Optical Transmission Characteristics of Fiber Ring Resonators

John E. Heebner, Vincent Wong, Aaron Schweinsberg, Robert W. Boyd, and Deborah J. Jackson

***Abstract*—We present the results of an experimental investigation of the transfer characteristics of a fiber ring resonator for various values of the resonator finesse. In particular, we measure the spectral dependence of the intensity transmission and the induced phase shift in the undercoupled, critically coupled, and overcoupled regimes. We also demonstrate tunable optical (true time) group delay via a fiber ring resonator and show that a high finesse is unnecessary. Our laboratory results are in excellent agreement with theoretical predictions.**

***Index Terms*—Delay effects, interferometry, optical fiber coupling, optical fiber delay lines, resonators.**

I. Introduction

THERE are many proposed and demonstrated applications of optical ring or disk resonators [1]–[5]. These devices can be constructed in a variety of sizes with distance scales varying from meters to micrometers, with spectral characteristics that scale with the size and finesse $\mathcal{F}$ of the device. Fiber ring resonators [6], [7] can be readily constructed from standard optical fiber components, which allows their properties to be studied in a systematic manner. In this paper, we describe our studies of the optical transfer characteristics of a family of such devices. Optical ring resonators hold great promise for a variety of applications, including optical switching [4], [8], optical time delay [9], photonic biosensors [10], [11], laser resonators [1], add–drop filters [12], and filters with tailored response [13], [14].

II. Theory of Ring Resonators

We analyze the device illustrated in Fig. 1 as follows [8], [10]. We describe the coupling of light into and out of the resonator in terms of generalized beam splitter relations of the form

$$E_2 = rE_1 + itE_3 \tag{1}$$

$$E_4 = rE_3 + itE_1 \tag{2}$$

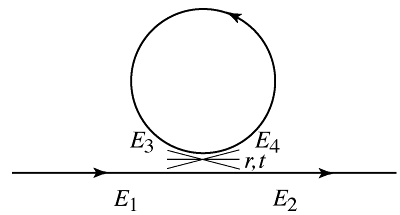

Fig. 1. Geometry of a fiber ring resonator.

where r and t are taken to be real quantities that satisfy the relation $r^2+t^2=1$ and the fields are defined with respect to the reference points indicated in Fig. 1. In addition, we describe the circulation of light within the resonator in terms of the round-trip phase shift ϕ and the amplitude transmission factor τ such that

$$E_2 = \tau \exp(i\phi)E_4. \tag{3}$$

The round-trip phase shift ϕ can be interpreted as $\phi = kL$, where $k = 2\pi n/\lambda$, n is the effective refractive index of the fiber mode, λ is the vacuum wavelength of the incident light, and L is the circumference of the fiber ring. Equations (1)–(3) can be solved simultaneously to find that the input and output fields are related by the complex amplitude transmission

$$\frac{E_2}{E_1} = \exp\left[i(\pi+\phi)\right]\frac{\tau - r\exp(-i\phi)}{1 - r\tau\exp(i\phi)}. \tag{4}$$

The intensity transmission factor T is given by the squared modulus of this quantity

$$T = \left|\frac{E_2}{E_1}\right|^2 = \frac{\tau^2 - 2r\tau\cos\phi + r^2}{1 - 2r\tau\cos\phi + r^2\tau^2}. \tag{5}$$

Note that the on-resonance transmission $(\phi = 0)$ drops to zero for the situation $r = \tau$. In this case, the internal losses are equal to the coupling losses, and the resonator is said to be critically coupled. For $r > \tau$, the resonator is said to be undercoupled and for $r < \tau$ the resonator is said to be overcoupled. The phase of the transmitted light is given by the argument of (4) as follows:

$$\begin{aligned}\Phi &= \arg\left(\frac{E_2}{E_1}\right)\\ &= \pi + \phi + \operatorname{atan}\left(\frac{r\sin\phi}{\tau - r\cos\phi}\right)\\ &\quad + \operatorname{atan}\left(\frac{r\tau\sin\phi}{1 - r\tau\cos\phi}\right).\end{aligned} \tag{6}$$

Near each resonance, the phase undergoes a rapid variation with respect to the round-trip phase shift. This round-trip phase

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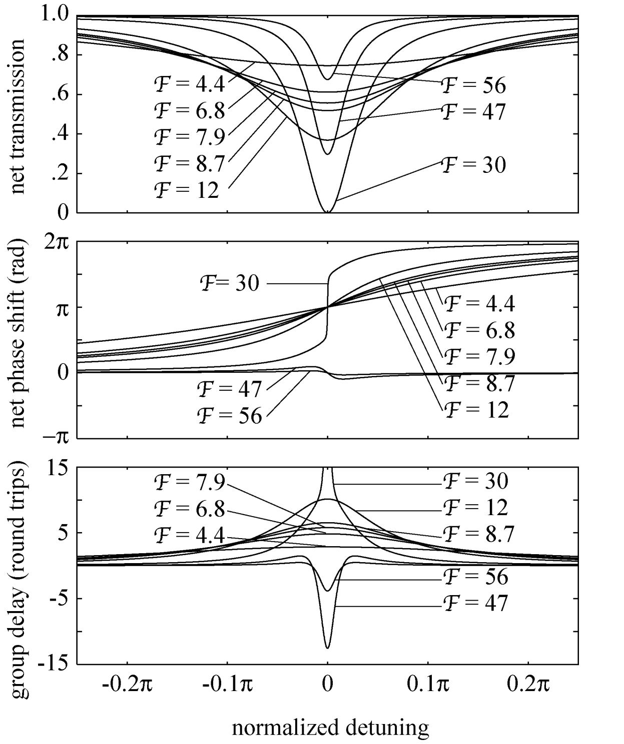

Fig. 2. Theoretically predicted net transmission, phase shift, and group delay for an ideal ring resonator with single-pass transmissivity $\tau = 0.95$ and varying values of the finesse. The normalized detuning is the angular frequency difference between the light field and the nearest cavity resonance multiplied by the ring circulation time.

shift may be interpreted as a frequency detuning normalized through multiplication by the round-trip time. The radian-frequency derivative of the phase shift is the group delay which is proportional to both the finesse $(\mathcal{F} \approx \pi/(1 - r * \tau))$ and the quality factor $(Q = n\mathcal{F}L/\lambda)$ in the overcoupled regime. In the undercoupled regime, the group delay can take on positive or negative values. Fig. 2 displays the transmission, phase, and group delay for a ring resonator with $\tau = 0.95$ and varying values of the finesse.

The fractional delay FD is defined as the group delay T_D normalized by the [full-width at half-maximum (FWHM)] pulse duration T_P, that is, $(FD = T_D/T_P)$. It is a convenient measure of the number pulsewidths by which a pulse can be delayed. Because for a single resonator there is a tradeoff between group delay and bandwidth given by $\Delta\nu \approx 2/(\pi T_D)$, the fractional delay imparted by a resonator is limited. For a Gaussian pulse with a bandwidth equaling the resonator bandwidth, the fractional delay is limited to $(FD)_{\max} \approx (2/\pi)[\pi/(2\ln 2)] = 1/\ln 2$. In reality, however, when the pulse bandwidth exactly matches the resonator bandwidth, it becomes distorted due to a nonuniform group delay across the pulse spectrum. Practically, for operation below a fractional delay of unity, a pulse is delayed and transmitted with low distortion. Larger fractional delays may be obtained by cascading resonators in a serial manner. It should be noted that this result is independent of the scale size and nearly independent of the finesse of the resonator. Thus, in order to maximize the fractional delay for a given pulse, the resonator bandwidth should be chosen appropriately. This still leaves a choice between a small, high-finesse resonator or a larger and proportionally lower finesse resonator. If both suffer the same loss per round trip dictated by splices and coupling-related losses, the lower finesse device will more closely approximate a phase-only device having a more uniform transmission across a free-spectral range. A cascade of N *low-finesse* ring resonators is thus most appropriate for the design of tunable optical delay lines for applications requiring the delay of pulse-trains by N pulsewidths.

III. Spectral Characterization of a Fiber Ring Resonator

An external-cavity diode laser tunable from 1506 to 1586 nm was used to measure the spectral response associated with resonances of the fiber ring resonator. The resonator was formed by fusion-splicing one output of a variable optical coupler to one input port, producing a ring with a circumference of 31 cm. The splitting ratio of the fused-glass directional coupler was controlled by applying bending stress with a micrometer stub. A polarization controller was placed before the resonator and adjusted so as to excite one of the eigenpolarizations of the resonator. The transmission of the resonator was measured as a function of carrier frequency for a variety of coupler settings. The laser wavelength was swept at 0.5 nm/s and the ring resonator output was measured with an InGaAs detector and captured on a digital oscilloscope. Because of the nonvanishing single-pass loss of the ring resulting from losses in the coupler and fusion splice, the coupler could be tuned to produce undercoupled, critically coupled, or overcoupled operation. In previous studies [7], workers varied the gain of an amplifier placed within the fiber ring resonator to achieve overcoupoing, undercoupoing, and critical coupling through variation of the parameter τ of our equation (3). In contrast, in this study, we varied the strength of the coupling into the ring by varying the parameter r of our equations (1) and (2).

Fig. 3 displays the measured transmission spectra for a variety of coupler settings. For one setting, the coupler was set to give maximum intensity extinction at the resonances. Based on a measured finesse of 30 for this case of critical coupling, we estimate that the coupling and single-pass transmission parameter were $r \approx \tau \approx 0.95$. The single-pass loss resulting from the insertion loss of the coupler and splice loss is thus estimated to be $10\ \log_{10}(.95^2)$ =0.45 dB. A loss-limited finesse of 60 is thus predicted. In the first five traces, the finesse is low and governed primarily by strong coupling into the ring. In this overcoupled regime, the resonator produces a nearly-flat-amplitude, phase-only response for low values of finesse and degrades as the finesse is increased. In the remaining two traces past critical coupling, narrow dips appear at each resonance. In this undercoupled regime, the finesse is determined primarily by the internal attenuation and thus approaches the predicted value of 60.

Next, the phase response of the ring resonator was measured by inserting the resonator into one arm of a nearly balanced Mach–Zehnder interferometer. The transmission of the interferometer was then measured near 1.55 μm as a function of carrier frequency. Fig. 4 displays the measured transmission spectra for the same coupler settings as in Fig. 3. The phase variation as fitted to this spectra is displayed in Fig. 5. The traces in Fig. 4

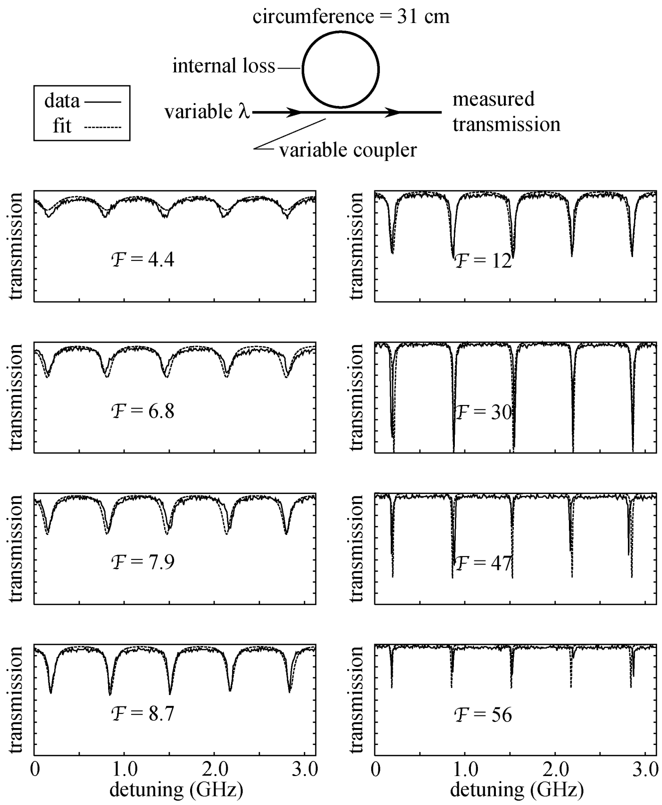

Fig. 3. Measured transmission spectra near 1.55 μm for a fiber ring resonator 31 cm in circumference with nonnegligible loss for a variety of coupler settings. The setup used to perform the measurement is shown at the top. The dotted lines are a theoretical fits to the data. The resonator is overcoupled, critically coupled, and undercoupled for $\mathcal{F}$ less than, equal to, and greater than 30, respectively.

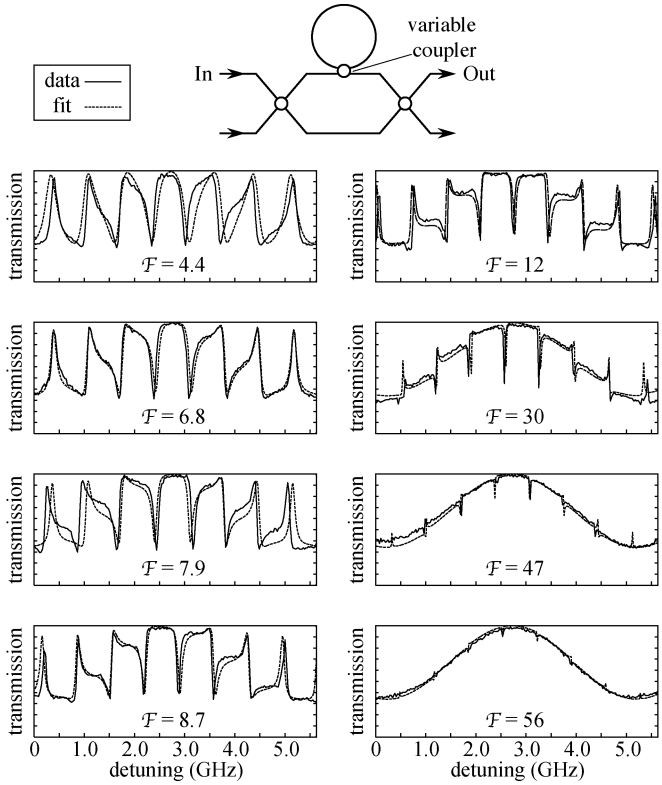

Fig. 4. Measured transmission spectra of a nearly balanced Mach–Zehnder interferometer with the 31-cm fiber ring resonator inserted into one arm. The setup is shown at the top. Data were taken near 1.55 μm for the same coupler settings as in Fig. 3. The dotted lines are a theoretical fit to the data.

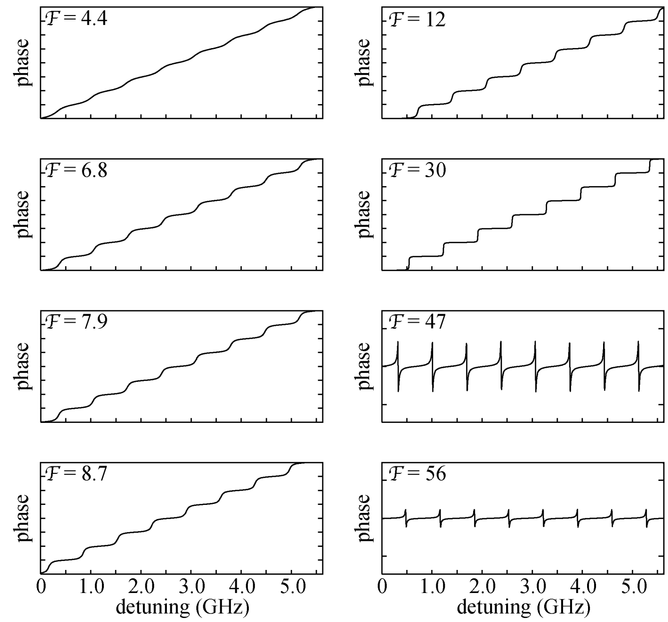

Fig. 5. Spectral dependence of the phase shift acquired by a light wave in interacting with the fiber ring resonator as determined from the fits to the data shown in Fig. 4. For better visibility, the vertical gradations are in units of 2π for the first six overcoupled resonator plots and units of $\pi/6$ for the remaining two plots.

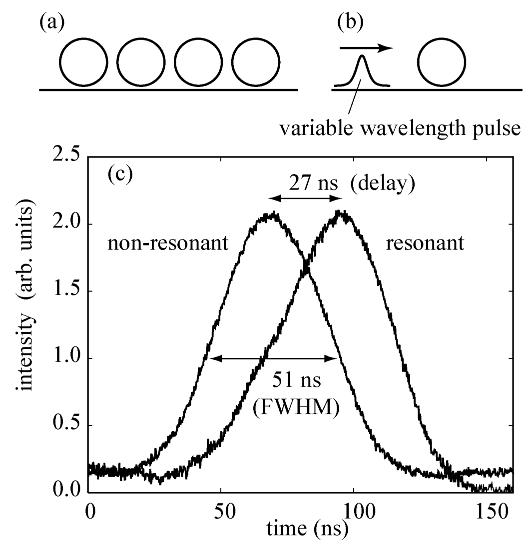

Fig. 6. (a) A ring-resonator optical delay line. (b) One element of the delay line. (c) Demonstration of controllable optical time delay in the setup of (b). A 51-ns pulse is delayed by 27 ns.

show features separated by a free-spectral range of approximately 660 MHz due to the resonances of the ring resonator as well as features on a much broader frequency scale of approximately 5.0 GHz due to a small imbalance of about 4.1 cm between the lengths of the two arms of the interferometer. The ring resonances thus occur 7.5 times within the background Mach–Zehnder transmission periodicity. Near overcoupled resonances (lower finesse values), the phase increases monotonically as a function of detuning frequency, leading to a π radian phase advance at each resonance. Conversely, near undercoupled resonances (finesse values 47 and 56), the phase undergoes a wiggle which does not produce a net advance in phase. The region of negative slope near each resonance frequency corresponds to a region of negative group velocity which can lead to superluminal light propagation in a serially distributed resonator device [15].

IV. True Time Delay Using a Fiber Ring Resonator

A second ring resonator (larger by a factor of 10) was constructed for the purpose of demonstrating tunable optical true time delays. A dye laser tuned near 589 nm was coupled into one port of a fixed four-port single-mode (single mode in the visible spectral region) directional coupler. One output port of the coupler was directly fusion spliced to the other input port to form a fiber ring resonator with a 2.8-m circumference. A coupling coefficient t^2 of 3/4 was chosen to produce a resonator with a low finesse of 5. A low finesse was used to approximate a phase-only device by maintaining a nearly uniform transmission across a free-spectral range. The light emerging from the output port was directed onto a silicon PIN detector and the waveform was collected by a digital oscilloscope. Thermal drifts in the ambient temperature caused the resonance frequencies to drift on a slow (millisecond) time scale. These drifts were partially stabilized by immersing the fiber ring into a room-temperature water bath. An acoustooptic modulator was used to generate a continuous train of 50-ns pulses from the dye laser output. A beamsplitter picked off a fraction of the pulse train for use as a trigger reference. As the dye laser frequency was swept through a free-spectral range (70 MHz) of the fiber ring resonator, the pulses emerging from the ring resonator showed a variable time delay. Fig. 6 compares the timing of an off-resonance pulse to that of an on-resonance pulse. In this experiment, a 51-ns pulse was delayed by 27 ns for a maximum fractional delay slightly greater than 1/2. By concatenating a sequence of resonance-locked fiber ring resonators, as shown in (a) of the figure, it would be possible to create much longer fractional time delays [9], [15].

V. Summary

We have presented the results of a systematic study of the transfer characteristics of fiber ring resonators and have shown how to construct a tunable true time delay element from such a resonator. In addition to their intrinsic value, these results can be useful in predicting the behavior of micro-ring and micro-disk resonators. Ring resonators show considerable promise as building blocks for the construction of a variety of compact photonic devices [16], [17].

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