Weak Values and Direct Measurement of the Quantum Wavefunction

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VOLUME 60, NUMBER 14 PHYSICAL REVIEW LETTERS 4 APRIL 1988

How the Result of a Measurement of a Component of the Spin of a Spin-$\frac{1}{2}$ Particle Can Turn Out to be 100

Yakir Aharonov, David Z. Albert, and Lev Vaidman
Physics Department, University of South Carolina, Columbia, South Carolina 29208, and School of Physics and Astronomy, Tel-Aviv University, Ramat Aviv 69978, Israel
(Received 30 June 1987)

We have found that the usual measuring procedure for preselected and postselected ensembles of quantum systems gives unusual results. Under some natural conditions of weakness of the measurement, its result consistently defines a new kind of value for a quantum variable, which we call the weak value. A description of the measurement of the weak value of a component of a spin for an ensemble of preselected and postselected spin-$\frac{1}{2}$ particles is presented.

PACS numbers: 03.65.Bz

standard expectation value: $< A >=< \Psi|\hat{A}|\Psi >$

weak value: $A_w \equiv \langle \psi_f | A | \psi_{\text{in}} \rangle / \langle \psi_f | \psi_{\text{in}} \rangle.$

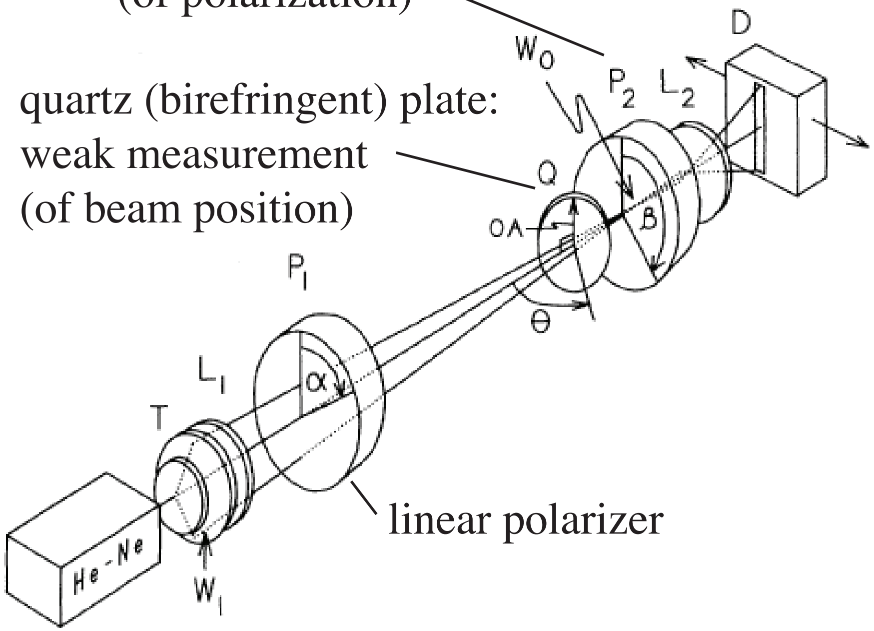
Why are weak values important?
can lead to amplification of small signals
can lead to direct measurement of the quantum wavefunction

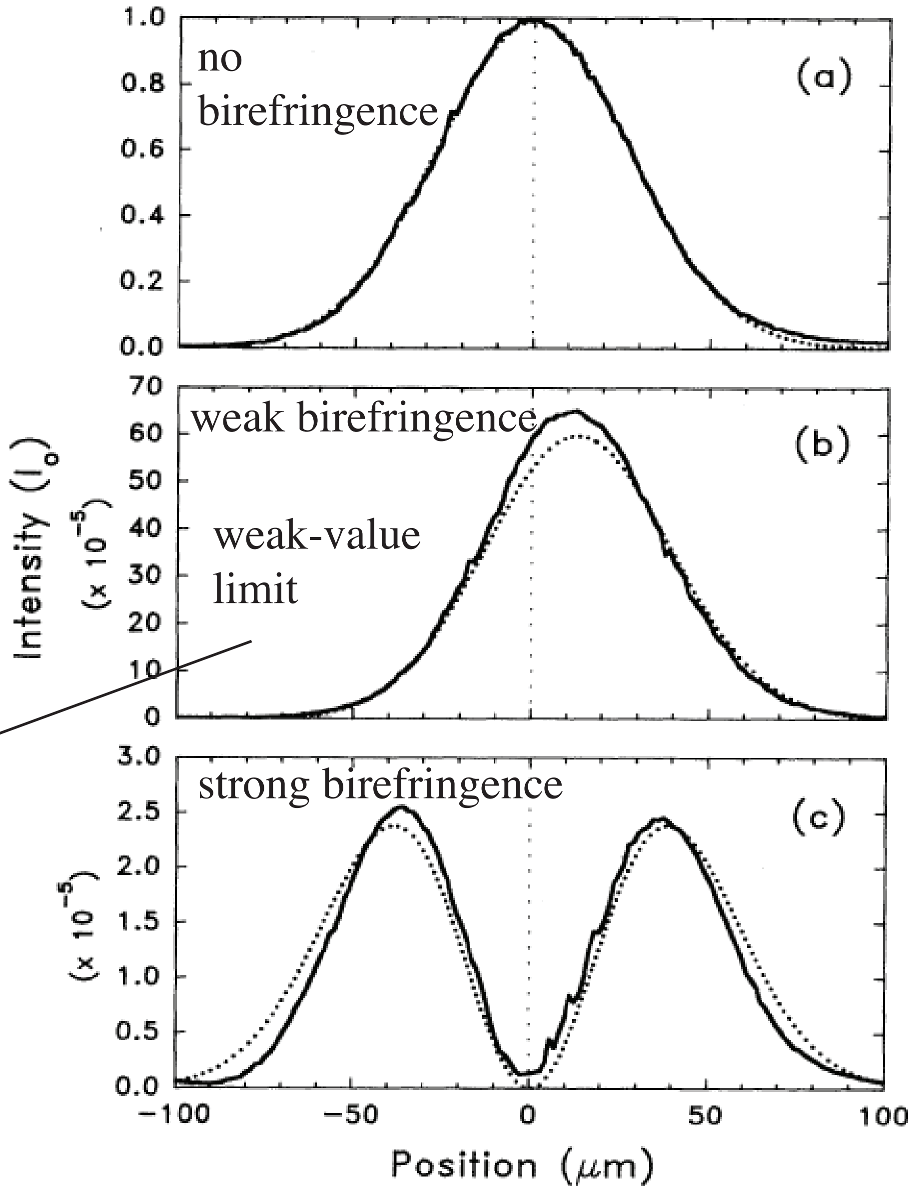
Realization of a Measurement of a "Weak Value"

N. W. M. Ritchie, J. G. Story, and Randall G. Hulet
Department of Physics and Rice Quantum Institute, Rice University, Houston, Texas 77251-1892
(Received 7 December 1990)

linear polarizer:
strong measurement
(of polarization)

quartz (birefringent) plate:
weak measurement
(of beam position)





Birefringence separates polarized beams by 0.64 μm, but gaussian in (b) is displaced by 12 μm.

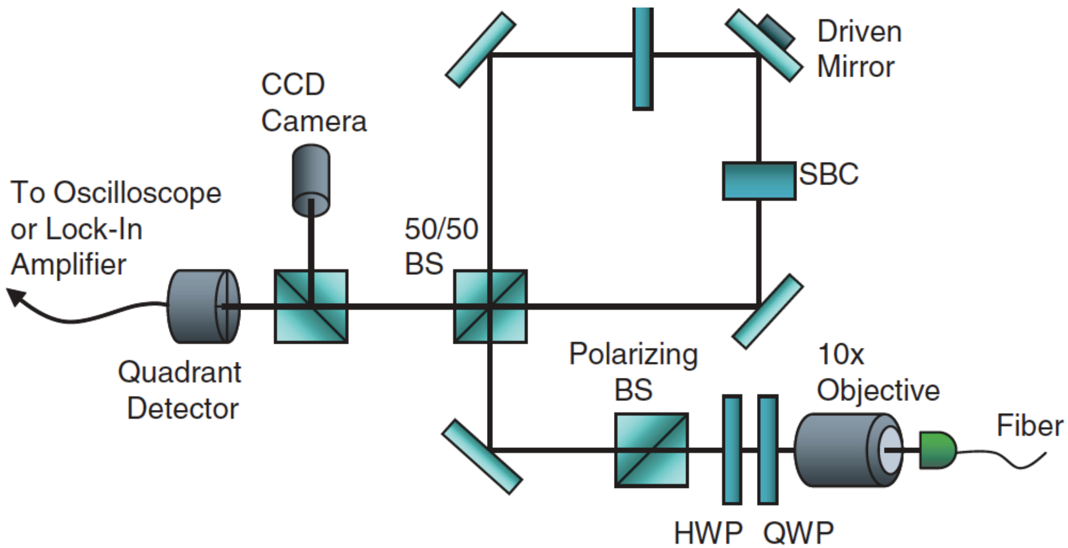
PRL 1991

Selected for a Viewpoint in *Physics*
PRL **102,** 173601 (2009) PHYSICAL REVIEW LETTERS week ending 1 MAY 2009

Ultrasensitive Beam Deflection Measurement via Interferometric Weak Value Amplification

P. Ben Dixon, David J. Starling, Andrew N. Jordan, and John C. Howell
Department of Physics and Astronomy, University of Rochester, Rochester, New York 14627, USA
(Received 12 January 2009; published 27 April 2009)

We report on the use of an interferometric weak value technique to amplify very small transverse deflections of an optical beam. By entangling the beam's transverse degrees of freedom with the which-path states of a Sagnac interferometer, it is possible to realize an optical amplifier for polarization independent deflections. The theory for the interferometric weak value amplification method is presented along with the experimental results, which are in good agreement. Of particular interest, we measured the angular deflection of a mirror down to 400 ± 200 frad and the linear travel of a piezo actuator down to 14 ± 7 fm.



Direct Measurement of the Quantum Wavefunction

The wavefunction of quantum mechanics is notoriously difficult to measure

Measurement at any one position causes the entire wavefunction to "collapse"

(The problem is related to the Heisenberg Uncertainty Relation; if you measure position you cannot also know momentum.)

Historically, the wavefunction has been measured only indirectly and inefficiently, using "quantum state tomography."

Recent work has demonstrated how to measure the wavefunction directly.

The idea is to perform a "weak measurement" on one variable (which thus only minimally disturbs the system), followed by a "strong measurement."

J. Lundeen et al., Nature 474, 188 (2011)
J. Z. Salvail et al., Nature Photonics, 10.1038 (2013)

LETTER

doi:10.1038/nature10120

Direct measurement of the quantum wavefunction

Jeff S. Lundeen[1], Brandon Sutherland[1], Aabid Patel[1], Corey Stewart[1] & Charles Bamber[1]

$$\langle A \rangle_{\mathrm{W}} = \frac{\langle c|A|\Psi\rangle}{\langle c|\Psi\rangle}$$

Returning to our example of a single particle, consider the weak measurement of position ($A = \pi_x \equiv |x\rangle\langle x|$) followed by a strong measurement of momentum giving $P = p$. In this case, the weak value is:

$$\langle \pi_x \rangle_{\mathrm{W}} = \frac{\langle p|x\rangle\ \langle x|\Psi\rangle}{\langle p|\Psi\rangle} \tag{2}$$

$$= \frac{e^{ipx/\hbar}\,\Psi(x)}{\Phi(p)} \tag{3}$$

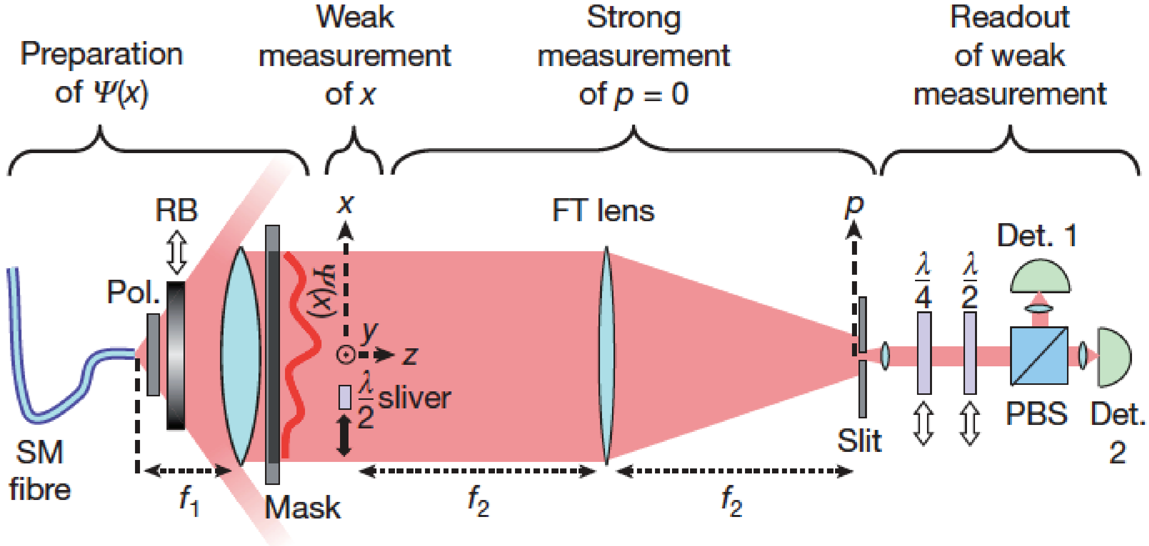
In the case $p = 0$, this simplifies to

$$\langle \pi_x \rangle_{\mathrm{W}} = k\Psi(x) \tag{4}$$

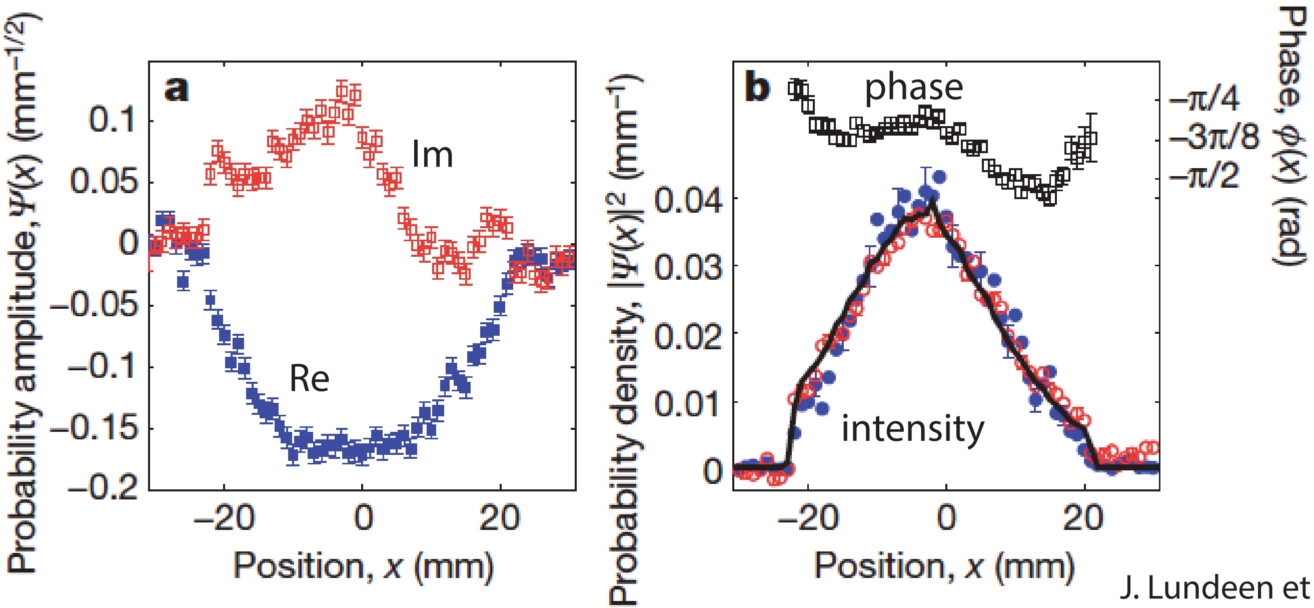
where $k = 1/\Phi(0)$ is a constant (which can be eliminated later by normalizing the wavefunction). The average result of the weak mea-

Direct Measurement of the Photon "Wavefunction"

Measurement setup



Typical results



J. Lundeen et al., Nature 474, 188 (2011)

Wavefunction of the Photon?

Many people feel it is inaccurate to speak of the "wavefunction" of the photon.

I personally try to avoid the term wavefunction of the photon. A photon is an excitation of a mode of the field. I prefer to distinguish the photon from the mode in which it lives.

Lundeen et al. use the term "wavefunction of the photon," but comment that it is sometimes called the "spatial mode of the photon."

See also Iwo Bialynicki-Birula, The Photon Wave Function, Coherence and Quantum Optics VII, Eds. J. H. Eberly, L. Mandel, and E. Wolf Plenum, NY 1996, p. 313

Work of My Own Group

1. We have made a direct measurement of the state of polarization of the photon. This is thus the first direct measurement of a qubit.

2. We have measured the statevector of a state imbedded in a 27-dimensional OAM Hilbert space. One expects direct measurement to be increasingly useful with increasing size of the Hilbert space.

nature photonics

ARTICLES

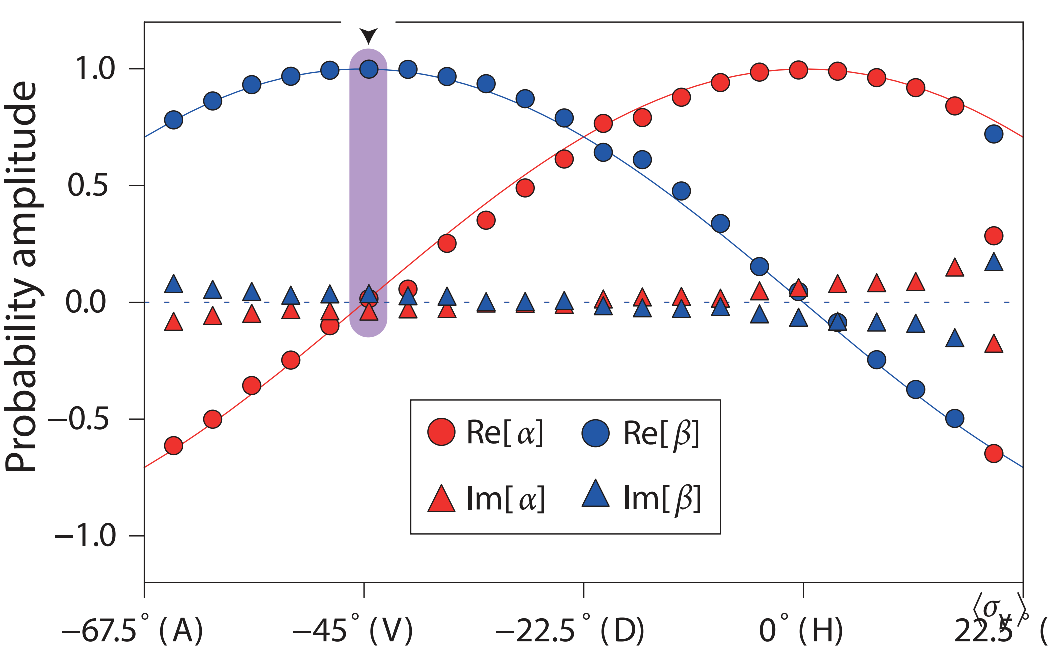
PUBLISHED ONLINE: 3 MARCH 2013 | DOI: 10.1038/NPHOTON.2013.24

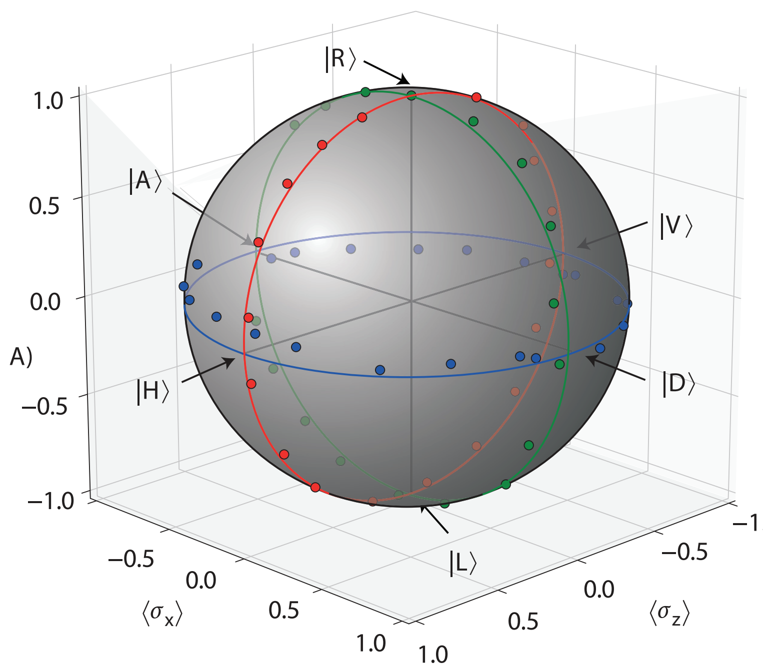
Full characterization of polarization states of light via direct measurement

Jeff Z. Salvail[1]*, Megan Agnew[1], Allan S. Johnson[1], Eliot Bolduc[1], Jonathan Leach[1] and Robert W. Boyd[1,2]

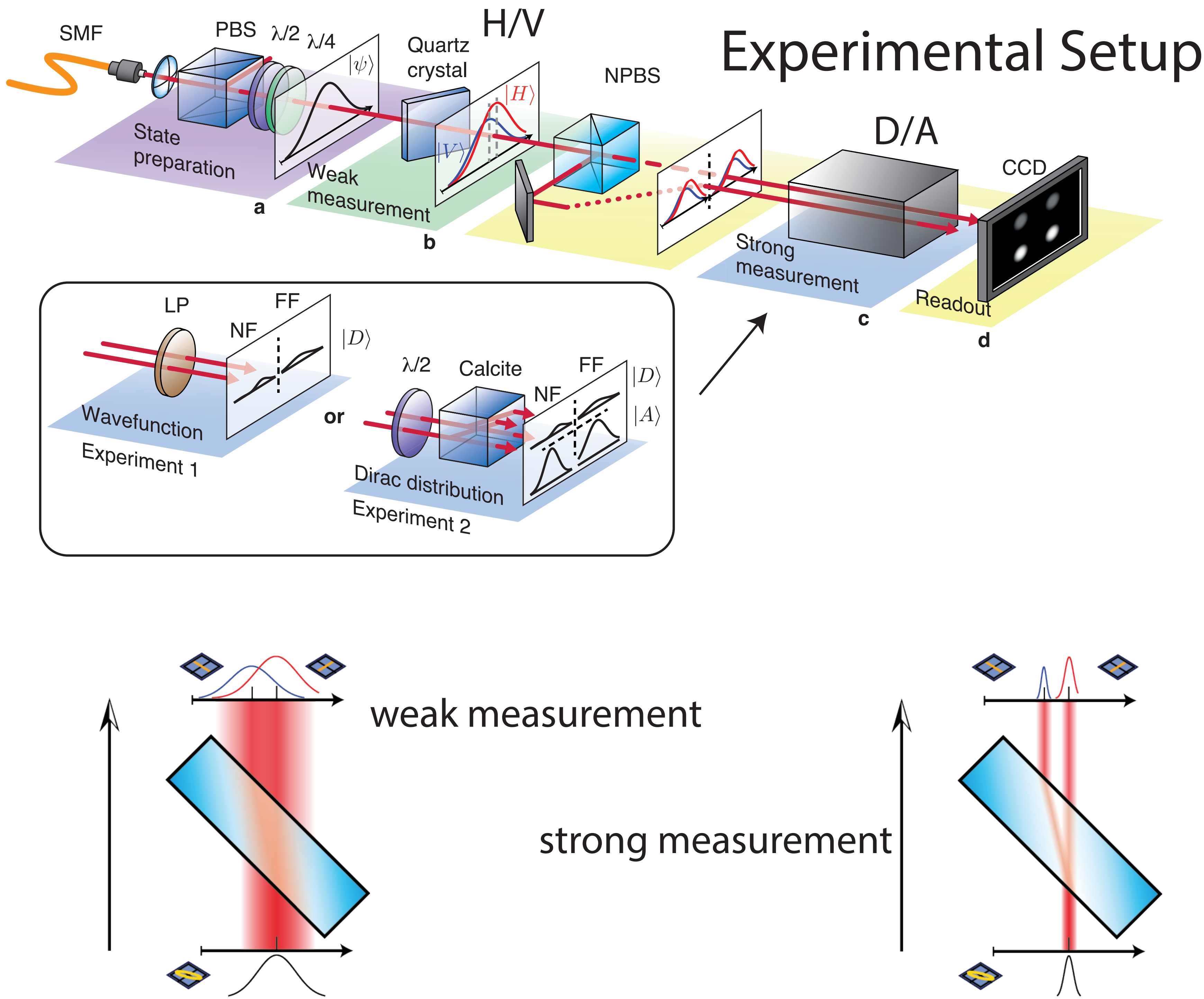
points on Poincaré sphere

$$|\psi\rangle = \alpha|H\rangle + \beta|V\rangle.$$





Half-wave plate angle (input polarization state)

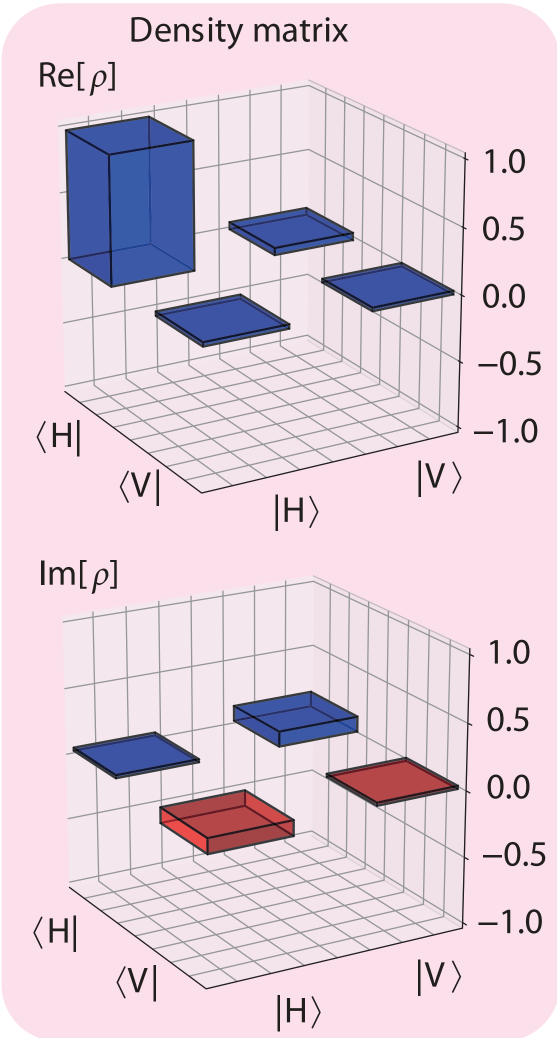


Experimental Setup
SMF
PBS
λ/2
λ/4
|ψ⟩
Quartz crystal
H/V
|H⟩
|V⟩
NPBS
D/A
CCD
State preparation
a
Weak measurement
b
Strong measurement
c
Readout
d
LP
FF
NF
|D⟩
Wavefunction
Experiment 1
or
λ/2
Calcite
FF
NF
|D⟩
|A⟩
Dirac distribution
Experiment 2
weak measurement
strong measurement

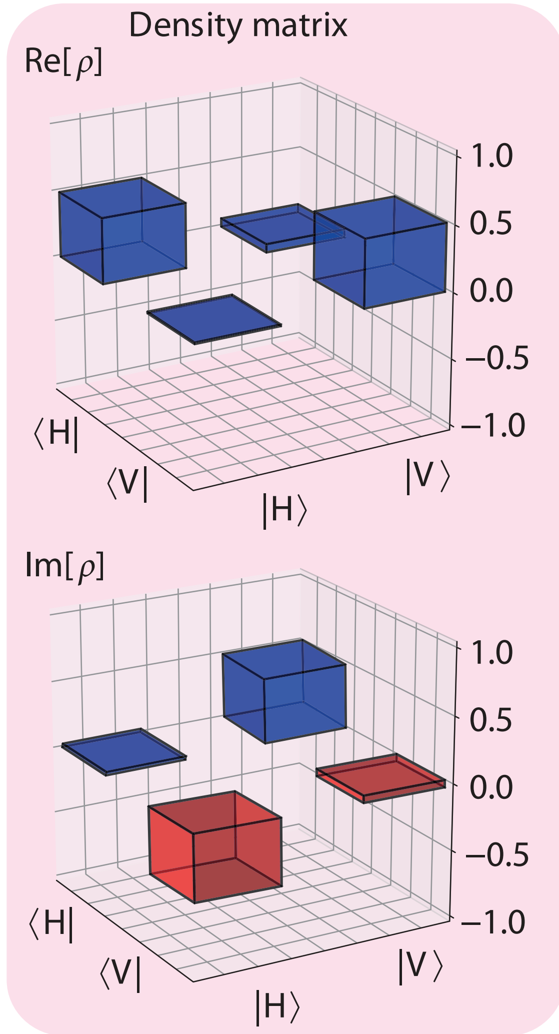
Direct Measurement of the Full Density Matrix

Meaurement provides enough information to determine full density matrix, as shown by J.S. Lundeen & C. Bamber, Phys. Rev. Lett. 108, 070402 (2012).

Horizontal polarization



Left-hand-circular polarization



Salvail et al, 2013

News Accounts of Our Work

physicsworld.com

First weak measurements made on optical polarization states

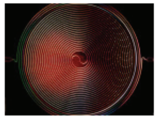
Mar 11, 2013 4 comments

Physicists in Canada and the US claim to be the first to make a direct measurement of the polarization quantum state of light – a feat that at first glance appears to defy Heisenberg's uncertainty principle. The technique, which relies on a process known as weak measurement, could help in fundamental studies on quantum mechanics or in the development of quantum computing.

NATIONAL POST

JEFF SALVAIL

Canadian researchers take a sneak peek at Schrödinger's Cat and a step toward a quantum computer



In an Ottawa lab, scientists have succeeded in side-stepping an obstacle of Heisenberg's Uncertainty Principle, a strange law of the quantum world

Lawrence Krauss: The Large Hadron Collider helps science do a quantum leap

Neil Turok: Furthering our understanding of the universe

Freezing antimatter could allow scientists to study the strangest stuff in existence: Canadian researcher

SCIENCE RECORDER

Scientists discover a way around Heisenberg's Uncertainty Principle

A new technique for quantum mechanics.

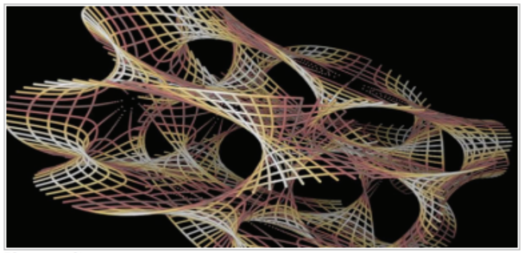


Photo credit:

Science Recorder | Mark Petillo | **Monday, March 04, 2013**

It could be big news of the world of quantum physics. According to a pair of scientists from the University of Rochester and the University of Ottawa, there may be a way around Heisenberg's famous Uncertainty Principle. According to a report published this week in *Nature Photonics*, a recently developed technique that allows scientists to directly measure the polarization states of light could be the key... **To continue reading, subscribe to *Science Recorder* today.**

Work of My Own Group

1. We have made a direct measurement of the state of polarization of the photon. This is thus the first direct measurement of a qubit.

2. We have measured the statevector of a state imbedded in a 27-dimensional OAM Hilbert space. One expects direct measurement to be increasingly useful with increasing size of the Hilbert space.

Malik, Mirhosseini, Lavery, Leach, Padgett, Boyd, Nature Communications, 5:3115 (2014).

High Dimensional QKD Protocol

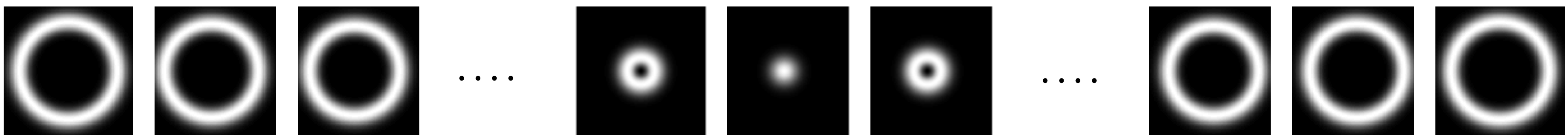
We are constructing a QKD system in which each photon carries many bits of information

We encode in states that carry OAM such as the Laguerre-Gauss states

As a diagnostic, we need to be able to measure the statevector of OAM states

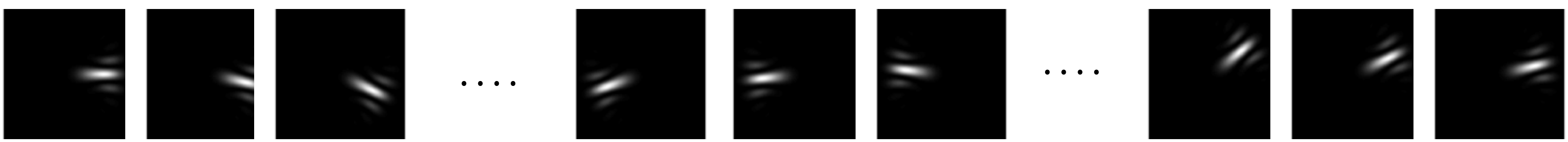
Single Photon States

Laguerre-Gaussian Basis $\ell = -13, \ldots, 13$



0 1 2 12 13 14 25 26 27

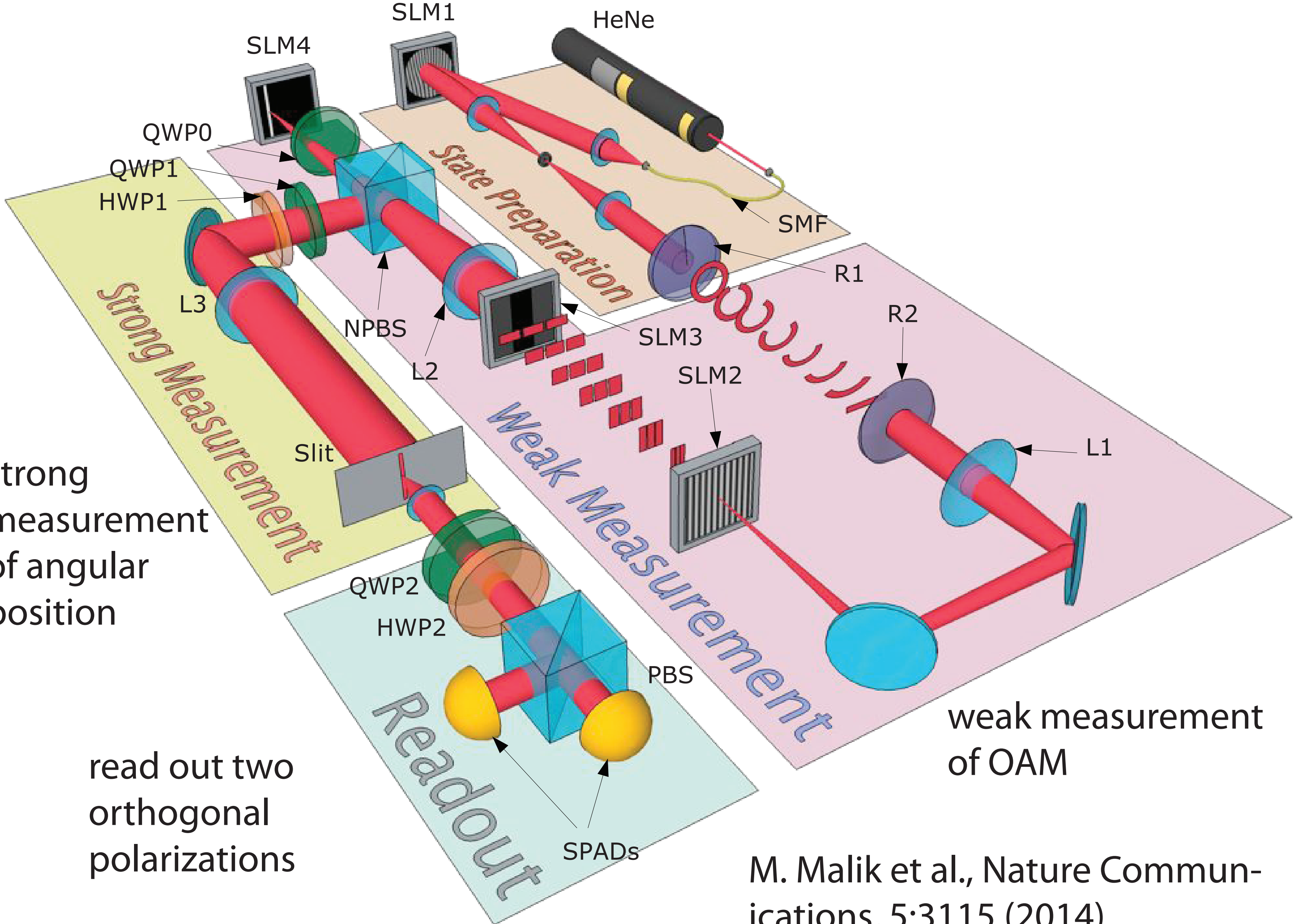
"Angular" Basis (mutually unbiased with respect to LG)



0 1 2 12 13 14 25 26 27

$$\Psi_{\mathrm{AB}}^{N} = \frac{1}{\sqrt{27}} \sum_{l=-13}^{13} \mathrm{LG}_{l,0} \exp\left(i 2\pi N l / 27\right)$$

Direct Measurement of a High-Dimensional OAM State



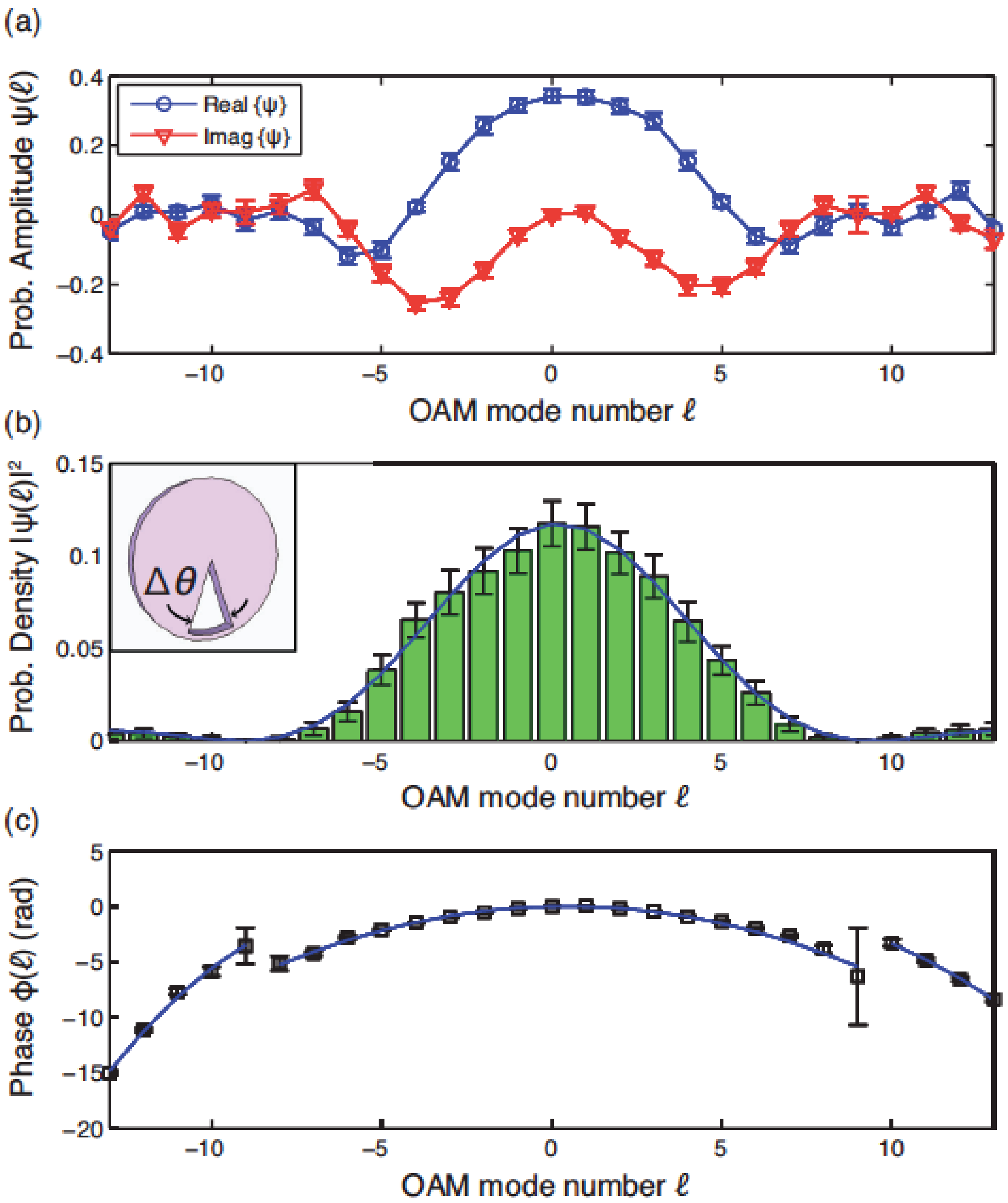
strong measurement of angular position

read out two orthogonal polarizations

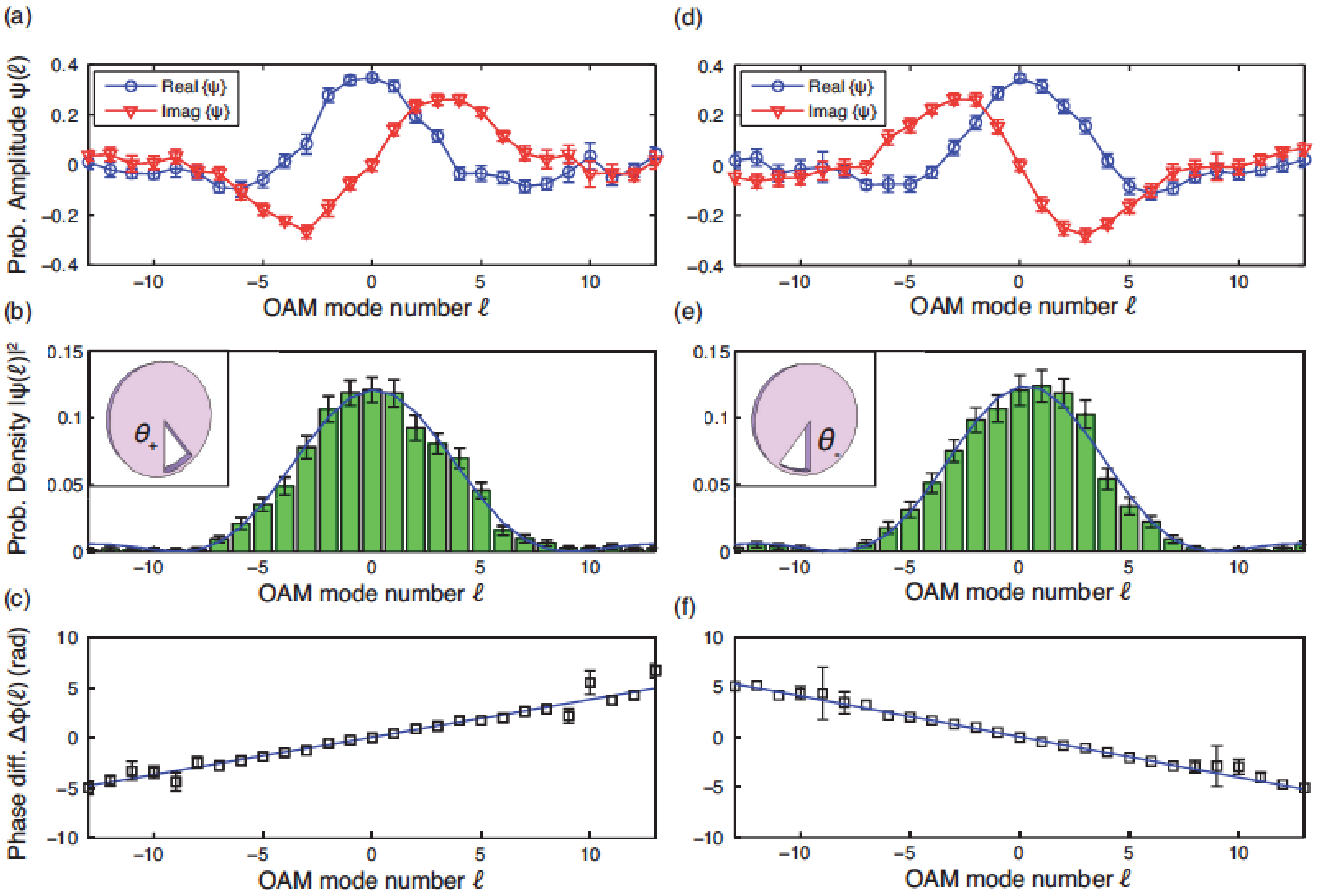
weak measurement of OAM

M. Malik et al., Nature Communications, 5:3115 (2014).

Results: Direct Measurement of a High-Dimensional OAM State

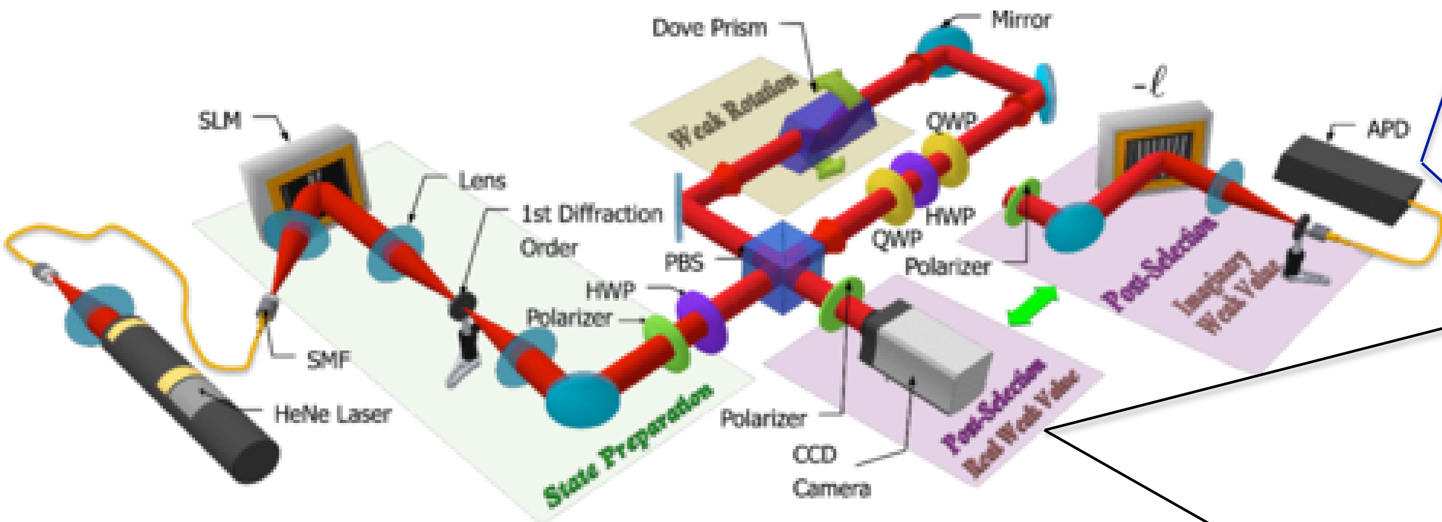


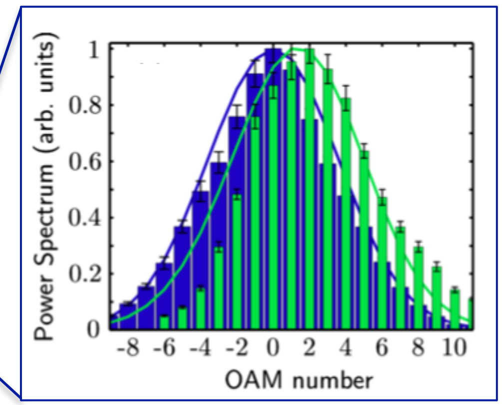
Direct Measurement Procedure Properly Measures Phase

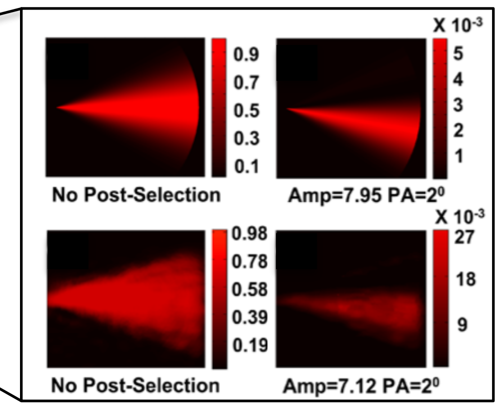


Amplification of Angular Rotations using Weak Measurements

Omar S. Magaña-Loaiza, Mohammad Mirhosseini, Brandon Rodenburg, and Robert W. Boyd, Phys. Rev. Lett 112, 200401 (2014)



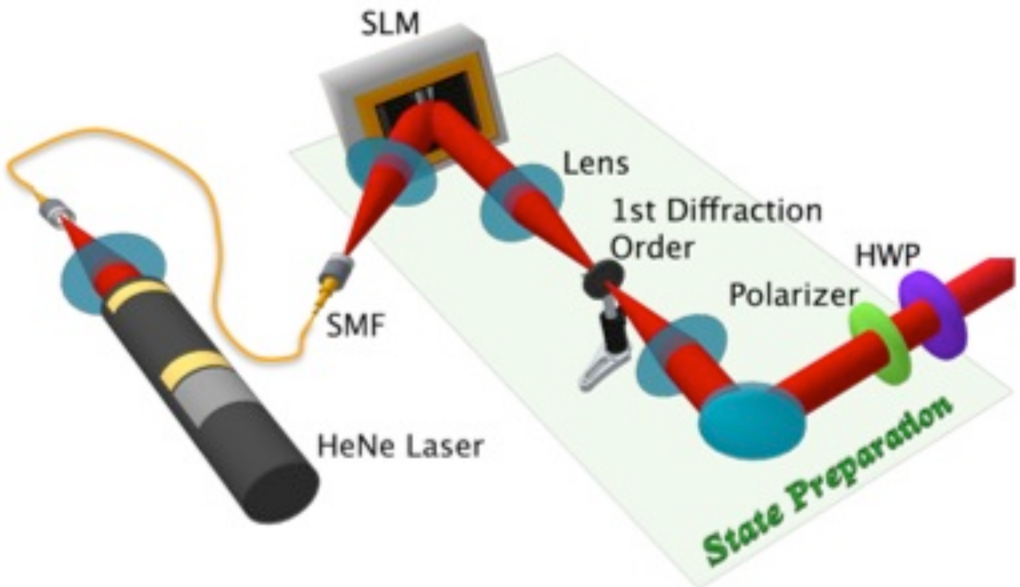




First demonstration of weak--value amplification in the azimuthal variables of angular position and orbital angular momentum.

Amplification of angular rotations using WM

State preparation or pre-selection



Pointer   Probe

$$f(\phi) = e^{\frac{\phi^2}{2\sigma_\phi^2}} \qquad \left|\Psi_{pr}\right\rangle = \frac{1}{\sqrt{2}}\left(\left|H\right\rangle + \left|V\right\rangle\right)$$

$$\boxed{\left|\Psi_i\right\rangle = \left|f(\phi)\right\rangle\left|\Psi_{pr}\right\rangle}$$

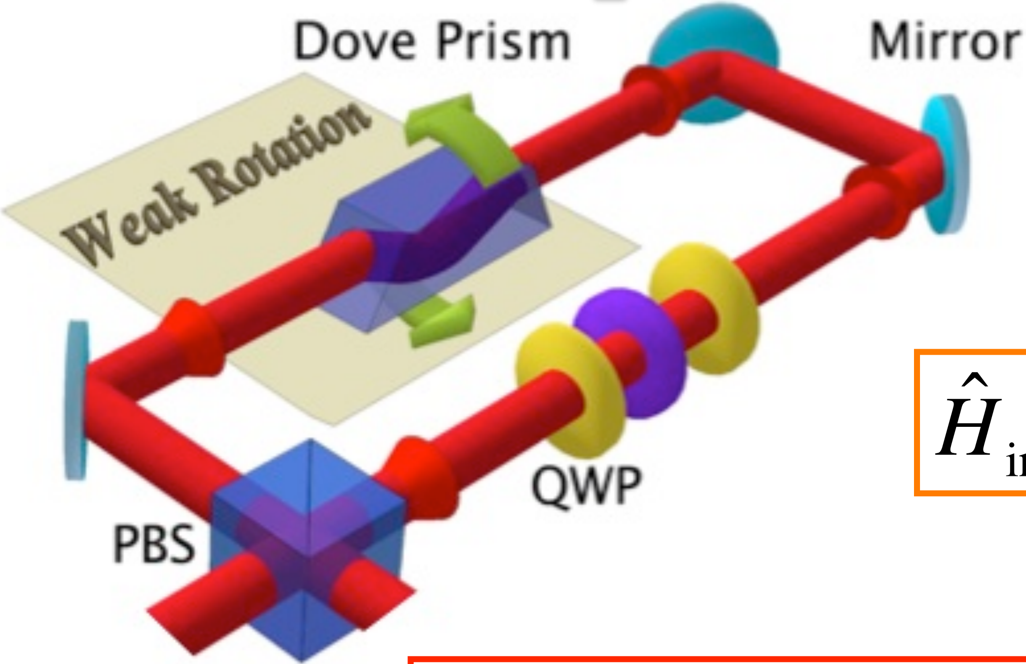
O. S. Magana-Loaiza et. al., Phys. Rev. Lett., 112, 200401 (2014)

Amplification of angular rotations using WM

Weak Interaction

Polarization sensitive
Sagnac interferometer

This is the rotation that we want to amplify!



$$\left|\Psi_i\right\rangle = \left|f(\phi)\right\rangle \left|\Psi_{pr}\right\rangle$$

$$\boxed{\hat{H}_{\text{int}}} = \boxed{\hat{H}_{SO}} + \boxed{\hat{H}_g}$$

$$\boxed{\left|\Psi_f\right\rangle = e^{-i\theta}\left|f(\phi - \Delta\phi/2)\right\rangle\left|H\right\rangle + e^{i\theta}\left|f(\phi+\Delta\phi/2)\right\rangle\left|V\right\rangle}$$

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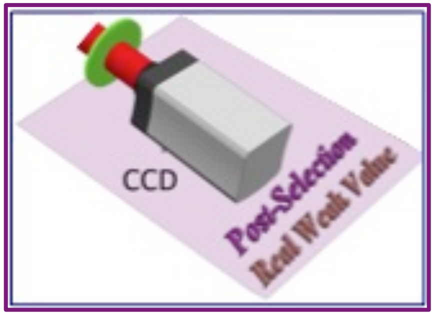
O. S. Magana-Loaiza et. al., Phys. Rev. Lett., 112, 200401 (2014)

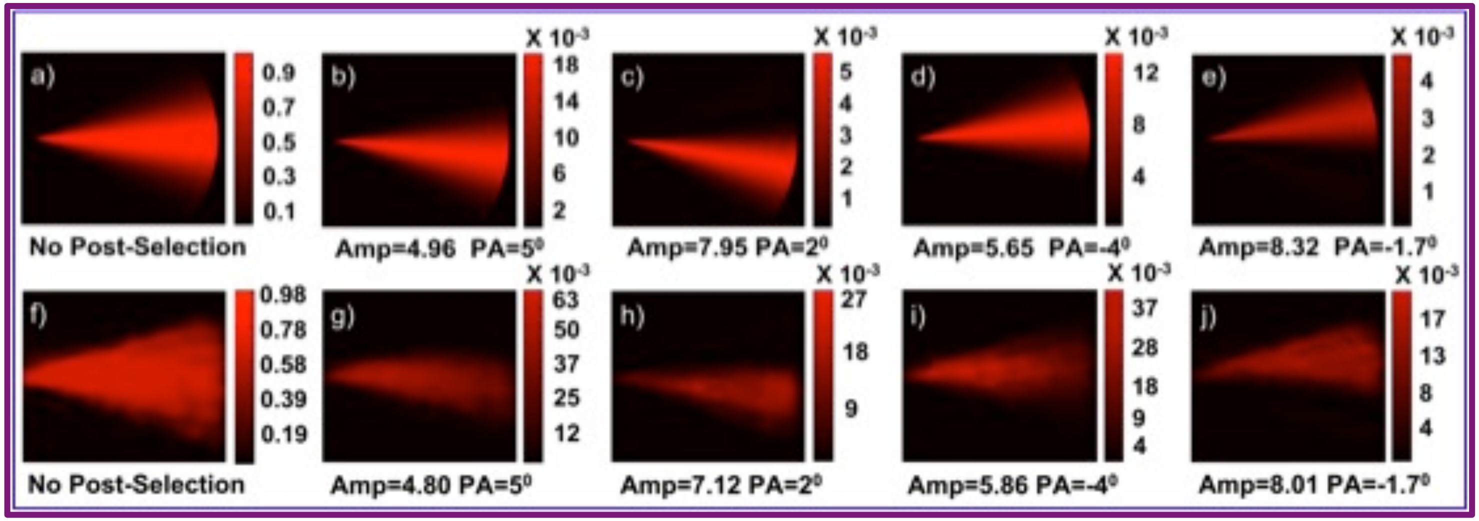
Real Weak Values

Experimental data for the real part of the weak value



Amp: Amplification of rotation
PA: Post-selection angle

The expected value of the angular position $\langle \hat{\phi} \rangle_f \propto \dfrac{\Re(\sigma_w)\Delta\phi}{2}$

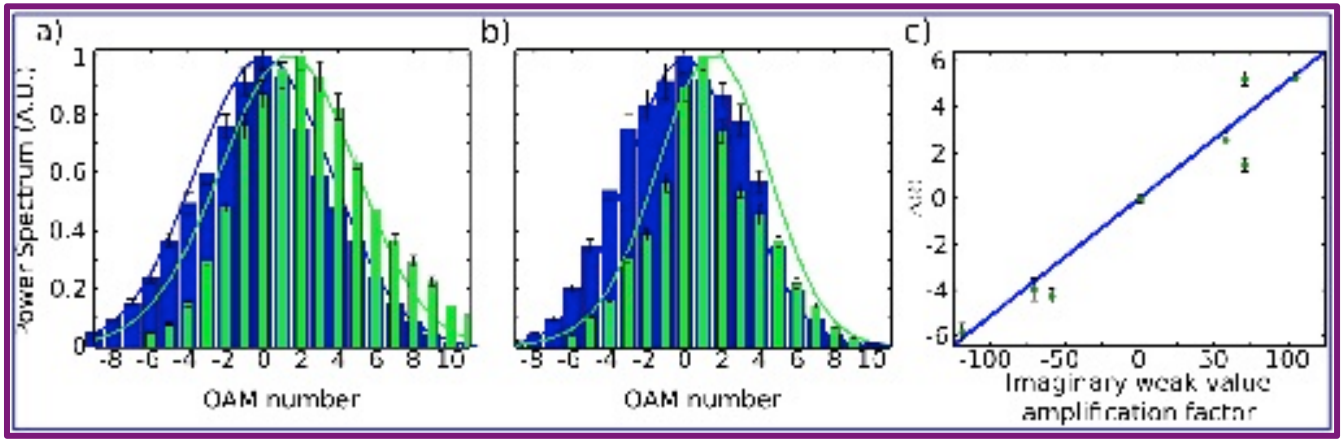


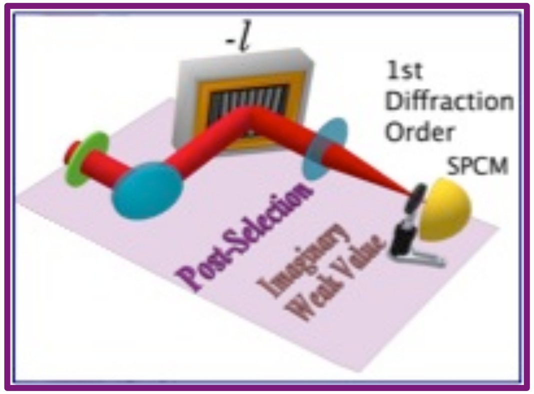
O. S. Magana-Loaiza et. al., Phys. Rev. Lett., 112, 200401 (2014)

Amplification of angular rotations using WM

Imaginary Weak Values

The expected value of the orbital angular momentum $\left\langle \hat{l} \right\rangle_f \propto \frac{\Im(\sigma_w)\Delta\phi}{2\eta_\phi^2}$





Gaussian in angular position

Fourier Pairs

Projection measurements

Gaussian in OAM

S. Franke-Arnold, et. al., New J. Phys. 6, 103 (2004).

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O. S. Magana-Loaiza et. al., Phys. Rev. Lett., 112, 200401 (2014)

Conclusions

- We have made the first step towards the study of WVA in the azimuthal degree of freedom.
- The shift in angular position is related to the real part of the weak value of a polarization operator.
- The OAM spectrum is shifted as a consequence of the breakup in the polarization symmetry realized by a differential geometric phase, which produces imaginary weak values.
- We believe that our protocol opens the possibility for new schemes in optical metrology.