

Effects of atmospheric turbulence on the entanglement of spatial two-qubit states

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We study the effects of atmospheric turbulence on the entanglement of spatial two-qubit states that are prepared using the signal and idler photons produced by parametric down-conversion. Such states are the basic ingredients of quantum information protocols and can be prepared, for example, by making down-converted photons pass through a pair of double-apertures. We make use of the Kolmogorov model for atmospheric turbulence and quantify the entanglement of the two-qubit state in terms of Wootters's concurrence. We restrict our analysis to the two-qubit states that can be represented by density matrices having only two nonzero diagonal elements.

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I. INTRODUCTION

There has recently been great interest in the use of quantum information methods for free-space optical communication [1]. Potential applications include both secure communication [2] and the development of quantum repeaters for swapping quantum states over perhaps large distances [3]. For many applications it is necessary for the photons that carry the quantum state to pass through the Earth's atmosphere, where turbulence constitutes a potential loss of quantum coherence. Several publications have dealt with the influence of atmospheric turbulence on the information content of single photons [4–8]. These treatments quantify the rate at which quantum information impressed on an individual photon is lost as the strength of the turbulence is increased. More recent works have started to investigate the effects of atmospheric turbulence on communication schemes that rely on quantum entanglement [9,10]. In the present paper, we deal with a situation in which the entangled photons produced by parametric down-conversion (PDC) are launched into free-space optical links and a two-qubit state is prepared by placing a pair of double-apertures in their paths. We wish to determine how the entanglement of such a two-qubit state is degraded by atmospheric turbulence in the communication link. We use the Kolmogorov model for atmospheric turbulence [11,12] and quantify the entanglement of the state in terms of Wootters's concurrence [13,14]. We restrict our analysis to two-qubit states that can be represented by a density matrix having only two nonzero diagonal elements, because for such states the concurrence can be determined directly by measuring the visibility of two-photon interference fringes [15,16]; also, due to the position correlations of down-converted photons, a very close approximation to such states is easily prepared in many experimental circumstances.

II. ENTANGLEMENT OF SPATIAL TWO-QUBIT STATES

Entangled two-qubit states are very important for quantum information technology, as they are the necessary ingredients for many quantum-information-based applications, such as quantum cryptography [17,18], quantum dense coding [19],

and quantum teleportation [20]. Two-qubit states have been realized through the use of photons entangled in variables including polarization [21], time bin [22,23], frequency [24], position [25,26], transverse momentum [27,28], orbital angular momentum [29–31], and angular position [15]. Two-qubit states that are based on the position correlations of entangled photons are referred to as spatial two-qubit states [16,25,26,28]. Figure 1 depicts a generic scheme for preparing spatial two-qubit states using the entangled photons produced by PDC. This scheme has been analyzed in detail in Ref. [16], but without the influence of atmospheric influence, and throughout this paper we use the theoretical framework worked out in that reference. In the scheme in Fig. 1, the down-converted photons are made to pass through a pair of signal and a pair of idler apertures, with the apertures being very small. The transverse positions of the apertures define the qubit spaces. Thus $\{|s1\rangle, |s2\rangle\}$ and $\{|i1\rangle, |i2\rangle\}$ form the two-dimensional orthonormal bases for the signal and idler photons, respectively, where $|s1\rangle$ represents the state of the signal photon passing through the aperture located at position $\mathbf{r}_{s1} \equiv (\boldsymbol{\rho}_{s1}, z)$, etc. The four-dimensional basis set for the two-qubit state can then be represented by $\{|s1\rangle|i1\rangle, |s1\rangle|i2\rangle, |s2\rangle|i1\rangle, |s2\rangle|i2\rangle\}$, where $|s1\rangle|i1\rangle$, represents the joint state of the signal and idler photons when they pass through the apertures located at positions $\mathbf{r}_{s1} \equiv (\boldsymbol{\rho}_{s1}, z)$ and $\mathbf{r}_{i1} \equiv (\boldsymbol{\rho}_{i1}, z)$, respectively. We now make the explicit assumption that the PDC phase matching is such that the probabilities of finding the signal and idler photons in states $|s1\rangle|i2\rangle$ and $|s2\rangle|i1\rangle$ are negligibly low. This can usually be ensured by keeping the distances between the two signal and the two idler apertures greater than the length scale over which the down-converted photons are correlated at the double-aperture plane [16,26]. We can now write the density matrix ρ_{qubit} of the two-qubit state in the basis $\{|s1\rangle|i1\rangle, |s1\rangle|i2\rangle, |s2\rangle|i1\rangle, |s2\rangle|i2\rangle\}$ as

$$\rho_{\text{qubit}} = \begin{pmatrix} a & 0 & 0 & c \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ d & 0 & 0 & b \end{pmatrix}. \quad (1)$$

Here a and b are the probabilities that the signal and idler photons are detected in states $|s1\rangle|i1\rangle$ and $|s2\rangle|i2\rangle$, respectively, with $a + b = 1$. The off-diagonal term c is a measure of coherence between state $|s1\rangle|i1\rangle$ and state $|s2\rangle|i2\rangle$,

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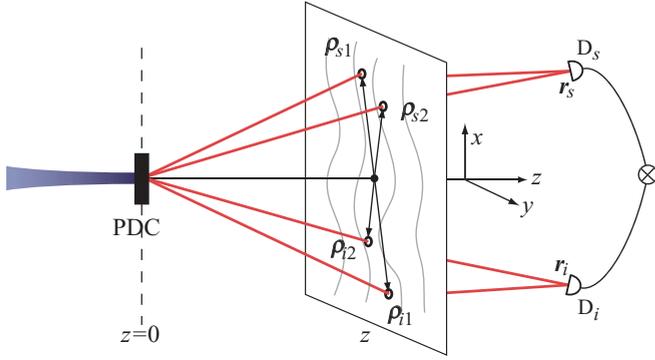


FIG. 1. (Color online) A generic scheme of the preparation of spatial two-qubit states. The state of the two photons through the four apertures is represented by the density matrix ρ_{qubit} of Eq. (1). The turbulence-induced wavefront errors are located primarily near the plane of the apertures that define the qubit space.

with $c = d^*$. We note that in an experimental situation, the probabilities a and b are maximum when the two pairs of signal and idler apertures are placed at symmetric positions, that is, $\rho_{s1} = -\rho_{i1}$ and $\rho_{s2} = -\rho_{i2}$.

A well-established method for quantifying the degree of entanglement of a two-qubit state is by means of Wootters's concurrence [13,14]. The concurrence provides a means of quantifying the degree of entanglement between two independent two-state particles. Specifically, it gives the amount of information for one particle that is lost when the information regarding the state of the other particle is discarded. For example, if the particles are not entangled, no information is lost and thus the concurrence is 0. Conversely, if the two particles are maximally entangled, the concurrence is defined in such a manner that its value is unity. For a given two-qubit density matrix ρ , the concurrence $C(\rho)$ is given by $C(\rho) = \max\{0, \sqrt{\lambda_1} - \sqrt{\lambda_2} - \sqrt{\lambda_3} - \sqrt{\lambda_4}\}$. Here the λ_i 's are the non-negative eigenvalues, in descending order, of the matrix $\zeta = \rho(\sigma_y \otimes \sigma_y)\rho^*(\sigma_y \otimes \sigma_y)$, with

$$\sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

being the usual Pauli operator and ρ^* the complex conjugate of ρ . For the density matrix ρ_{qubit} , the concurrence $C(\rho_{\text{qubit}})$ can therefore be shown to be

$$C(\rho_{\text{qubit}}) = 2|c|. \quad (2)$$

We find it useful to perform our analysis in terms of the displacement parameters introduced in Ref. [16], which are defined in terms of the transverse position vectors of the signal and idler photons as

$$\begin{aligned} \rho_1 &\equiv \frac{\rho_{s1} + \rho_{i1}}{2}, & \rho_2 &\equiv \frac{\rho_{s2} + \rho_{i2}}{2}, & \Delta\rho &= \rho_1 - \rho_2, \\ \rho'_1 &\equiv \rho_{s1} - \rho_{i1}, & \rho'_2 &\equiv \rho_{s2} - \rho_{i2}, & \Delta\rho' &= \rho'_1 - \rho'_2. \end{aligned} \quad (3)$$

Here $\rho_{1(2)}$ and $\rho'_{1(2)}$ are the two-photon transverse position vector and the two-photon position-asymmetry vector when the signal and idler photons are in state $|s1\rangle|i1\rangle(|s2\rangle|i2\rangle)$. In what follows the quantity of central interest is the two-photon cross-spectral density function $W^{(2)}(\mathbf{r}_{s1}, \mathbf{r}_{i1}, \mathbf{r}_{s2}, \mathbf{r}_{i2})$ [16], which quantifies the coherence between the two-photon

fields at the two pairs of positions $(\mathbf{r}_{s1}, \mathbf{r}_{i1})$ and $(\mathbf{r}_{s2}, \mathbf{r}_{i2})$, and is defined as

$$\begin{aligned} W^{(2)}(\mathbf{r}_{s1}, \mathbf{r}_{i1}, \mathbf{r}_{s2}, \mathbf{r}_{i2}) \\ = \langle \text{tr}\{\rho_{\text{tp}} \hat{E}_{s1}^{(-)}(\mathbf{r}_{s1}) \hat{E}_{i1}^{(-)}(\mathbf{r}_{i1}) \hat{E}_{i2}^{(+)}(\mathbf{r}_{i2}) \hat{E}_{s2}^{(+)}(\mathbf{r}_{s2})\} \rangle. \end{aligned} \quad (4)$$

Here $\hat{E}_{s2}^{(+)}(\mathbf{r}_{s2})$ is the positive frequency part of the field at position \mathbf{r}_{s2} , etc. The symbol tr stands for the trace, and ρ_{tp} is the density matrix of the two-photon field produced by PDC. The ensemble average $\langle \dots \rangle$ is to account for the statistical fluctuations introduced by the turbulent medium. The two-photon cross-spectral density function reduces to the two-photon spectral density function when the two signal and the two idler positions coincide. Thus $S^{(2)}(\mathbf{r}_{s1}, \mathbf{r}_{i1}) \equiv W^{(2)}(\mathbf{r}_{s1}, \mathbf{r}_{i1}, \mathbf{r}_{s1}, \mathbf{r}_{i1})$ is the two-photon spectral density at the pair of positions $(\mathbf{r}_{s1}, \mathbf{r}_{i1})$; it is proportional to the probability of finding the signal and idler photons at the pair of positions $(\mathbf{r}_{s1}, \mathbf{r}_{i1})$. To keep the notations simpler, we do not show the frequency arguments in the definitions of the two-photon spectral density and the two-photon cross-spectral density functions. We now express the matrix elements of ρ_{qubit} in terms of the two-photon correlation functions and thus write

$$a = \eta k_1^2 S^{(2)}(\mathbf{r}_{s1}, \mathbf{r}_{i1}), \quad (5)$$

$$b = \eta k_2^2 S^{(2)}(\mathbf{r}_{s2}, \mathbf{r}_{i2}), \quad (6)$$

$$c = d^* = \eta k_1 k_2 W^{(2)}(\mathbf{r}_{s1}, \mathbf{r}_{i1}, \mathbf{r}_{s2}, \mathbf{r}_{i2}), \quad (7)$$

where $\eta = 1/[k_1^2 S^{(2)}(\mathbf{r}_{s1}, \mathbf{r}_{i1}) + k_2^2 S^{(2)}(\mathbf{r}_{s2}, \mathbf{r}_{i2})]$ is a constant of proportionality. The constant factors k_1 and k_2 depend on the sizes of the apertures and the geometry of the arrangement [16]. We assume that in the scheme in Fig. 1 the phase fluctuations incurred in the propagation to the double-apertures can, in effect, be accounted for by replacing the turbulent medium by a single phase screen located primarily near the double-apertures [4,12]. Therefore, the total field $\hat{E}_{s1}^{(+)}(\mathbf{r}_{s1})$ at position \mathbf{r}_{s1} can be written as

$$\hat{E}_{s1}^{(+)}(\mathbf{r}_{s1}) = \hat{E}_{s1}^{(+)}(\mathbf{r}_{s1}) e^{i\phi(\mathbf{r}_{s1})}. \quad (8)$$

Here $\hat{E}_{s1}^{(+)}(\mathbf{r}_{s1})$ represents a deterministic field, whereas $\phi(\mathbf{r}_{s1})$ has a statistical character and represents the turbulence-induced wavefront errors [12]. We also assume that the statistical fluctuations induced by the turbulence are independent of the fluctuations induced due to the pump field. Equation (4) for the two-photon cross-spectral density then becomes

$$\begin{aligned} W^{(2)}(\mathbf{r}_{s1}, \mathbf{r}_{i1}, \mathbf{r}_{s2}, \mathbf{r}_{i2}) = \mu_{\text{turb}}(\mathbf{r}_{s1}, \mathbf{r}_{i1}, \mathbf{r}_{s2}, \mathbf{r}_{i2}) \text{tr}\{\rho_{\text{tp}} \hat{E}_{s1}^{(-)}(\mathbf{r}_{s1}) \\ \times \hat{E}_{i1}^{(-)}(\mathbf{r}_{i1}) \hat{E}_{i2}^{(+)}(\mathbf{r}_{i2}) \hat{E}_{s2}^{(+)}(\mathbf{r}_{s2})\}, \end{aligned} \quad (9)$$

where

$$\mu_{\text{turb}}(\mathbf{r}_{s1}, \mathbf{r}_{i1}, \mathbf{r}_{s2}, \mathbf{r}_{i2}) = \langle e^{-i[\phi(\mathbf{r}_{s1}) - \phi(\mathbf{r}_{s2}) + \phi(\mathbf{r}_{i1}) - \phi(\mathbf{r}_{i2})]} \rangle. \quad (10)$$

For pump beams that are of Gaussian Schell-model type, $W^{(2)}(\mathbf{r}_{s1}, \mathbf{r}_{i1}, \mathbf{r}_{s2}, \mathbf{r}_{i2})$ can be written, using the definitions in Eq. (3), as

$$\begin{aligned} W^{(2)}(\mathbf{r}_{s1}, \mathbf{r}_{i1}, \mathbf{r}_{s2}, \mathbf{r}_{i2}) = \mu_{\text{turb}}(\mathbf{r}_{s1}, \mathbf{r}_{i1}, \mathbf{r}_{s2}, \mathbf{r}_{i2}) \\ \times \sqrt{S^{(2)}(\rho_1, z) S^{(2)}(\rho_2, z)} \mu^{(2)}(\Delta\rho, z), \end{aligned} \quad (11)$$

where $\mu^{(2)}(\Delta\rho, z)$ is the degree of spatial two-photon coherence. Using Eqs. (2), (5), (6), (7), and (11), and considering the special case where $k_1 S^{(2)}(\rho_1, z) = k_2 S^{(2)}(\rho_2, z)$, that is, $a = b$, we obtain the following formula for the concurrence $C(\rho_{\text{qubit}})$ of the spatial two-qubit state:

$$C(\rho_{\text{qubit}}) = \mu_{\text{turb}}(\mathbf{r}_{s1}, \mathbf{r}_{i1}, \mathbf{r}_{s2}, \mathbf{r}_{i2}) \mu^{(2)}(\Delta\rho, z). \quad (12)$$

The effects of the spatial coherence properties of the pump beam on the entanglement of spatial two-qubit states have already been studied in detail [16]; in the present paper our main aim is to study the influence of turbulence. So we take the pump beam to be fully coherent, that is, $\mu^{(2)}(\Delta\rho, z) = 1$, in which case the Eq. (12) reduces to

$$C(\rho_{\text{qubit}}) = \mu_{\text{turb}}(\mathbf{r}_{s1}, \mathbf{r}_{i1}, \mathbf{r}_{s2}, \mathbf{r}_{i2}). \quad (13)$$

III. INFLUENCE OF KOLMOGOROV TURBULENCE ON THE ENTANGLEMENT

Assuming the Kolmogorov model [4,11,12] for atmospheric turbulence, we now derive an explicit expression for $\mu_{\text{turb}}(\mathbf{r}_{s1}, \mathbf{r}_{i1}, \mathbf{r}_{s2}, \mathbf{r}_{i2})$ and, thereby, for the concurrence $C(\rho_{\text{qubit}})$. We take the fluctuations introduced by turbulence to be a Gaussian random variable with zero mean and, using the standard result $\langle e^{-ix} \rangle = e^{-(1/2)\langle x^2 \rangle}$ for Gaussian variables, write Eq. (10) in the following form:

$$\begin{aligned} \mu_{\text{turb}}(\mathbf{r}_{s1}, \mathbf{r}_{i1}, \mathbf{r}_{s2}, \mathbf{r}_{i2}) &= \exp \left\{ -\frac{1}{2} [\langle [\phi(\mathbf{r}_{s1}) - \phi(\mathbf{r}_{s2})]^2 \rangle + \langle [\phi(\mathbf{r}_{i1}) - \phi(\mathbf{r}_{i2})]^2 \rangle \right. \\ &\quad + \langle [\phi(\mathbf{r}_{s1}) - \phi(\mathbf{r}_{i2})]^2 \rangle + \langle [\phi(\mathbf{r}_{s2}) - \phi(\mathbf{r}_{i1})]^2 \rangle \\ &\quad \left. - \langle [\phi(\mathbf{r}_{s1}) - \phi(\mathbf{r}_{i1})]^2 \rangle - \langle [\phi(\mathbf{r}_{s2}) - \phi(\mathbf{r}_{i2})]^2 \rangle \right\}. \quad (14) \end{aligned}$$

The quantity $\langle [\phi(\mathbf{r}_{s1}) - \phi(\mathbf{r}_{s2})]^2 \rangle$, etc., is known as the phase structure function, and in the Kolmogorov theory [11] of atmospheric turbulence it is given by

$$\langle [\phi(\mathbf{r}_{s1}) - \phi(\mathbf{r}_{s2})]^2 \rangle = 6.88 \left| \frac{\rho_{s1} - \rho_{s2}}{r_0(z)} \right|^{5/3}. \quad (15)$$

Here $r_0(z)$ is Fried's coherence diameter at plane z , which is a measure of the transverse scale over which refractive index correlations remain correlated. We evaluate the other terms in Eq. (14) in a similar manner. Next, using Eq. (13), and in terms of the displacement parameters defined in Eq. (3), we obtain the following expression for the concurrence:

$$\begin{aligned} C(\rho_{\text{qubit}}) &= \exp \left[-\frac{3.44}{[r_0(z)]^{5/3}} (|\Delta\rho + \Delta\rho'/2|^{5/3} \right. \\ &\quad + |\Delta\rho - \Delta\rho'/2|^{5/3} + |\mathbf{d} + \Delta\rho|^{5/3} + |\mathbf{d} - \Delta\rho|^{5/3} \\ &\quad \left. - |\mathbf{d} + \Delta\rho'/2|^{5/3} - |\mathbf{d} - \Delta\rho'/2|^{5/3} \right], \quad (16) \end{aligned}$$

where we have substituted \mathbf{d} for $(\rho'_1 + \rho'_2)/2$. Equation (16) is the main result of this paper; it quantifies the effects of atmospheric turbulence on the entanglement of spatial two-qubit states. Here $|\mathbf{d}| = d$ can be taken as a measure of the separation between the two signal and the two idler apertures, whereas $|\Delta\rho'| = \Delta\rho'$ can be taken as a measure of the effective physical size of the two-qubit state. For conceptual clarity, we now consider the one-dimensional version of Eq. (16) and

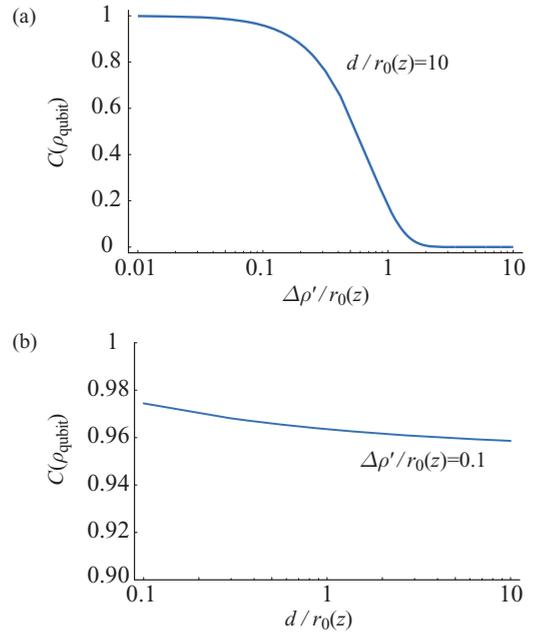


FIG. 2. (Color online) Plot of the concurrence $C(\rho_{\text{qubit}})$ (a) versus $\Delta\rho'/r_0(z)$ for $d/r_0(z) = 10$ and (b) versus $d/r_0(z)$ for $\Delta\rho' = 0.1$. Here d is a measure of the separation between the two signal and two idler apertures, whereas $\Delta\rho'$ can be taken as a measure of the effective physical size of the two-qubit state. Note that the concurrence depends much more strongly on $\Delta\rho'$ than on d .

restrict our further analysis to two-qubit states that have $\Delta\rho = 0$. Such states are prepared by placing the two signal and the two idler apertures at symmetric positions, that is, by keeping $\rho_{s1} = -\rho_{i1}$ and $\rho_{s2} = -\rho_{i2}$. This situation is experimentally most relevant because, as noted previously, in this situation the probabilities a and b in Eq. (1), that the signal and idler photons pass through the apertures, are maximum. The concurrence of the two-qubit state can now be written as:

$$\begin{aligned} C(\rho_{\text{qubit}}) &= \exp \left\{ -\frac{3.44}{[r_0(z)]^{5/3}} [2(\Delta\rho'/2)^{5/3} + 2d^{5/3} \right. \\ &\quad \left. - (d + \Delta\rho'/2)^{5/3} - (d - \Delta\rho'/2)^{5/3} \right\}. \quad (17) \end{aligned}$$

In writing Eq. (17) we have used the convention that $\rho'_1 > \rho'_2 > 0$, which implies that $d > \Delta\rho'/2 > 0$. Figure 2(a) shows a plot of the concurrence as a function of $\Delta\rho'/r_0(z)$ for $d/r_0(z) = 10$, and Fig. 2(b) shows a plot of the concurrence as a function of $d/r_0(z)$ for $\Delta\rho'/r_0(z) = 0.1$. We find that the concurrence of a spatial two-qubit state remains close to unity as long as $\Delta\rho'$ is much smaller than Fried's coherence diameter $r_0(z)$. We also find that the effects of increasing the separation d on the concurrence saturates once d becomes much larger than Fried's coherence diameter. Here, since we are dealing with two-qubit states with only two nonzero diagonal elements, we note that in an experiment the concurrence of Eq. (17) can be measured directly, by measuring the visibility of the two-photon interference fringes [15,16].

IV. DISCUSSION AND CONCLUSIONS

In conclusion, we have studied the effects of atmospheric turbulence on degrading the entanglement of spatial two-qubit

states that are prepared by making down-converted photons pass through a pair of double-apertures. We have used the Kolmogorov model for atmospheric turbulence and have quantified the entanglement of the two-qubit state in terms of its concurrence. Our analysis is presented for two-qubit states that can be represented by density matrices having only two nonzero diagonal elements. We have found that, as a general rule, the concurrence of spatial two-qubit states remains close to unity as long as the magnitude of the

two-photon position-asymmetry vector is much smaller than Fried's coherence diameter.

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