeezing by a process analogous to post selection. In this paper we perform a careful examination of how the quantum features of light fields are degraded upon propagation through attenuating and amplifying media, with the goal of determining the sense in which certain quantum states are more useful than others at performing certain types of measurements.

## 2. Review of earlier work

As a first step, we present a review of some of the relevant scientific literature.

Shimoda et al. [1] and Haus and Mullen [2] have presented analyses that treat the amount of noise added to a light wave by a laser amplifier. The second of these papers establishes the oft-quoted result that the noise figure of a high-gain optical amplifier can be no smaller than 2.0.

Hong et al. [3] describe how quantum features (sub-Poissonian statistics and squeezing) are modified by the

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doi:10.1016/j.optcom.2008.03.022

amplification achieved by a "three-level" laser amplifier. This analysis does not necessarily apply to other types of amplification, such as that resulting from optical paramatreic amplification. Their conclusion is the following: an amplitude gain $G$ greater than $\sqrt{2}$ will always remove squeezing and sub-Poissonian statistics from a light field. If the amplifier is fully inverted, a gain of $\sqrt{2}$ is required. If the amplifier is partially inverted ($N_2 > N_1$ but $N_1$ non-vanishing; where $N_1$ and $N_2$ are the lower and upper population densities, respectively) the gain required to remove quantum features is $(2N_2/(N_1 + N_2))^{1/2}$ which can be much smaller than $\sqrt{2}$. They also address the more general issue of the conditions under which the output cannot show any non-classical features. They take as the sufficient condition for classical behavior that the phase space density $\phi(v)$ be positive semi-definite. They find that the condition for a purely classical output, for any input, is $G > \sqrt{N_2/N_1}$. Note that for a four-level laser (that is, a laser for which $N_1 = 0$) this condition can never be achieved. Thus, for such a laser amplifier, the gain can be arbitrarily large and still allow the possibility of some quantum features in the output of the amplifier.

Agarwal and Tara [4] present an analysis of the transformation of non-classical states of light by an optical amplifier. They make use of the $P$ distribution. Their primary interest is the situation in which the output of the amplifier shows neither squeezing nor sub-Poissonian statistics but still possesses non-classical features. Thus, the condition for removing squeezing and sub-Poissonian statistics from a beam of light is different from that of removing all quantum features of a beam of light. In particular, more noise must be added (that is, there must be more amplification) to remove all quantum features than simply to remove squeezing or sub-Poissonian features. It should be borne in mind that the conditional removal of photons can lead to a whole new class of states. For example, in a more recent work [5], it has been shown that if a single photon is removed from a squeezed vacuum state, the resulting state is still highly non-classical.

Loudon [6] presents an analysis of the propagation of non-classical light. His results are best summarized by quoting several sentences from his conclusion: "Non-classical features tend to survive transmission through an attenuating slab, although their magnitudes may be reduced. The effects of transmission through an amplifying slab are more drastic, and non-classical features are usually lost, even for modest values of the gain. Thus for the phase-insensitive amplification considered here, both the antibunching and squeezing effects are lost for intensity gains with maximum values of order 2." A more detailed analysis of the same system was published by Artoni and Loudon [7] the following year. Their approach is to determine the consequences of a Langevin noise source that must accompany the presence of a gain or loss term.

Kim et al. [8] note that a beam splitter can be used to create entangled output fields. They show that the necessary condition for the output fields of a beam splitter to show entanglement is that the input fields show non-classical behavior. In an earlier work, Diament and Teich [9] study in great detail the changes in photon statistics of a light beam as it propagates through a travelling-wave amplifier. However, they deal explicitly with more traditional states.

Leuchs and Andersen [10] present a review of the effects of dissipation on quantum states of the radiation field. The first two sentences of their abstract are quoted here: "We point out similarities in the evolution of different types of non-classical light fields. Generally, Fock and Cat states are considered to decay much faster under dissipation than do squeezed states."

## 3. Review of some simple quantum optics systems

### 3.1. Conversion from squeezing to entanglement by a beam splitter

Several recent studies have made use of the properties of a beam splitter to convert squeezed light into entangled light beams. For example, Furusawa et al. [11] made use of this conversion in their demonstration of unconditional quantum teleportation. In this experiment, two squeezed vacuum fields were made to interfere on a beam splitter. This situation (along with other related processes) has been analyzed in detail by Leuchs et al. [12]. This paper presents a very good intuitive explanation of why a beam splitter converts squeezing at the input into entanglement of the fluctuations at the output. In addition, Silberhorn et al. [13] have performed an experiment demonstrating that it is possible to create two independent, amplitude-squeezed light fields by means of the Kerr nonlinearity of an optical fiber and then allow these fields to interfere at a beam splitter to form two entangled light fields. In this experiment, the authors demonstrated entanglement by showing that if they measured the amplitude fluctuations in one output port, they were strongly correlated (that is, to better than the shot-noise limit) to those in the other output port. Conversely, if they chose to measure the phase noise in one output beam, it was strongly correlated to that in the other output beam.

### 3.2. Influence of attenuation on squeezing

The influence of attenuation on squeezing is well treated in the book of Bachor and Ralph [14]. Here we summarize their conclusions. Let the variance along each principal axis of the squeezing ellipse be $V_1$ and $V_2$ so that $V_1 = \exp(-2r)$ and $V_2 = \exp(2r)$, where $r$ is the squeezing parameter. Let $\eta$ be the intensity transmission of the linear, phase-insensitive attenuator, which is conveniently modeled as a beam splitter. Then it is easy to show that

$$V_1(\text{out}) = \eta V_1(\text{in}) + (1 - \eta) \qquad (1)$$

Here the second term can be interpreted as the influence of vacuum fluctuations entering through the additional port of the beam splitter. This result can be rewritten as $[1 - V_1(\text{out})] = \eta[1 - V_1(\text{in})]$. This form is particularly useful, as the quantity $1 - V$ can be interpreted as the degree of noise reduction, which is seen to decrease linearly with attenuation. Note that some degree of squeezing will remain even for arbitrarily small values of $\eta$, but the effect is unlikely to be useful for losses greater than about 50%. One can also ask whether the output remains a minimum uncertainty state if the input is a minimum uncertainty state. One finds by multiplying the equation for $V_1(\text{out})$ by an analogous equation for $V_2(\text{out})$ that $V_1(\text{out})\,V_2(\text{out}) = [\eta V_1(\text{in}) + (1-\eta)][\eta V_1(\text{in}) + (1-\eta)]$. A plot of this form is shown in the book. One finds that the uncertainty product exceeds unity except for $\eta = 1$ (no loss) and for $\eta = 0$ (complete loss, because the vacuum state is itself a minimum uncertainty state). For all other values of $\eta$, the output is a mixed state and clearly is not a minimum uncertainty state.

## 4. Are some quantum states more robust against attenuation than others?

We first address this question at a conceptual level by considering two different representative applications of quantum states of light:

1. *Enhanced measurement sensitivity using squeezed light.* This is the situation analyzed by Bachor and Ralph that is described above. We saw that there is no fundamental limit to how much loss squeezed light can experience, but that the benefits of using squeezed light are largely eliminated for a loss of greater than about 50%.
2. *Ghost imaging (and other measurements involving coincidence detection).* Let us consider the standard configuration for ghost imaging, as illustrated in Fig. 1. Here pairs of entangled photons are created in a nonlinear crystal by means of the process of spontaneous parametric downconversion. For the present, we assume that the pump intensity is sufficiently low that there is negligible chance that more than two photons will be generated during the resolution time of the detection system.

We consider the influence of loss (for instance, less-than unity quantum efficiency of the detectors) in either of the two pathways. Because the procedure utilizes coincidence detection, if either of the detectors does not fire, the result will be that no count is registered for that event. Thus, the

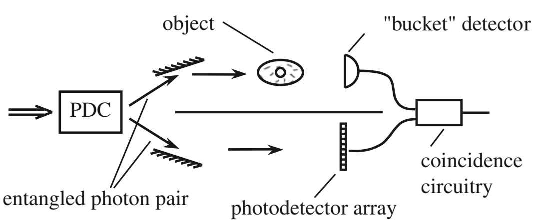


Fig. 1. Ghost (or coincidence) imaging.

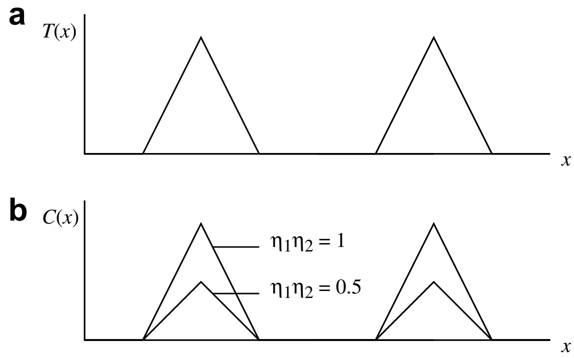


Fig. 2. (a) Transmission of object as a function of the transverse coordinate. (b) Coincidence count rate as a function of position in the lower arm of the setup of Fig. 1. Note that the only consequence of loss in either arm is a decrease in count rate.

only consequence of loss will be to increase the integration time required to obtain an image of prescribed quality. The integration time will be increased by the factor $(\eta_1\eta_2)^{-1}$, where $\eta_1$ and $\eta_2$ are the transmissions of the two arms. This conclusion should be examined with some care, because it is perhaps not immediately clear that excess noise is not introduced by the loss of quantum efficiency. For instance, if the object is an amplitude object and the bucket detector does not fire, this could mean either that the photon was absorbed by the object (which conveys information about the object) or that the photon hit the detector but the detector did not fire (which seems to convey inaccurate information about the object). Since there is uncertainty as to which of these events occurred, it seems as if noise is introduced into the measurement process. But in fact this noise, if we are to call it noise, is simply accounted for by a decreased count rate. This conclusion can be seen from the argument presented in Fig. 2. We see that the dark regions of the object will show up dark in the coincidence image, because no signal photons hit the bucket detector in this case. But we also see that, the bright regions will show up bright, but with a decreased count rate.

## 5. Mathematical modeling of the influence of attenuation on squeezing

In this section we develop a mathematical model of the two prototypical quantum detection systems shown in Fig. 3. In both cases, the light source is an unseeded optical parametric amplifier (OPA) of arbitrary gain. The common situation of spontaneous parametric downconversion is thus included as the limit of low gain. We treat the situation of arbitrary OPA gain both for generality and for the practical reason that many applications require the use of an intense light source. In part (a) of the figure we consider detection based on coincidence count rates and in (b) we consider detection based on squeezed light generation. In both cases, we are concerned with how the statistical properties of the detection process are modified by the presence of attenuator or amplifiers placed at various loca-

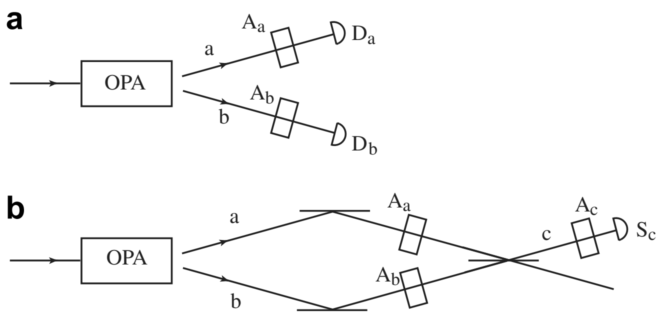


Fig. 3. (a) Parametric downconversion leads to the generation of entangled photons in each of modes $a$ and $b$. The nature of the coincidence counts is modified by placing an attenuator A in either output arm. (b) The two output beams are combined at a beamsplitter to produce squeezed light in mode $c$. We study the influence of attenuation on the degree of squeezing, which is measured by the circuit shown symbolically as $S_c$. The attenuators can be placed in modes $a$, $b$, or $c$.

tions in the optical system. We denote the amplitude transmission of the attenuator/amplifier as $t$, where $|t|^2 < 1$ for an attenuator and $|t|^2 > 1$ for an amplifier.

We represent the quantum state of the optical field leaving the OPA as [15,16]

$$|\Psi\rangle = (1-\rho^2)^{1/2} \sum_{n=0}^{\infty} \rho^n |n,n\rangle \tag{2}$$

where the notation $|n,n\rangle$ indicates that there are $n$ photons in the signal mode and $n$ photons in the idler mode and where

$$\rho = \mathrm{i}e^{\mathrm{i}\phi} \tan\mathrm{h}|G| \tag{3}$$

with $G = |G|\exp(\mathrm{i}\phi)$ is the gain parameter proportional to the intensity of the pump field.

Our primary interest is in how a quantum light field becomes modified upon passing through an attenuator or an amplifier. We represent the action of an attenuator as

$$a_{\mathrm{out}} = t\,a_{\mathrm{in}} + f_a \tag{4}$$

Here $f_a$ is an operator that represent the noise added to the light beam as a consequence of the attenuation process. To determine the nature of this noise operator, we note that the photon operators must obey standard commutation relations, that is,

$$[a_{\mathrm{out}}, a_{\mathrm{out}}^\dagger] = 1 \quad \text{and} \quad [a_{\mathrm{in}}, a_{\mathrm{in}}^\dagger] = 1 \tag{5}$$

The simultaneous validity of these relations requires that the noise operator $f_a$ be present and obey the commutation relation

$$[f_a, f_a^\dagger] = (1 - |t|^2) \tag{6}$$

where we assume that

$$\langle f_a^\dagger f_a\rangle = 0 \quad \text{and consequently that} \quad \langle f_a f_a^\dagger\rangle \neq 0 \tag{7}$$

We are now ready to determine how the presence of loss modifies each of the detection processes. We first consider the situation shown in part (a) of Fig. 3. In this situation, the count rate at detector $D_a$ is given by $\langle a^\dagger a\rangle$, the count rate at detector $D_b$ is given by $\langle b^\dagger b\rangle$, and the joint detection rate is given by $\langle a^\dagger a b^\dagger b\rangle$, where the field operators are to be evaluated at the detector after passing through any attenuators or amplifiers, and where we are using the notation $\langle a^\dagger a\rangle = \langle\psi \mid a^\dagger a \mid \psi\rangle$, etc, where $|\psi\rangle$ is the state of the light field. Using Eqs. (2), (6), and (7) we find that

$$\langle a_{\mathrm{out}}^\dagger a_{\mathrm{out}}\rangle = |t_a|^2\langle a_{\mathrm{in}}^\dagger a_{\mathrm{in}}\rangle = |t_a|^2(1-|\rho|^2)\sum_{n=0}^{\infty} n|\rho|^{2n} = |t_a|^2\left(\frac{|\rho|^2}{1-|\rho|^2}\right) \tag{8}$$

with a similar result for mode $b$. We also find for the coincidence count rate that

$$\langle a_{\mathrm{out}}^\dagger a_{\mathrm{out}} b_{\mathrm{out}}^\dagger b_{\mathrm{out}}\rangle = |t_a|^2|t_b|^2\langle a_{\mathrm{in}}^\dagger a_{\mathrm{in}} b_{\mathrm{in}}^\dagger b_{\mathrm{in}}\rangle = |t_a|^2|t_b|^2\left[\frac{|\rho|^2(1+|\rho|^2)}{(1-|\rho|^2)^2}\right] \tag{9}$$

The key conclusions are that the individual count rate for mode $i$ for $i = a, b$ scales with the transmission amplitude $t_i$ as $|t_i|^2$ and that the coincidence count rate scales with the amplitude transmissions $t_a$ and $t_b$ as $|t_a|^2|t_b|^2$.

The analysis for the situation in which an amplifier is placed in the light path is different from that of an attenuator in that an amplifier truly "adds" noise. This situation can be described by taking [17]

$$a_{\mathrm{out}} = t_a\,a_{\mathrm{in}} + f_a^\dagger \tag{10}$$

where now $|t_a|^2 > 1$ and where commutation relations now require that

$$[f_a, f_a^\dagger] = (|t_a|^2 - 1) \tag{11}$$

where we assume that

$$\langle f_a^\dagger, f_a\rangle = 0 \quad \text{and consequently that} \quad \langle f_a, f_a^\dagger\rangle \neq 0 \tag{12}$$

We then find that

$$\langle a_{\mathrm{out}}^\dagger a_{\mathrm{out}}\rangle = \langle\psi|(t_a^* a_{\mathrm{in}}^\dagger + f_a)(t_a a_{\mathrm{in}} + f_a^\dagger)|\psi\rangle \tag{13}$$

$$= |t_a|^2\langle a_{\mathrm{in}}^\dagger a_{\mathrm{in}}\rangle + (|t_a|^2 - 1) \tag{14}$$

We thus see that there can be an output even in the presence of only a vacuum input. We similarly find that the coincidence count rate is given by

$$\begin{aligned}\langle a_{\mathrm{out}}^\dagger a_{\mathrm{out}} b_{\mathrm{out}}^\dagger b_{\mathrm{out}}\rangle &= \langle\psi|(t_a^* a_{\mathrm{in}}^\dagger + f_a)(t_a a_{\mathrm{in}} + f_a^\dagger)(t_b^* b_{\mathrm{in}}^\dagger + f_b)(t_b b_{\mathrm{in}} + f_b^\dagger)|\psi\rangle \\ &= |t_a|^2|t_b|^2\langle a_{\mathrm{in}}^\dagger a_{\mathrm{in}} b_{\mathrm{in}}^\dagger b_{\mathrm{in}}\rangle + (|t_a|^2-1)|t_b|^2\langle b_{\mathrm{in}}^\dagger b_{\mathrm{in}}\rangle \\ &\quad + (|t_b|^2-1)|t_a|^2\langle a_{\mathrm{in}}^\dagger a_{\mathrm{in}}\rangle + (|t_a|^2-1)(|t_b|^2-1)\end{aligned} \tag{15}$$

For the detection of squeezing (part (b) of Fig. 3), the analysis is somewhat more involved. To describe the degree of squeezing, we first define the quadrature amplitude operator $x_\theta$ as

$$x_\theta = c e^{\mathrm{i}\theta} + c^\dagger e^{-\mathrm{i}\theta} \tag{16}$$

We then define the squeezing parameter $S$ as

$$S = [\langle x_\theta^2\rangle - \langle x_\theta\rangle^2 - 1] \tag{17}$$

which can alternatively be written as

$$S = \langle : (c\mathrm{e}^{\mathrm{i}\theta} + c^\dagger \mathrm{e}^{-\mathrm{i}\theta})^2 : \rangle - \langle c\mathrm{e}^{\mathrm{i}\theta} + c^\dagger \mathrm{e}^{-\mathrm{i}\theta}\rangle^2 \tag{18}$$

According to this prescription, squeezing occurs if $S$ is negative, and for $\langle x_\theta\rangle = 0$ perfect squeezing occurs if $\langle x_\theta^2\rangle = 0$ implying that $S = -1$.

For the situation of Fig. 3b, we need to relate the squeezed mode $c$ to the output modes $a$ and $b$ of the OPA. We define

$$c = \frac{a + ib}{\sqrt{2}} \tag{19}$$

We then find (ignoring attenuation, for the present) that

$$S = \langle : (c + c^\dagger)^2 : \rangle = \langle a^\dagger a + b^\dagger b\rangle - \mathrm{Im}\langle ab\rangle \tag{20}$$

We next note that for the $\theta = 0$ quadrature the quantity $\langle ab\rangle$ for the output mode can be expressed as

$$\begin{aligned}\langle a_{\mathrm{out}} b_{\mathrm{out}}\rangle &= t_a t_b \langle a_{\mathrm{in}} b_{\mathrm{in}}\rangle \\ &= t_a t_b (1 - |\rho|^2)\rho \sum_{n=0}^{\infty} (n+1)|\rho|^{2n} \\ &= t_a t_b \left(\frac{\rho}{1 - |\rho|^2}\right)\end{aligned} \tag{21}$$

This result is obtained by means of a calculation similar to that leading to Eq. (8). Thus for $t_a = t_b = t$,

$$S = t^2\left(\frac{2|\rho|^2}{1 - |\rho|^2} - \frac{\rho}{1 - |\rho|^2}\right) = t^2\left[\frac{(2\rho - 1)\rho}{1 - \rho^2}\right] \tag{22}$$

Note that, for $\rho < 1/2$, $S$ is negative, implying the presence of squeezing. We can now determine how the degree of squeezing depends on the presence of attenuators in modes $a$ and $b$. For the special case in which $t_a = t_b = t$ and $t$ is real, we find that

$$S_{\mathrm{out}} = |t|^2 S_{\mathrm{in}} \tag{23}$$

Thus, squeezing degrades less rapidly with decreasing system transmission than does the coincidence count rate of Eq. (9). The reason is simply that squeezing depends on the second power of the field whereas the coincidence count rate is fourth-order in the field.

Similar conclusions apply if the attenuator is placed in mode $c$. In this case, we find that

$$c_{\mathrm{out}} = t c_{\mathrm{in}} + f_c \tag{24}$$

where

$$\langle f_c^\dagger f_c\rangle = 0 \qquad \text{and} \qquad \langle f_c f_c^\dagger\rangle = (1 - |t|^2) \tag{25}$$

It follows that

$$\langle : (c_{\mathrm{out}} + c_{\mathrm{out}}^\dagger)^2 : \rangle = |t|^2 \langle : (c_{\mathrm{in}} + c_{\mathrm{in}}^\dagger)^2 : \rangle \tag{26}$$

or again that

$$S_{\mathrm{out}} = |t|^2 S_{\mathrm{in}} \tag{27}$$

We thus have verified the statement mentioned above that squeezing is never lost by attenuation.

The analysis proceeds differently if an amplifier rather than attenuator is placed in the light beam. We consider the situation in which the amplifier is placed in mode $c$. Through use of Eqs. (11) and (12) we find that $\langle c_{\mathrm{out}}^2\rangle = t_c^2\langle c_{\mathrm{in}}^2\rangle$, $\langle c_{\mathrm{out}}^{\dagger 2}\rangle = t_c^{*2}\langle c_{\mathrm{in}}^{\dagger 2}\rangle$, and

$$\langle c_{\mathrm{out}}^\dagger c_{\mathrm{out}}\rangle = |t_c|^2\langle c_{\mathrm{in}}^\dagger c_{\mathrm{in}}\rangle + (|t_c|^2 - 1) \tag{28}$$

We then find that

$$\begin{aligned} S_{\mathrm{out}} &= \langle c_{\mathrm{out}}^2 + c_{\mathrm{out}}^{\dagger 2} + 2c_{\mathrm{out}}^\dagger c_{\mathrm{out}}\rangle &(29)\\ &= |t_c|^2\langle : (c_{\mathrm{in}} + c_{\mathrm{in}}^\dagger)^2 : \rangle + (|t_c|^2 - 1) &(30)\end{aligned}$$

or

$$S_{\mathrm{out}} = |t_c|^2 S_{\mathrm{in}} + 2(|t_c|^2 - 1) \tag{31}$$

Recall that $|t_c| > 1$. We see that even for the case of perfect squeezing at the input, $S_{\mathrm{in}} = -1$, $S_{\mathrm{out}}$ will be positive for $|t_c|^2 \geqslant 2$. Thus amplification by a factor of two is a sufficient condition for removing squeezing from a beam of light.

## 6. Integration time and signal-to-noise considerations

We now consider how the integration time required to achieve a prescribed signal-to-noise ratio depends upon the transmission losses of a particular detection system. Let us first consider a detection system that utilizes the properties of squeezed light, as in Fig. 3b. For definiteness, we assume that we desire to make an accurate measurement of the noise properties of one particular quadrature amplitude of a light field. The signal strength of the transmitted field clearly decreases with increasing attenuation. The noise in the transmitted field depends both on the amount of squeezing in the incident field and on the level of attenuation. Thus the signal-to-noise ratio of the transmitted field depends in a complex manner on the transmission losses of the optical system. We choose to quantify our results in terms of the required integration time because this metric allows direct comparison to schemes based on post selection, in which the integration time required to acquire a prescribed number of events increases as the coincidence count rate decreases.

For definiteness, we consider the situation in which the detector produces a voltage that is proportional to the applied optical power. For the case of balanced homodyne detection, which is the method routinely used in the detection of squeezed light, this voltage $v_s$ is proportional to the general quadrature amplitude $\langle x_\theta\rangle$ introduced above. In the following discussion, we consider a generalization of the situation shown in Fig. 3b in which modes $a$ and $b$ at the input to the OPA are not necessarily in the vacuum state, or more generally that the squeezed light is generated by some unspecified generic nonlinear interaction. We had earlier assumed for reasons of convenience that we were interested only in the case of a vacuum state input to the

OPA. However, $\langle x_\theta \rangle$ necessarily vanishes in this situation. For our present considerations, which include both the signal and the noise contained in the squeezed light, it is thus necessary to treat a situation more general than that shown in Fig. 3b. We then define the voltage signal-to-noise ratio $(S/N)$ through

$$(S/N)^2 = \frac{\overline{v_s^2}}{\overline{v_N^2}} \tag{32}$$

Here the numerator $\overline{v_s^2}$ is the mean-square signal voltage and the denominator is the mean-square noise voltage. We use this definition of the signal strength to allow the possibility that the signal voltage is modulated in time. We now introduce the detection efficiency $\eta$, which includes the influence of transmission losses through the optical system (attenuator $A_c$ in Fig. 3b) and detector quantum efficiency. If transmission losses are the dominant noise mechanism, the detection efficiency is given by $\eta = |t_c|^2 = |t|^2$. We now note that $\overline{v_s^2}$ will decrease with increasing losses as $(\overline{v_s^2})_{\text{out}} = \eta^2(\overline{v_s^2})_{\text{in}}$, whereas $\overline{v_N^2}$ will decrease with increasing losses as $(\overline{v_N^2})_{\text{out}} = \eta(\overline{v_N^2})_{\text{in}} + (1-\eta)$ (see also Eq. (1)). We thus find that

$$(S/N)^2_{\text{out}} = \frac{\eta^2(\overline{v_s^2})_{\text{in}}}{\eta(\overline{v_N^2})_{\text{in}} + (1-\eta)} \tag{33}$$

It is instructive to examine this result for various limiting cases. If the input is strongly squeezed so that $(\overline{v_N^2})_{\text{in}} = 0$, we find that

$$(S/N)^2_{\text{out}} = \frac{\eta^2(\overline{v_s^2})_{\text{in}}}{1-\eta} \tag{34}$$

Conversely, for the case of a coherent state (that is, an unsqueezed, shot-noise-limited) input and making use of our normalization convention that $(\overline{v_N^2})_{\text{in}} = 1$, we find that

$$(S/N)^2_{\text{out}} = \frac{\eta^2(\overline{v_s^2})_{\text{in}}}{1} = \eta^2(\overline{v_s^2})_{\text{in}} \tag{35}$$

Finally, if the input noise $(\overline{v_s^2})_{\text{in}}$ is dominated by technical noise, so that $(\overline{v_N^2})_{\text{in}}$ is much greater than unity, we find that

$$(S/N)^2_{\text{out}} = \frac{\eta(\overline{v_s^2})_{\text{in}}}{(\overline{v_N^2})_{\text{in}}} = \eta(S/N)^2_{\text{in}} \tag{36}$$

The dependence of the output signal-to-noise ratio on the detection efficiency $\eta$ is shown in Fig. 4 for each of these cases. Note that for unit detection efficiency the signal-to-noise ratio achievable through use of squeezed light becomes formally divergent, because we have assumed that the input is perfectly squeezed and that there is thus no noise in the quadrature of the input that is detected. Note also that the scaling law for a coherent state input is different from that of an input dominated by technical noise. These graphs should not be interpreted as implying that certain of these strategies perform better than the others for a given value of $\eta$. Rather, these curves simply indicate how the performance of each of these strategies scales with the efficiency $\eta$ of the detection process.

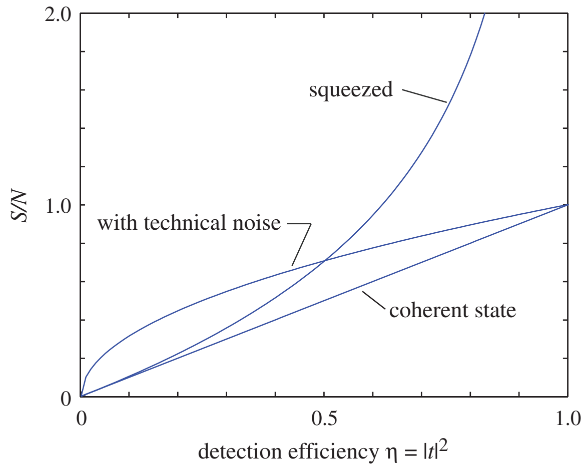


Fig. 4. Variation of the signal-to-noise ratio with detection efficiency for the various situations described in the text.

Let us now consider how the integration time required to achieve a specified signal-to-noise ratio varies with the efficiency parameter $\eta$. We recall the basic result of signal averaging which asserts that the signal-to-noise ratio $(S/N)_M$ obtained from averaging $M$ independent measurements is larger than that of a single measurement $(S/N)_{\text{out}}$ according to

$$(S/N)_M = \sqrt{M}\ (S/N)_{\text{out}} \tag{37}$$

Thus the number of measurements that need be averaged to achieve a specified signal-to-noise ratio, or equivalently the required integration time, depends on $\eta$ according to

$$M \propto \frac{\sqrt{1-\eta}}{\eta} \tag{38}$$

for a fully squeezed input, to

$$M \propto \frac{1}{\eta} \tag{39}$$

for a coherent state input, and

$$M \propto \frac{1}{\sqrt{\eta}} \tag{40}$$

for an input dominated by technical noise.

We next consider the case of coincidence detection, such as in the examples shown in Figs. 1, 2, and 3a. For definiteness, we assume that the transmissive portions of the object of Figs. 1 and 2 have unit transmission. If both arms possess the same detection efficiency, we find that the coincidence detection rate scales as $|t|^4$. Thus the detection time required to obtain a prescribed number of detection events scales with $\eta = |t|^2$ as $M \propto 1/\eta^2$.

These various situations are compared graphically in Fig. 5. These curves are normalized such that the integration time parameter $M$ is equal to unity for $\eta = 1$ for all cases except that of perfectly squeezed light, in which case $M = 0$ for $\eta = 1$ as there is assumed to be no noise in the detection process. In all cases, the required integration time increases as the detection efficiency decreases. But the rate

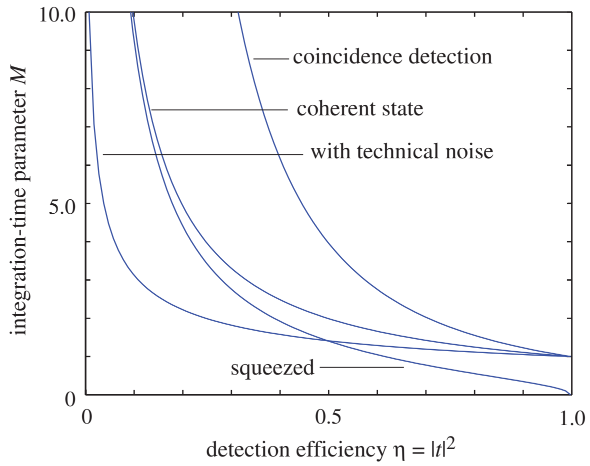


Fig. 5. Scaling laws for the dependance of integration on the detection efficiency for the various situations described in the text.

at which the required integration time increases is different for different measurement strategies. For $\eta$ only slightly smaller than unity, measurements based on squeezing are degraded most rapidly by attenuation: the integration time parameter $M$ increases from zero to a non-zero value. But for values of $\eta$ of 0.5 or smaller, the integration time parameter $M$ increases most rapidly with decreasing efficiency for measurements based on coincidence detection. Furthermore, we note that if only one arm of the coincidence detection setup possesses limited detection $|t|^2$ efficiency whereas the other arm possesses unit efficiency, the integration time parameter $M$ scales as $1/\eta = 1/|t|^2$, which is identical to that of the curve labeled "coherent state" in the figure.

## 7. Conclusions

We now summarize the conclusions that we have reached in our study.

Attenuation does not remove squeezing entirely from a beam of light. Squeezing is reduced at the same rate as the intensity of the light beam. This result is shown in Eq. (23). Amplification can remove squeezing completely from a beam of light. Amplification by a factor or two is sufficient to remove all squeezing from a light beam. This result follows from Eq. (31). Both of these results have been well known for many years (see, for instance [6,7]). Our intent has been to re-derive these results in terms of the more general study presented in this article.

In some ways, coincidence measurements are much less influenced by attenuation than are squeezing measurements. The only influence of attenuation on coincidence measurements is to decrease the count rate. No additional noise is introduced. This situation is quite different for the case of squeezed light detection. Here the influence of attenuation is two fold: it makes the beam weaker and it decreases the amount of noise reduction. Thus, the nature of the quantum state is fundamentally modified by the attenuation process. For both coincidence measurements and squeezing measurements, the integration time required to perform a desired measurement increases with increased optical attenuation. However, the functional dependance on the optical loss is very different in the two cases, as is illustrated in Fig. 5.

## Acknowledgements

We gratefully acknowledge useful discussion with J.C. Howell, P.W. Milonni, J.E. Sipe, C.R. Stroud, Jr., and G. Tyler. This work was supported by an ARO MURI Grant.

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