

Elimination of the band gap of a resonant optical material by electromagnetically induced transparency

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We consider the possibility of wave propagation in the normally disallowed (band-gap) region of a resonant optical medium, that is, in the band of frequencies near resonance where the real part of the frequency-dependent dielectric function is negative. We demonstrate that wave propagation can become allowed by the application of a strong electromagnetic field resonant with some additional transition of the material system. The frequencies at which these effects can occur are strongly influenced by local-field effects. [S1050-2947(99)51310-3]

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It is well known [1] that electromagnetic waves with wave vector \vec{k} cannot propagate in the frequency region where the dielectric function $\epsilon(\omega)$ of the medium is negative. This conclusion follows from the standard dispersion relation $k^2 = (\omega^2/c^2)\epsilon(\omega)$. By way of illustration, let us consider a material system characterized by a dielectric function of the form

$$\epsilon(\omega) = \epsilon_0 \left(1 + \frac{\omega_L^2 - \omega_0^2}{\omega_0^2 - \omega^2} \right). \quad (1)$$

Here ϵ_0 is the background dielectric constant, ω_L is a parameter [defined by $\epsilon(\omega_L) = 0$] that characterizes the strength of the optical response, and we ignore the effects of damping. We see from Eq. (1) that $\epsilon(\omega)$ is negative in the frequency region $\omega_0 < \omega < \omega_L$. This region corresponds to a band gap—a region in which only evanescent waves are possible. This band gap occurs even though in our model $\epsilon(\omega_L)$ is a real quantity. It is also known that the optical properties of a medium can be modified significantly by the application of resonant electromagnetic fields. In particular, resonant absorption can be made very small by the application of a control field leading to electromagnetically induced transparency (EIT) [2–5]. The question thus arises: Is it possible to eliminate the band-gap region by the application of electromagnetic fields? We examine this question in detail and produce an answer in the affirmative. We examine a model system that is of relevance in many applications and that could be generalized easily in several ways depending on need. We take full account of local-field effects [6–9] because band gaps can occur only in dense systems. We demonstrate that such a band gap can occur in alkali-metal vapors and can be removed by the application of a strong control field, although because of self-broadening effects alkali-metal vapors are not an ideal system for studying such effects.

It should be noted that the situation considered here is very different from those under which EIT has been previously studied. EIT is usually used to minimize absorption at frequencies for which $\text{Im } \epsilon(\omega)$ is large. In contrast, we consider the possibility of allowing propagation at frequencies where $\text{Im } \epsilon(\omega)$ is small but where propagation is usually prevented by the fact that $\text{Re } \epsilon(\omega)$ is negative. We also note that earlier Harris [10] showed how the application of an additional electromagnetic field can produce propagation at a frequency below the cutoff frequency in an ideal plasma. Harris utilized collective effects arising from nonlinearities in a plasma to produce EIT in plasma. In our work local-field effects play an important role—these effects arise from dipole-dipole interactions and are in a sense like collective effects. We note still further that Scully and co-workers [11] have produced schemes based on quantum interferences for the enhancement of the refractive index of a medium. In their work the region of interest is $\text{Re } \epsilon > 0, \text{Im } \epsilon \approx 0$. We note that in Zibrov *et al.*'s case [11] the susceptibility can take negative values but ϵ is still positive.

Let us now consider a more detailed theoretical model based on the coupling scheme shown in the inset of Fig. 1. We consider first the optical response to a single applied field at frequency ω ; we will later see how this response is modified by the application of an additional control field of frequency ω_c . The linear response at frequency ω is given by the standard “two-level” model as [12].

$$\chi = \frac{n|\vec{d}_{13}|^2}{\hbar(\Delta - i\Gamma)}, \quad \Delta = \omega_{13} - \omega. \quad (2)$$

Here, as usual, n is the atomic number density of the medium, \vec{d}_{13} is the transition dipole matrix element, and Γ is the half width at half maximum of the transition $|1\rangle \leftrightarrow |3\rangle$. The susceptibility as modified by local-field effects is given in Gaussian units by

$$\chi_l = \frac{\chi}{\left(1 - \frac{4\pi}{3}\chi\right)}. \quad (3)$$

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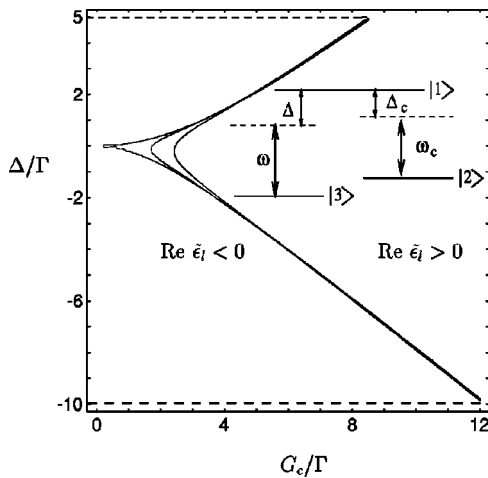


FIG. 1. Contour plots of $\text{Re } \tilde{\epsilon}_l(\omega) = 0$ in the band-gap region (indicated by dashed line) defined by relation (6). The three curves from left to right are for increasing values of the collisional parameter $\Gamma_d/\Gamma = 0, 0.5$, and 1.0 . The region to the right of the plot corresponds to positive values of $\text{Re } \tilde{\epsilon}_l$, where electromagnetic propagation is allowed. The inset shows the two-level scheme and its modification by the application of a control laser of frequency ω_c . We choose $\delta/\Gamma = 5$.

Let us introduce the *local-field parameter* δ with dimensions of frequency

$$\delta = \frac{4\pi n |d|^2}{3\hbar}. \quad (4)$$

On using Eqs. (2)–(4), the dielectric function with the inclusion of local-field effects can be expressed as

$$\epsilon_l(\omega) \equiv 1 + 4\pi\chi_l = 1 + \frac{3\delta}{\Delta - i\Gamma - \delta}. \quad (5)$$

For Γ sufficiently small (in particular, for $\Gamma/\delta < \frac{3}{2}$), Eq. (5) predicts the existence of a band-gap region in which $\epsilon_l(\omega)$ is negative. For the limiting case in which Γ is negligibly small, $\epsilon_l(\omega)$ is negative in the region

$$-2\delta < \Delta < \delta. \quad (6)$$

Note that the frequency ω_L of Eq. (1) for the response described by Eq. (5) is determined by the condition $\epsilon_l(\omega_L) = 0$ and hence is given by $\omega_L = \omega_0 + 2\delta$.

We next demonstrate that the band gap can be removed by the application of a resonant electromagnetic field [13] on a suitably chosen transition of the material system, such as the $|1\rangle \leftrightarrow |2\rangle$ transition shown in the inset of Fig. 1. Let us denote by $2G_c$ and Δ_c the Rabi frequency and the detuning of the electromagnetic control field of frequency ω_c on the transition $|1\rangle \leftrightarrow |2\rangle$. Let Γ_d be the dephasing rate of the dipole-forbidden transition $|2\rangle \leftrightarrow |3\rangle$. The modification of the susceptibility given by Eq. (2) as a result of the control field on has been derived earlier [3(a)] and is given by

$$\tilde{\chi} = \frac{n|d|^2}{\hbar \left(\Delta - i\Gamma - \frac{i|G_c|^2}{\Gamma_d + i\Delta - i\Delta_c} \right)}, \quad \Delta_c = \omega_{12} - \omega_c. \quad (7)$$

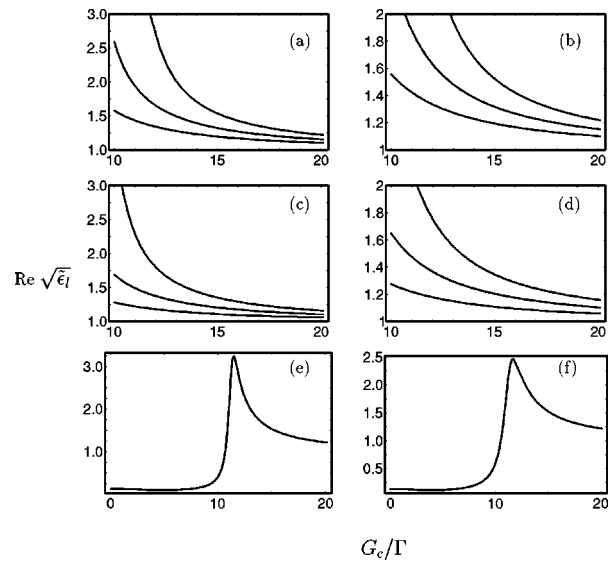


FIG. 2. Control field dependence of the real part of the complex refractive index; that is, the real part of the square root of the dielectric function (8) for $\delta/\Gamma = 5$. Only the region of interest is shown. The frames (a), (c) [(b), (d)] are for the dephasing parameter $\Gamma_d/\Gamma = 0(0.5)$, whereas the detuning parameter $\Delta_c = \omega_{12} - \omega_c$ is chosen as $0 (-2\Gamma)$ for (a), (b) [(c), (d)]. The Rabi frequency of the control field is $2G_c$. In all the frames (a)–(d) the curves from bottom to top are for $\Delta/\Gamma = -5, -7$, and -9 . The frames (e), (f) exhibit the behavior of $\text{Re } \sqrt{\tilde{\epsilon}_l}$ in the full range of the Rabi frequencies for $\Delta/\Gamma = -9$, $\Delta_c/\Gamma = 0$, and for $\Gamma_d/\Gamma = 0$ (e), 0.5 (f).

It should be borne in mind that we are concerned with a region that is distinct from the usual region of electromagnetic induced transparency (which for large $|G_c|$ and $\Delta_c \sim 0$ occurs in the region $\Delta \sim 0$). Using Eq. (7) in Eq. (3), we obtain the control-field-modified susceptibility and the dielectric response with local-field corrections given by

$$\tilde{\epsilon}_l(\omega) = 1 + \frac{3\delta}{\Delta - i\Gamma - \frac{i|G_c|^2}{\Gamma_d + i\Delta - i\Delta_c} - \delta}. \quad (8)$$

Note that if local-field corrections were ignored, then δ would not appear in the denominator in Eq. (8). Using Eq. (8), we can study the extent to which the band gap given by Eq. (6) can be eliminated by the application of a control field on the transition $|1\rangle \leftrightarrow |2\rangle$. Let us first make an approximate estimate by assuming that $\Gamma_d \sim 0, \Delta_c = 0$, and by dropping Γ . In that case Eq. (8) reduces to

$$\tilde{\epsilon}_l(\omega) = 1 + \frac{3\delta}{\Delta - \frac{|G_c|^2}{\Delta} - \delta}. \quad (9)$$

Clearly for $\Delta < 0, \tilde{\epsilon}_l(\omega)$ is positive and greater than 1 if

$$|G_c|^2 > (|\Delta| + \delta)|\Delta|. \quad (10)$$

Thus for Δ negative (but with $|\Delta| < 2\delta$) the band gap is removed by an electromagnetic field whose Rabi frequency obeys Eq. (10). For $\delta > \Delta > 0$, the condition for eliminating the band gap is

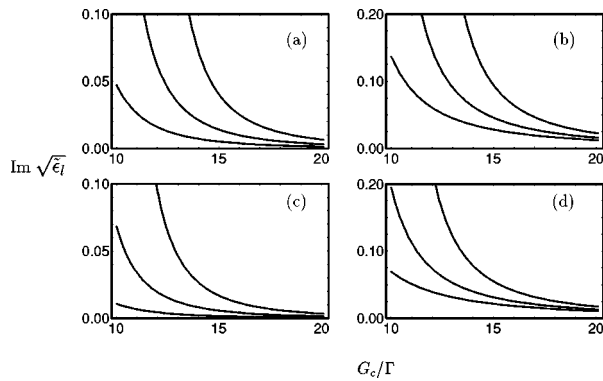


FIG. 3. Same as in Figs. 2(a), 2(b), 2(c), and 2(d) but for the imaginary part of the complex refractive index, i.e., $\sqrt{\tilde{\epsilon}_l}$.

$$|G_c|^2 > \Delta(2\delta + \Delta). \quad (11)$$

In Fig. 1 we show the regions of the parameter space where $\text{Re } \tilde{\epsilon}_l$ takes positive and negative values. We show in Figs. 2 and 3 the behavior of the real and imaginary parts of $\sqrt{\tilde{\epsilon}_l(\omega)}$ for different values of Δ and for a control field on resonance as well as detuned from resonance. The plots are shown as functions of the strength of the controlling field. The band gap is removed when $\text{Re } \sqrt{\tilde{\epsilon}_l} > 0$, and the region where $\text{Im } \sqrt{\tilde{\epsilon}_l}$ remains small is especially of interest. Note that, as the strength of the control field is changed, the system goes through a new resonance region in which $\text{Im } \tilde{\epsilon}_l$ becomes large. In the region of large G_c the collisional dephasing primarily affects the imaginary part of $\tilde{\epsilon}_l$.

Let us now explore the experimental feasibility of observing these effects. The primary experimental requirement is that there exist some region near the atomic resonance in which the dielectric function is negative. From Eq. (5) we deduce that the real part of the dielectric function reaches its minimum value for $(\Delta - \delta) = -\Gamma$, at which frequency its value is given by

$$\text{Re } \epsilon_l(\text{min}) = 1 - \frac{3\delta}{2\Gamma}. \quad (12)$$

For sufficiently large atomic number density n the linewidth Γ will be dominated by self-broadening effects, such that $\Gamma = C_s n$, where C_s is the self-broadening coefficient. For atomic sodium, $C_s = 1.5 \times 10^{-7} \text{ cm}^3/\text{s}$; the value for other alkali-metal atoms is comparable [14]. Thus for sodium vapor, for $n > 6 \times 10^{14} \text{ cm}^{-3}$ the collisional linewidth will exceed the natural linewidth of 10 MHz, and for $n > 1 \times 10^{17} \text{ cm}^{-3}$ the collisional linewidth will exceed the typical 2-GHz Doppler linewidth. Note from Eq. (4) that δ is also proportional to n , and hence for sufficiently large num-

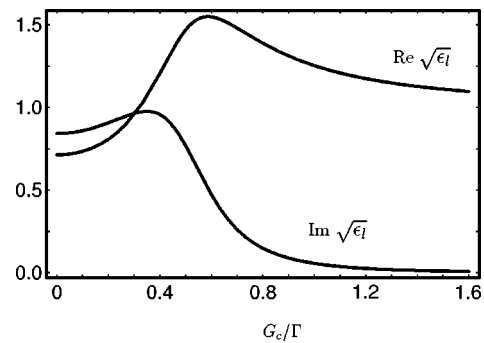


FIG. 4. Control-field-induced propagation for the situation described by Eq. (12) with $\Gamma_d/\Gamma = 0$, $\Delta = \delta - \Gamma$, $\Gamma/\delta = 1.25$, $\Delta_c = 0$. Note that for G_c/Γ greater than approximately 0.4, the real part of $\sqrt{\tilde{\epsilon}_l}$ is greater than unity. Note also that for G_c/Γ greater than approximately 0.7 the imaginary part of $\sqrt{\tilde{\epsilon}_l}$ becomes quite small. Under these conditions the propagation of optical fields has been induced.

ber density $\text{Re } \epsilon_l(\text{min})$ attains a density-independent value. If we evaluate Eqs. (4) and (12) in this limit, using $d = 5.5 \times 10^{-18} \text{ esu}$ (the value for the sodium resonance line), we find that the ratio Γ/δ [mentioned above in connection with Eq. (5)] has the value 1.25 and that $\text{Re } \epsilon_l(\text{min}) = -0.21$, which hence shows that a band gap exists under these circumstances. Note, however, that unlike in the idealized situation discussed above, where damping was neglected, here the imaginary part of ϵ_l is substantial. To study the influence of damping effects, in the Fig. 4 we show the real and imaginary parts of $\sqrt{\tilde{\epsilon}_l}$. We notice the important prediction that in the region $G_c/\Gamma \sim 1$, $\text{Im } \sqrt{\tilde{\epsilon}_l} \approx 0$, whereas the real part is positive, leading to the propagation of electromagnetic waves. Note that in order to obtain a Rabi frequency as large as 2 GHz so that the condition $G_c/\Gamma \approx 1$ can be satisfied for a medium in which the collisional broadening is comparable to Doppler broadening, the laser intensity need be only as large as 600 W/cm^2 .

In conclusion we have shown how the ideas of quantum interference can be used to get electromagnetic wave propagation in the region in which it is normally forbidden. These results thus constitute a new example of electromagnetically induced transparency. In the foregoing, we have demonstrated this idea by considering a Λ system. Similar results will apply to others like V and Ξ systems.

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