Quantum Properties of Twisted Light

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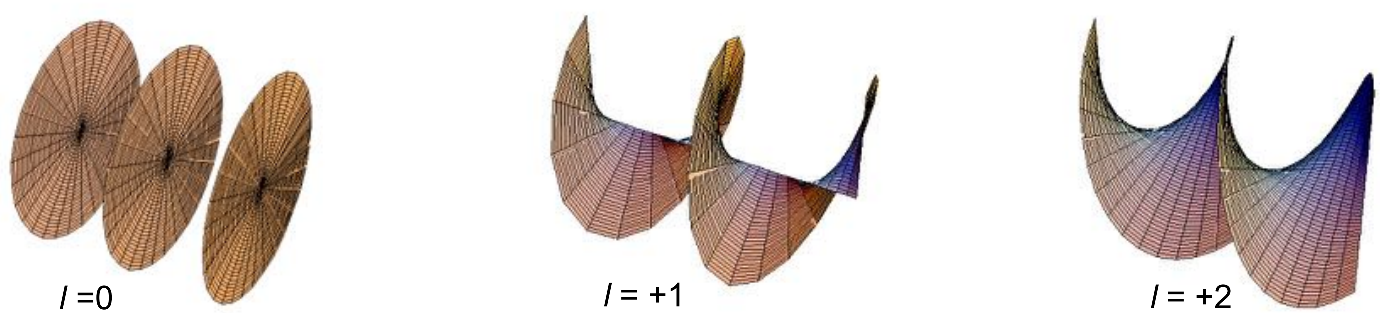
Quantum Information and Orbital Angular Momentum (OAM) of Light

- Utilize the transverse degree of freedom of light
  - Spatial division multiplexing
- In particular, encode in angular position and its conjugate variable, orbital angular momentum (OAM)
- Motivation: Encode more information per photon

What Are the OAM States of Light?

- Light can carry spin angular momentum (SAM) by means of its circular polarization.
- Light can also carry orbital angular momentum (OAM) by means of the phase winding of the optical wavefront.
- A well-known example are the Laguerre-Gauss modes. These modes contain a phase factor of $\exp(il\phi)$ and carry angular momentum of $l\hbar$ per photon. (Here ϕ is the azimuthal coordinate.)

Phase-front structure of some OAM states



See, for instance, A. M. Yao and M. J. Padgett, Advances in Photonics 3, 161 (2011)

Laguerre-Gauss Modes

The paraxial approximation to the Helmholtz equation $(\nabla^2 + k^2)E(\boldsymbol{k}) = 0$ gives the paraxial wave equation which is written in the cartesian coordinate system as

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + 2ik\frac{\partial}{\partial z}\right) E(x, y, z) = 0. \tag{1}$$

The paraxial wave equation is satisfied by the Laguerre-Gaussian modes, a family of orthogonal modes that have a well defined orbital angular momentum. The field amplitude $LG_p^l(\rho, \phi, z)$ of a normalized Laguerre-Gaussian modes is given by

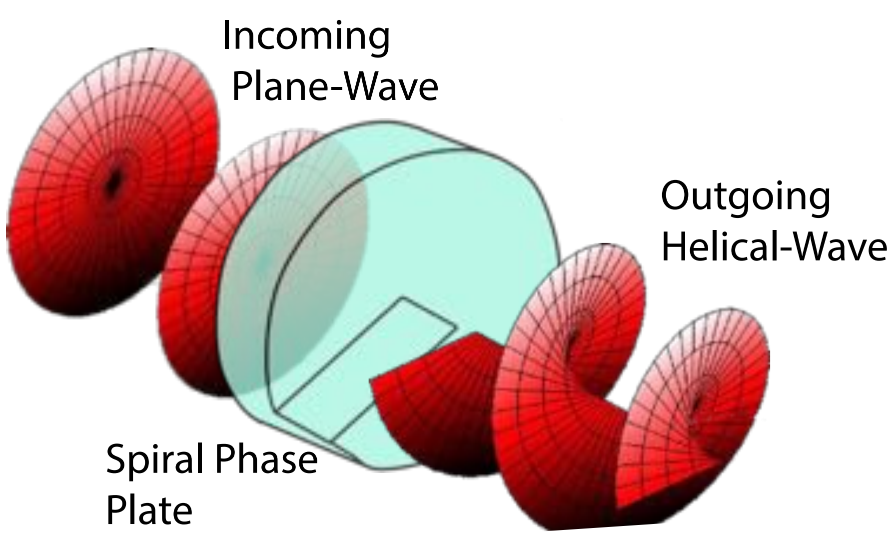
$$\begin{aligned} LG_p^l(\rho, \phi, z) = {} & \sqrt{\frac{2p!}{\pi(|l| + p)!}} \frac{1}{w(z)} \left[\frac{\sqrt{2}\rho}{w(z)}\right]^{|l|} L_p^l \left[\frac{2\rho^2}{w^2(z)}\right] \\ & \times \exp\left[-\frac{\rho^2}{w^2(z)}\right] \exp\left[-\frac{ik^2\rho^2 z}{2(z^2 + z_R^2)}\right] \exp\left[i(2p + |l| + 1)\tan^{-1}\left(\frac{z}{z_R}\right)\right] e^{-il\phi}, \end{aligned} \tag{2}$$

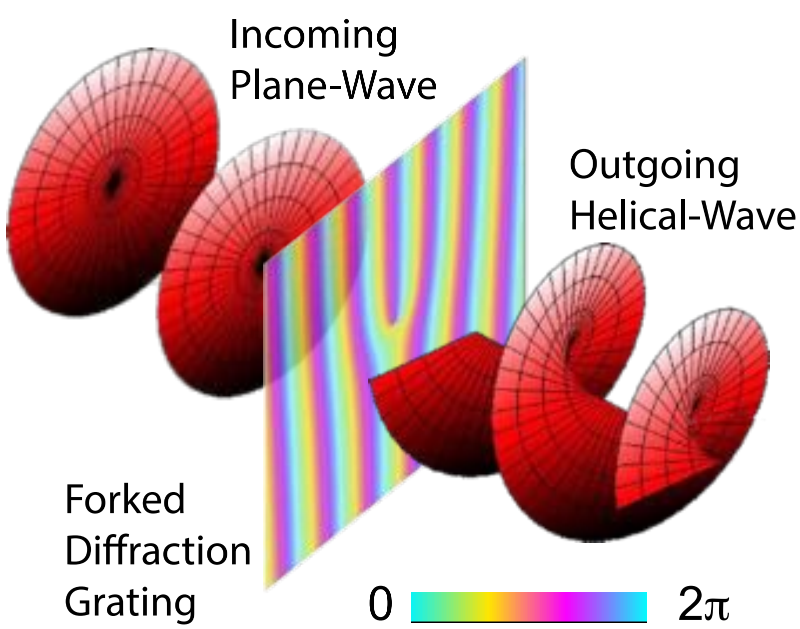
where k is the wave-vector magnitude of the field, z_R the Rayleigh range, $w(z)$ the radius of the beam at z, l is the azimuthal quantum number, and p is the radial quantum number. L_p^l is the associated Laguerre polynomial.

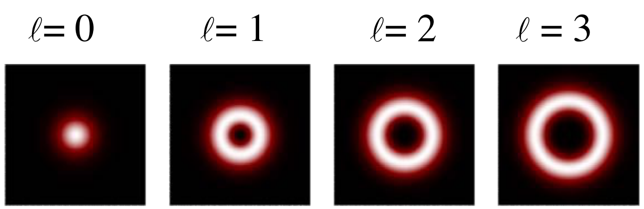
How to create a beam carrying orbital angular momentum?

- Pass beam through a spiral phase plate

- Use a spatial light modulator acting as a computer generated hologram (more versatile)





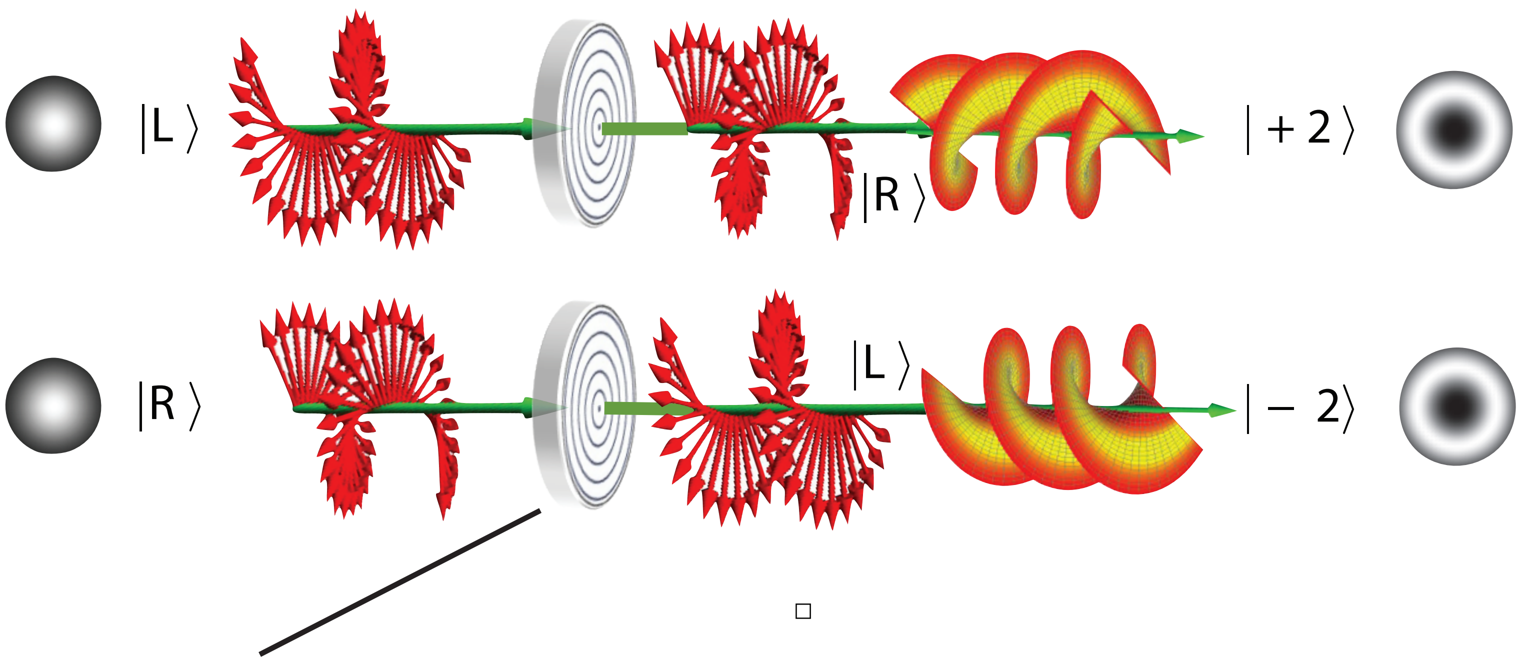


Exact solution to simultaneous intensity and phase masking with a single phase-only hologram, E. Bolduc, N. Bent, E. Santamato, E. Karimi, and R. W. Boyd, Optics Letters 38, 3546 (2013).

Photonic Q-Plates: A Quantum Interface

Spin angular momentum can be transferred to OAM through use of a Q-plate

Ability to change basis of encoding useful for quantum information processing

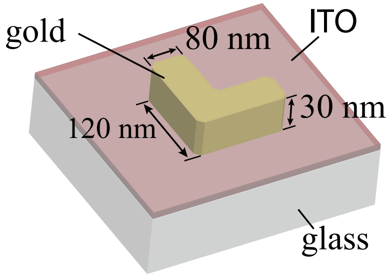


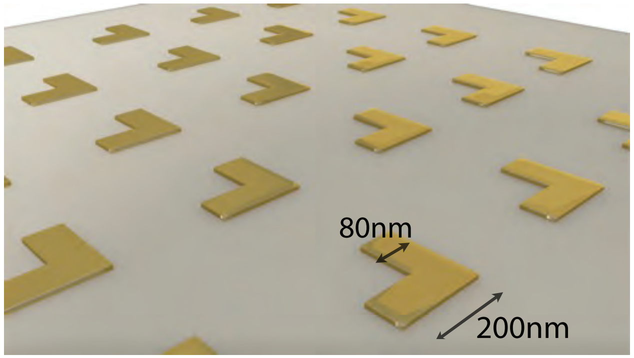
Q-plate. Usually a carefully constructed liquid-crystal cell

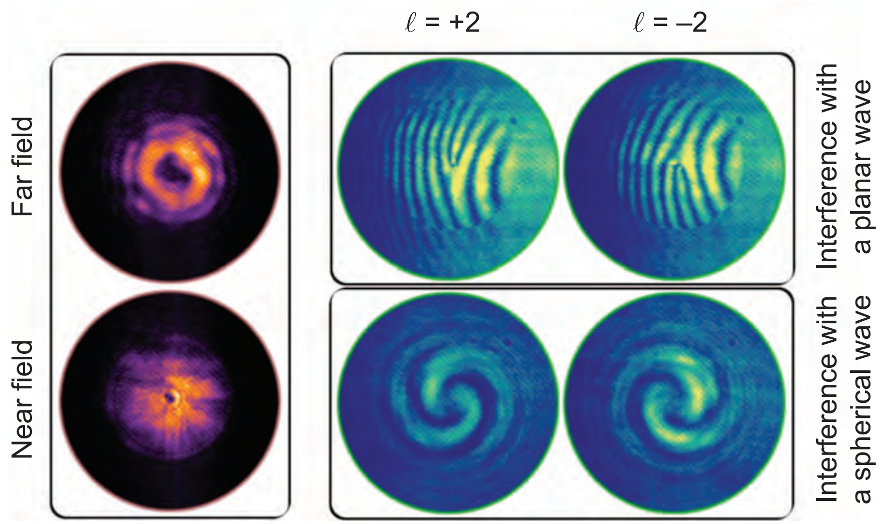
Marrucci et al., PRL 96 , 163905 (2006)
Karimi et al.,APL 98 , 231124 (2009)

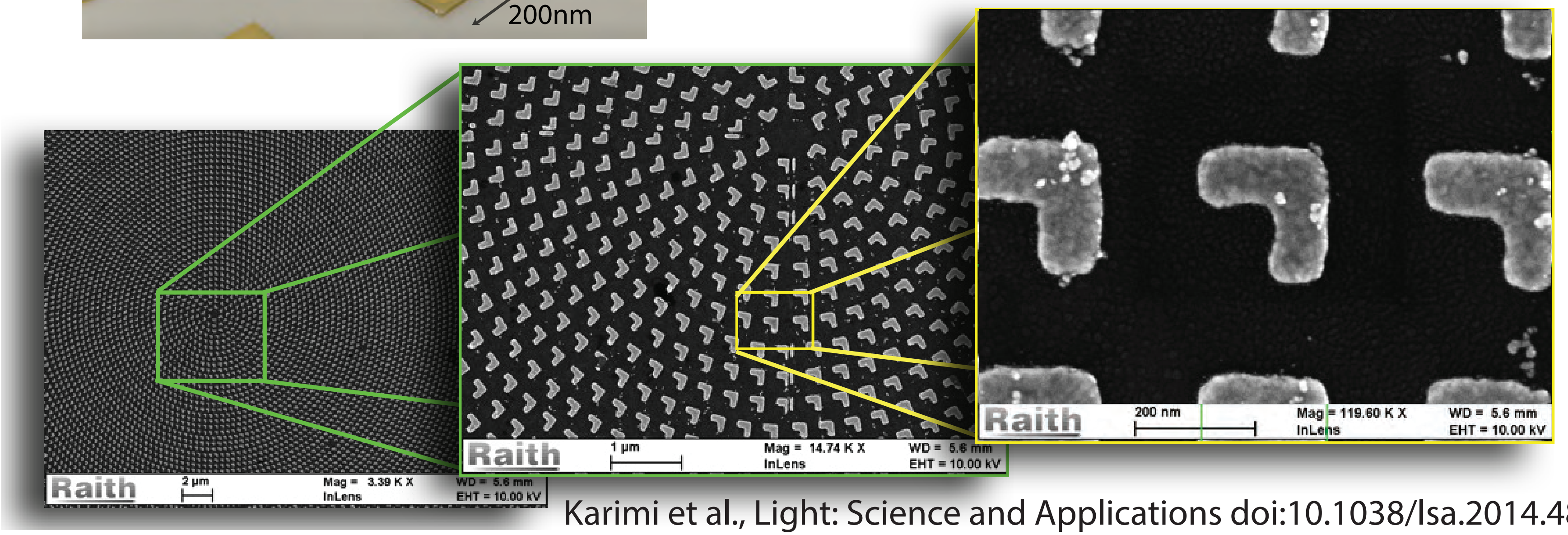
Fabrication of a Nano Plasmonic q-Plate

- A q-plate is a device that converts spin angular momentum into orbital angular momentum.
- It functions as a quantum interface.
- Fabricated device is only 30-nm thick and thus suitable for use in integrated quantum circuits.









Karimi et al., Light: Science and Applications doi:10.1038/lsa.2014.48 (2014)

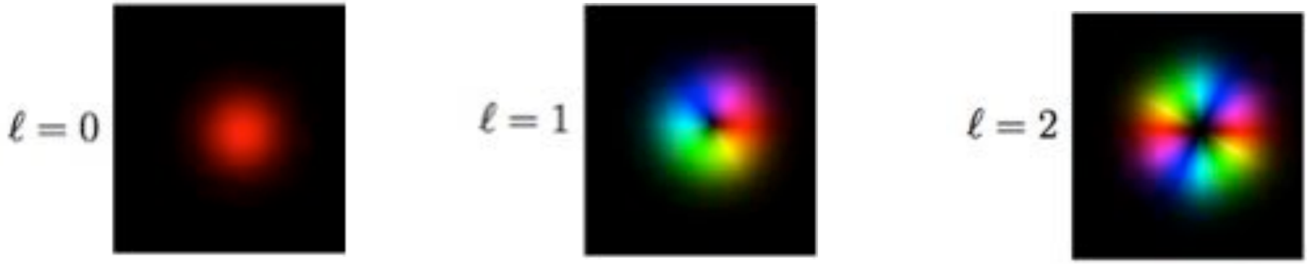
Background

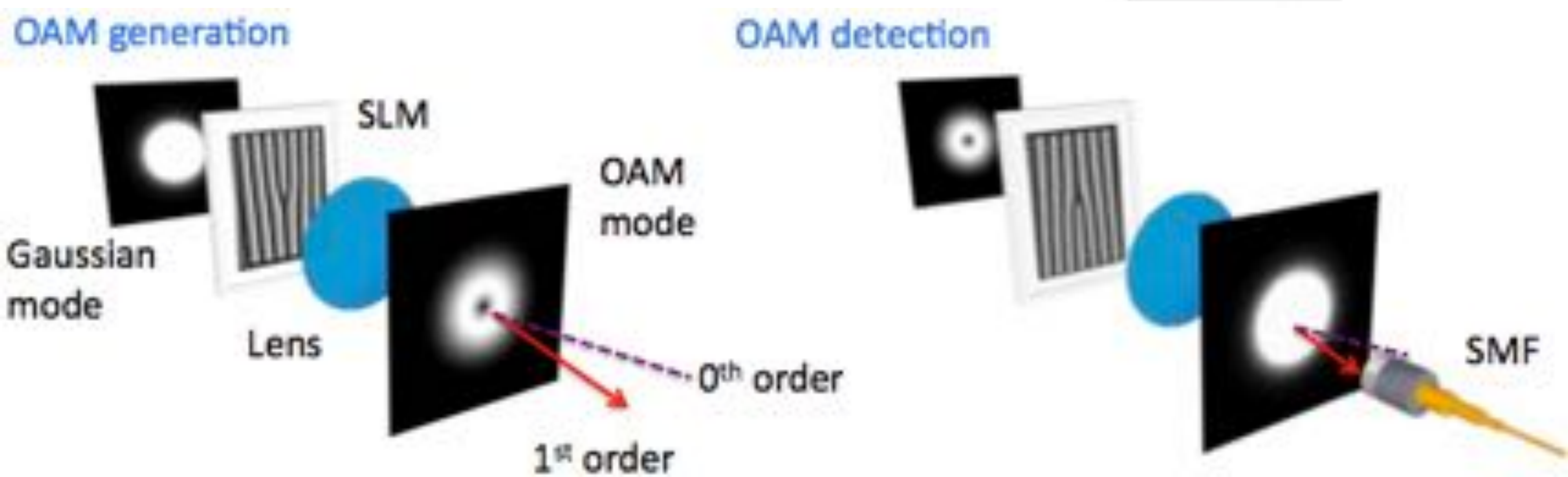
PHYSICAL REVIEW A VOLUME 45, NUMBER 11 1 JUNE 1992

Orbital angular momentum of light and the transformation of Laguerre-Gaussian laser modes

L. Allen, M. W. Beijersbergen, R. J. C. Spreeuw, and J. P. Woerdman
Huygens Laboratory, Leiden University, P.O. Box 9504, 2300 RA Leiden, The Netherlands
(Received 6 January 1992)

A beam of light with helicity of the phase front given by the azimuthal phase dependence of $e^{il\phi}$ carries orbital angular momentum (OAM).





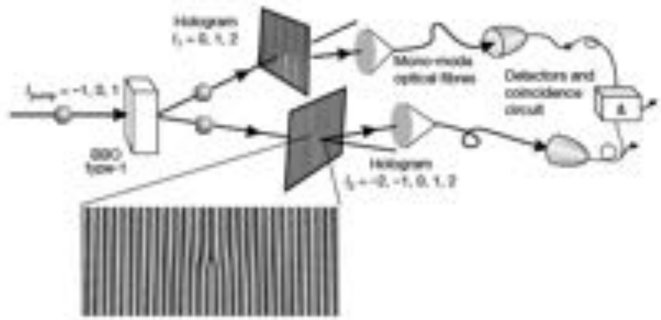
Background

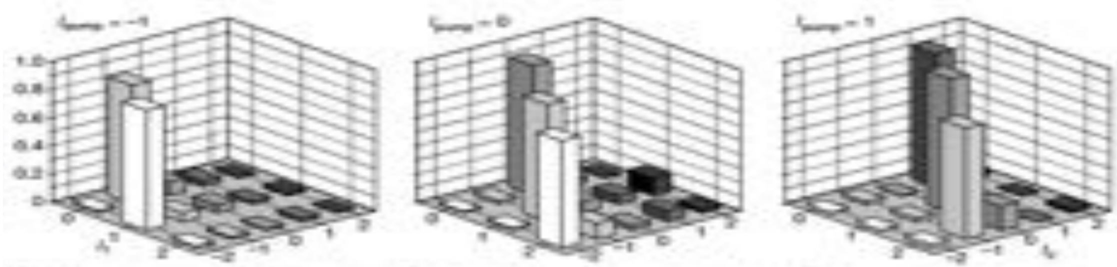
Entanglement of the orbital angular momentum states of photons

Alois Mair*, Alipasha Vaziri, Gregor Weihs & Anton Zeilinger

Institut für Experimentalphysik, Universität Wien, Boltzmanngasse 5, 1090 Wien, Austria

NATURE | VOL 412 | 19 JULY 2001 | www.nature.com

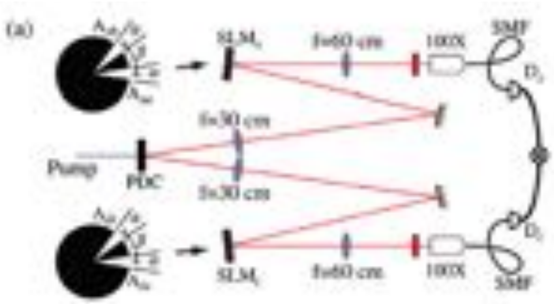


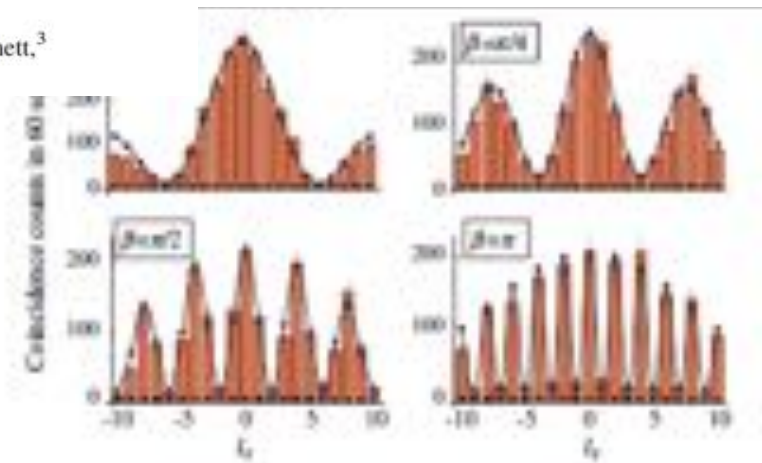


PRL **104,** 010501 (2010) PHYSICAL REVIEW LETTERS week ending 8 JANUARY 2010

Angular Two-Photon Interference and Angular Two-Qubit States

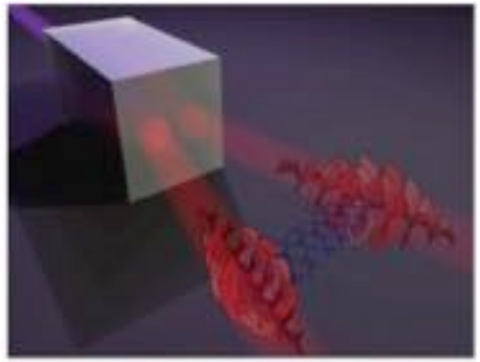
Anand Kumar Jha,[1] Jonathan Leach,[2] Barry Jack,[2] Sonja Franke-Arnold,[2] Stephen M. Barnett,[3] Robert W. Boyd,[1] and Miles J. Padgett[2]

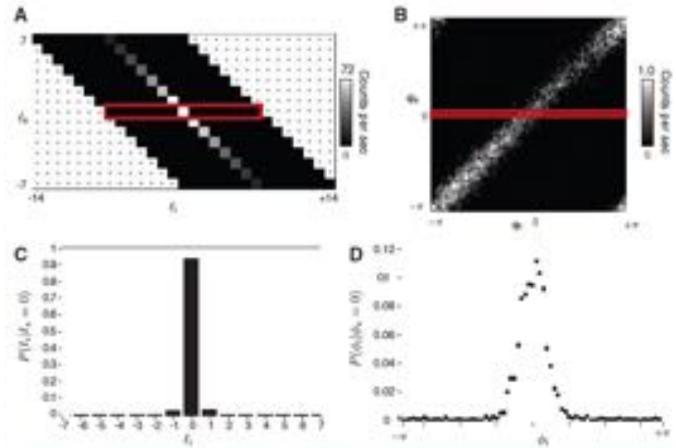




Background

Quantum Correlations in Optical Angle–Orbital Angular Momentum Variables

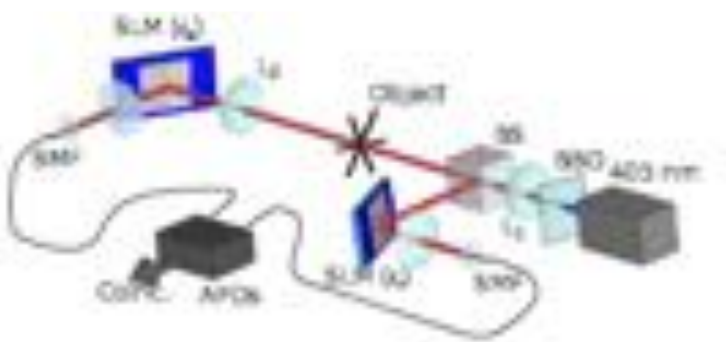




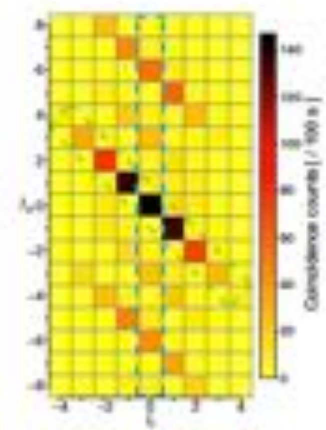
PRL 110, 043601 (2013) PHYSICAL REVIEW LETTERS week ending 25 JANUARY 2013

Object Identification Using Correlated Orbital Angular Momentum States

Néstor Uribe-Patarroyo,[1,*] Andrew Fraine,[1] David S. Simon,[1,2] Olga Minaeva,[3] and Alexander V. Sergienko[1,4]



(a)



(b)

Quantum Hilbert Hotel

Václav Potoček,[1,2] Filippo M. Miatto,[3,4,*] Mohammad Mirhosseini,[5,†] Omar S. Magaña-Loaiza,[5]
Andreas C. Liapis,[5] Daniel K. L. Oi,[6] Robert W. Boyd,[4,5,1] and John Jeffers[6]
[1]*SUPA School of Physics and Astronomy, University of Glasgow, Glasgow G12 8QQ, United Kingdom*
[2]*Department of Physics, Czech Technical University in Prague, Faculty of Nuclear Sciences and Physical Engineering, Břehová 7, 115 19 Praha 1, Czech Republic*
[3]*Institute for Quantum Computing and Department of Physics and Astronomy, University of Waterloo, 200 University Avenue West, Waterloo, Ontario N2L 3G1, Canada*
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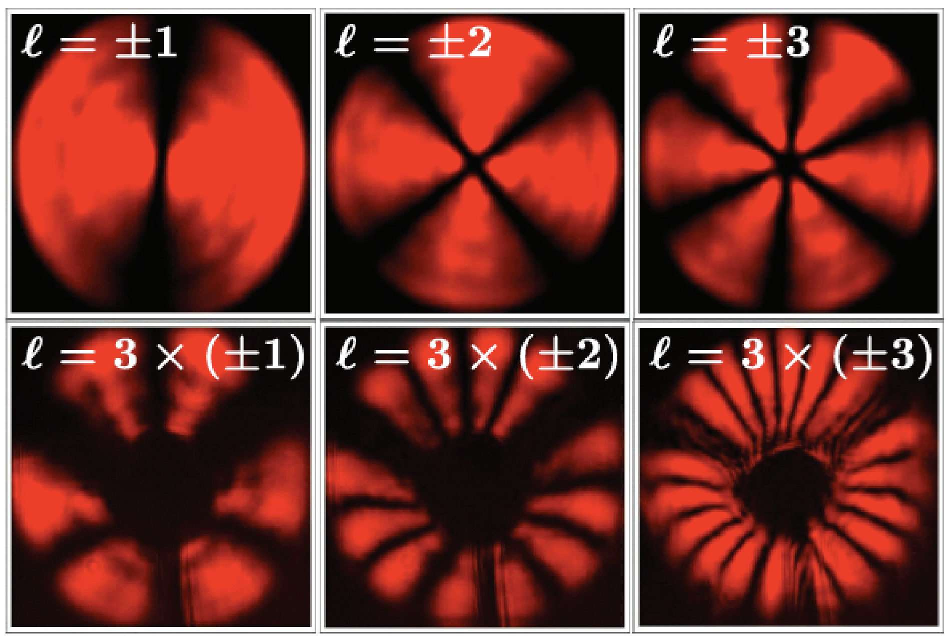
[6]*SUPA Department of Physics, University of Strathclyde, Glasgow G4 0NG, United Kingdom*
(Received 15 June 2015; published 15 October 2015)

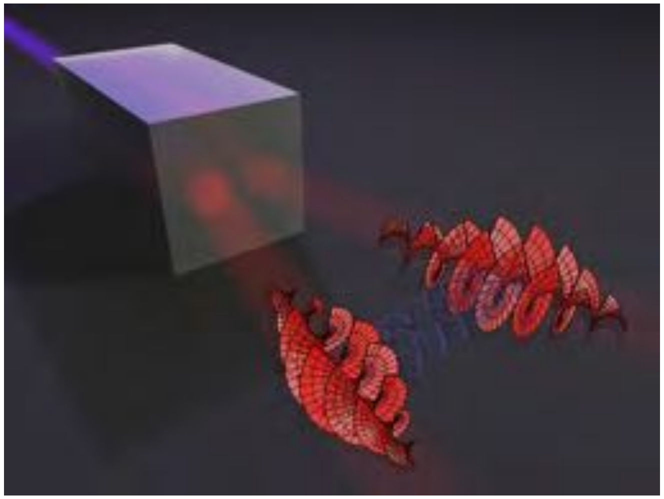


Use of Quantum States for Secure Optical Communication

- The celebrated BB84 protocol for quantum key distribution (QKD) transmits one bit of information per received photon

- We have built a QKD system that can carry more than one bit per photon.
  – Note that in traditional telecom, one uses many photons per bit!

- Our procedure is to encode using beams that carry orbital angular momentum (OAM), such as the Laguerre-Gauss states, which reside in an infinite dimensional Hilbert space.



QKD System Carrying Many Bits Per Photon

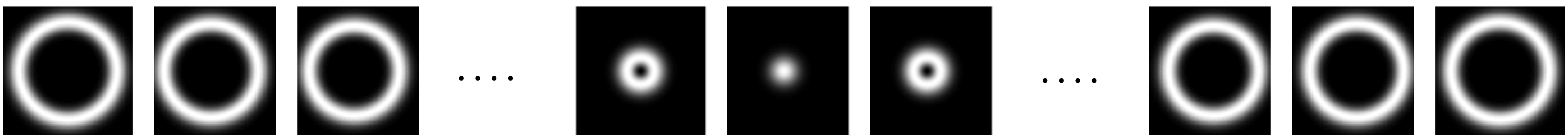
We are constructing a QKD system in which each photon carries many bits of information

We encode in states that carry OAM such as the Laguerre-Gauss states

As a diagnostic, we need to be able to measure the statevector of OAM states

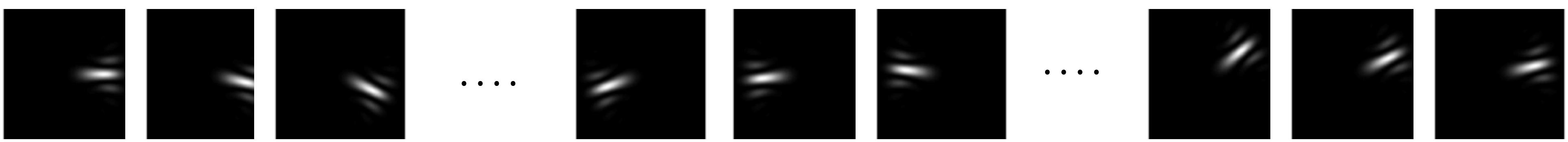
<u>Single Photon States</u>

Laguerre-Gaussian Basis $\ell = -13, \ldots, 13$



-13 -12 -11 -1 0 1 11 12 13

"Angular" Basis (mutually unbiased with respect to LG)



0 1 2 12 13 14 25 26 27

$$\Psi_{\mathrm{AB}}^{N} = \frac{1}{\sqrt{27}} \sum_{l=-13}^{13} \mathrm{LG}_{l,0} \exp\left(i 2\pi N l / 27\right)$$

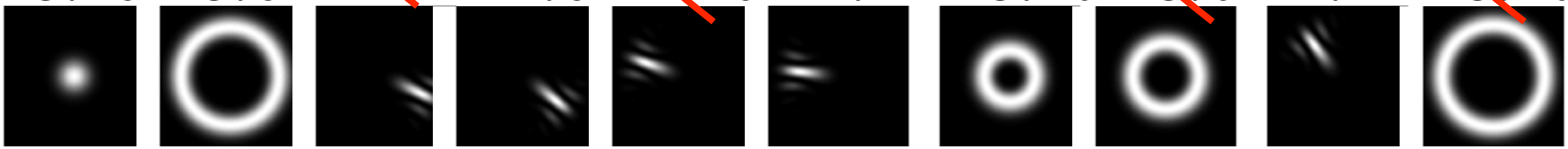
Alice (Alison) and Bob

(St)eavesdropper!

Is there an eavesdropper?

Protocol

Alice LG:13 LG:3 ~~AB:2~~ AB:3 ~~AB:15~~ AB:14 LG:16 ~~LG:8~~ AB:24 ~~LG:26~~



Bob LG LG ~~LG~~ AB ~~LG~~ AB LG ~~AB~~ AB ~~AB~~

Result 13 3 ~~15~~ 3 ~~15~~ 14 16 ~~17~~ 24 ~~10~~

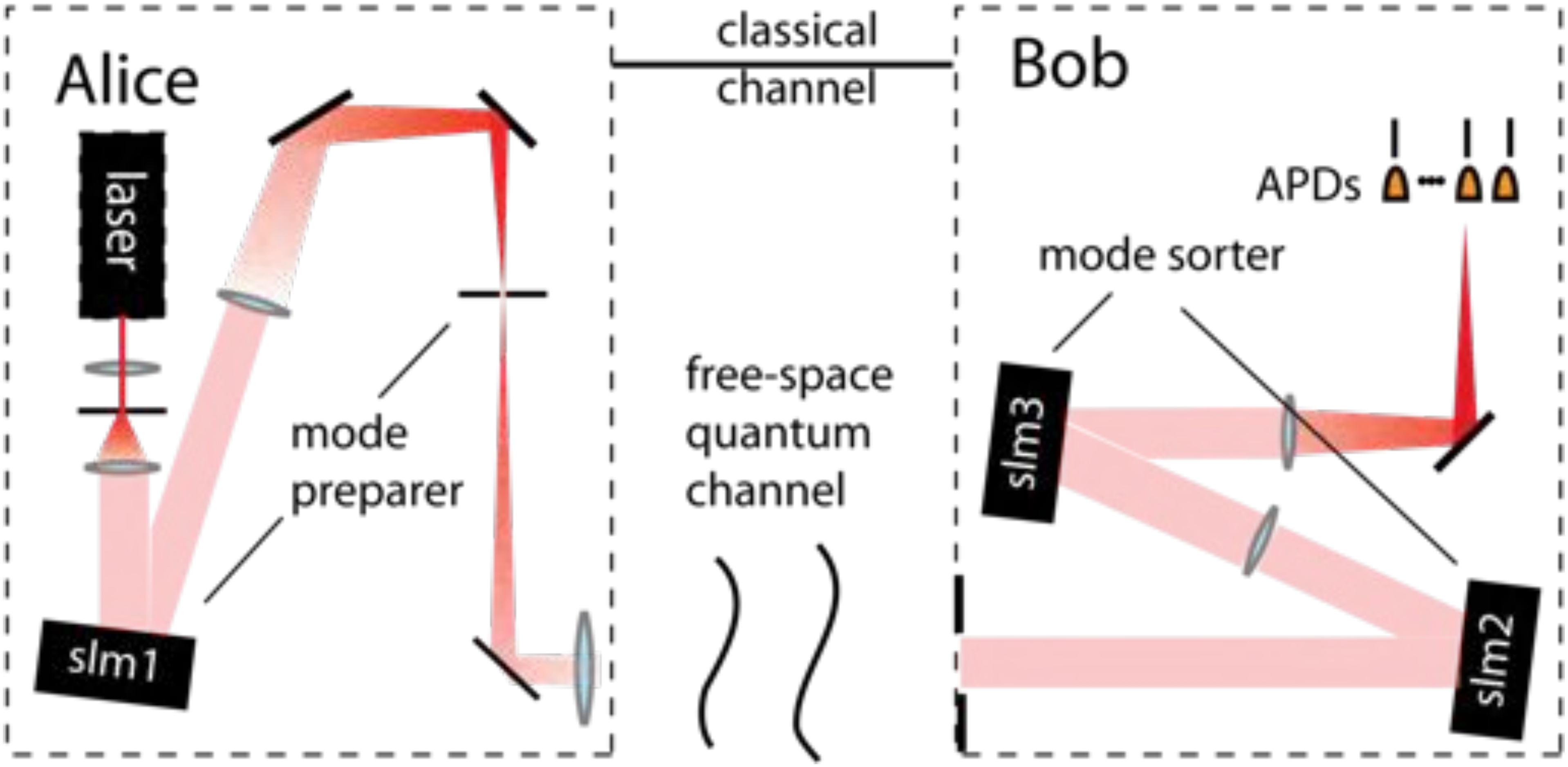
Sifted Key 13 3 3 14 16 24 … ← in principle contains no errors unless eavesdropper is present.

In any real system, Bob’s key will have errors due to system imperfections.

1. Error Correction (Cascade Protocol)
2. Privacy Amplification

Under many conditions, these protocols can be successfully implemented if Alice/Bob share more bits of information than Alice and Eve.

Spatially Based QKD System



Source

Weak Coherent Light
(Using decoy states)

Protocol

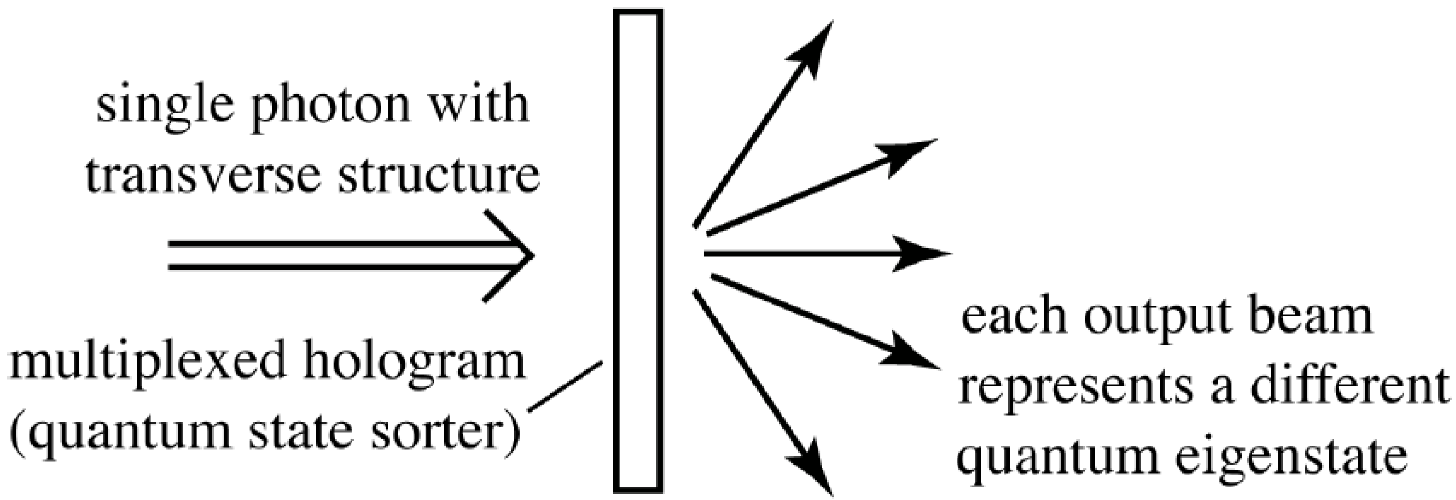
Modified BB84 as
discussed

Challenges

1. State Preparation
2. State Detection
3. Turbulence

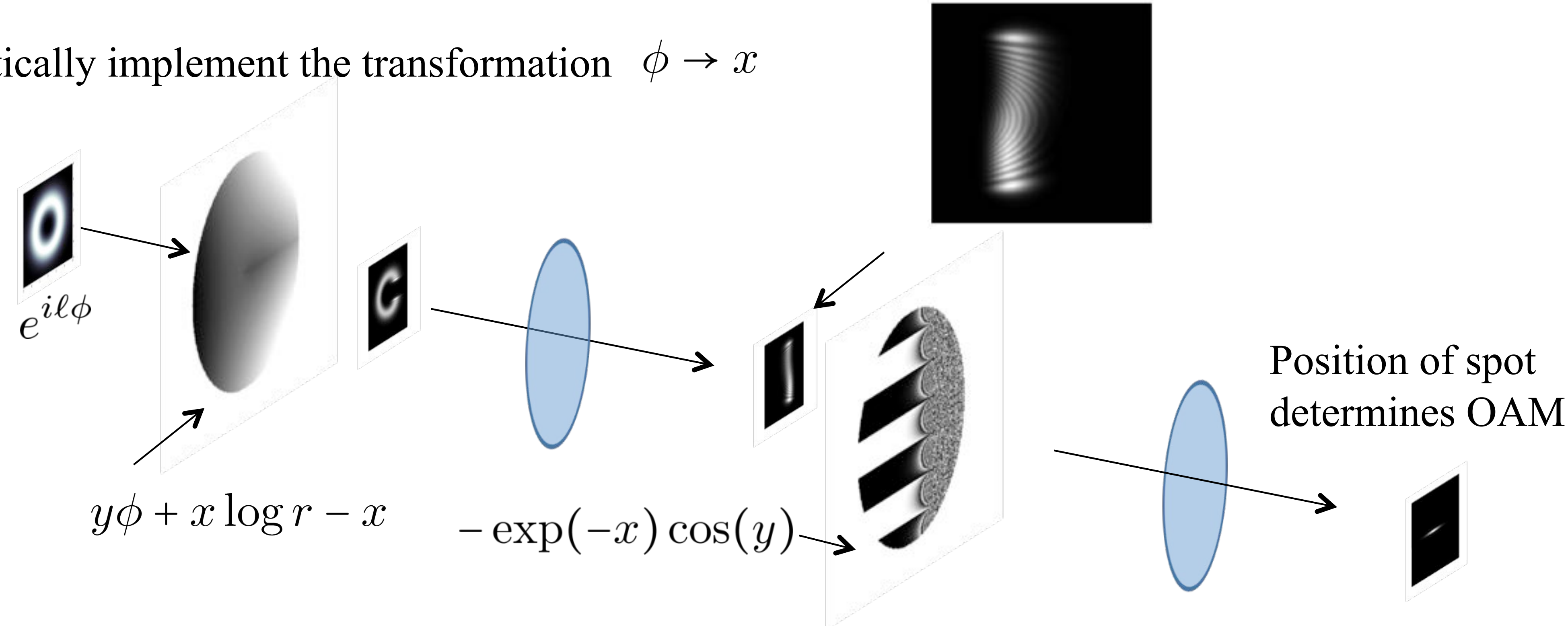
Mode Sorting

A mode sorter

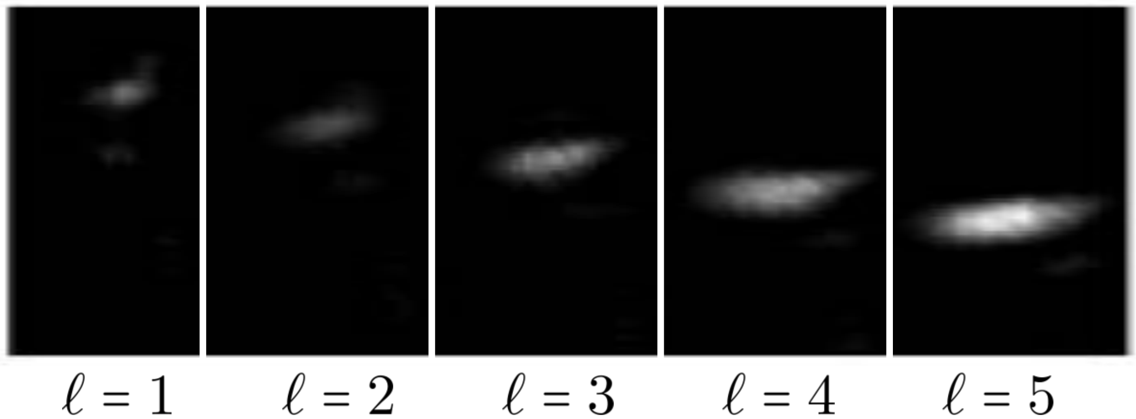


Sorting OAM using Phase Unwrapping

Optically implement the transformation $\phi \rightarrow x$



Experimental Results (CCD images in output plane)



-Can also sort angular position states.

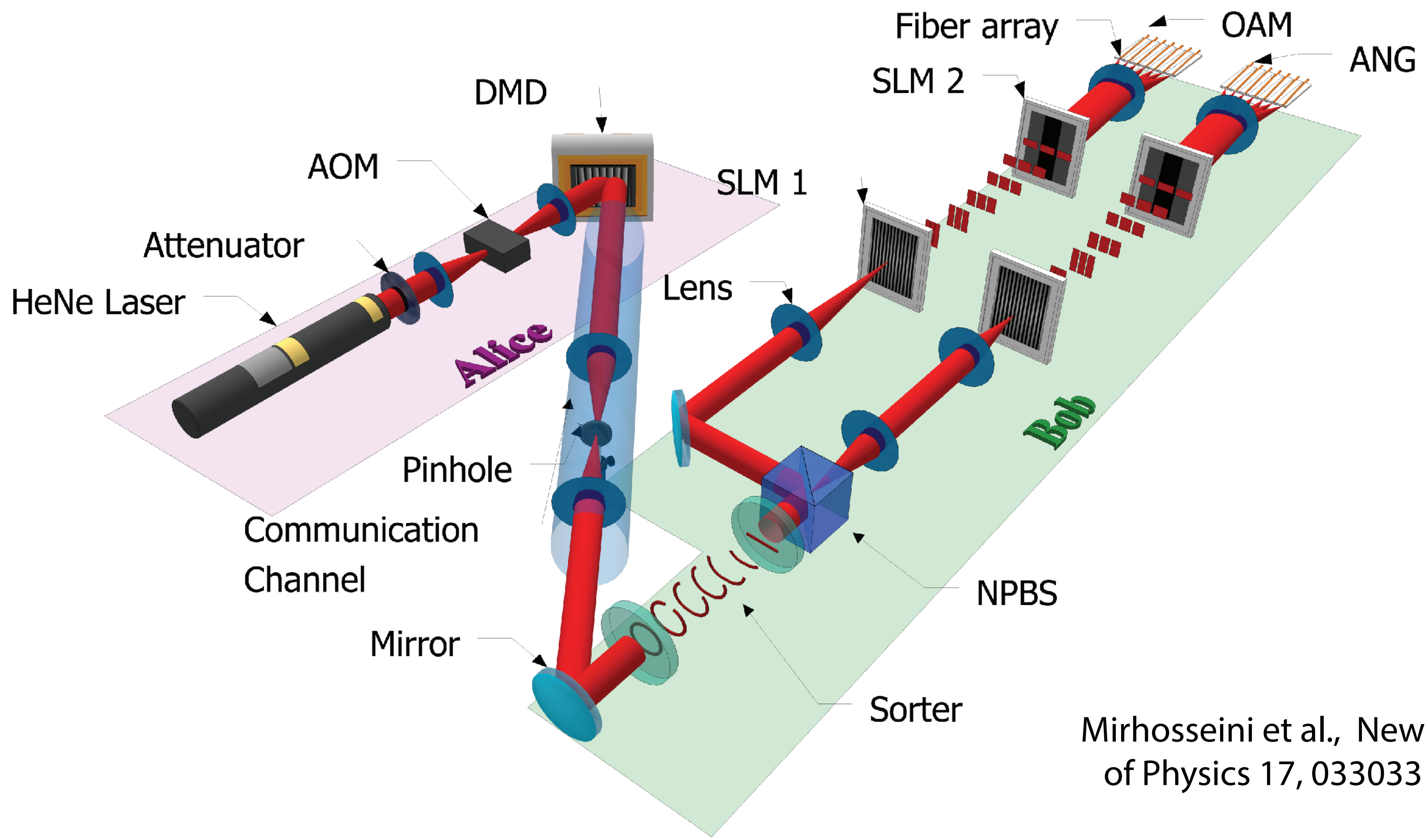
-Limited by the overlap of neighboring states.

UNIVERSITY of ROCHESTER

*Berkhout *et al.* *PRL* **105,** 153601 (2010).
O. Bryngdahl, *J. Opt. Soc. Am.* **64**, 1092 (1974).

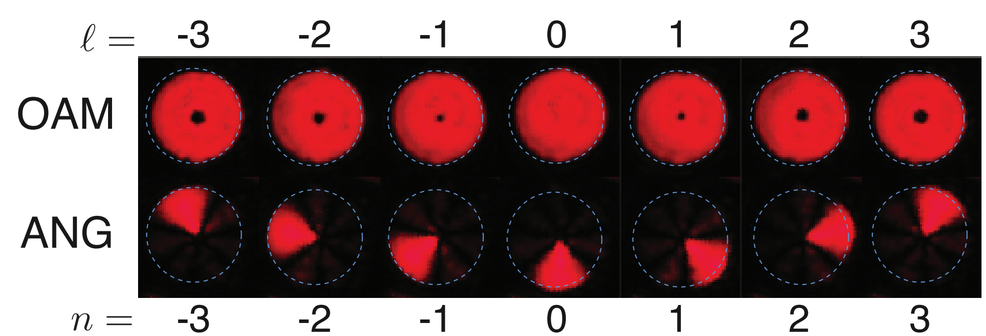
Nonlinear Optics Group

Our Laboratory Setup



Mirhosseini et al., New Journal of Physics 17, 033033 (2015).

We use a seven-dimensional state space.



Laboratory Results - OAM-Based QKD

Alice

063133602132045444456141026645
545050363603025261643215524164
230146602513401613222451551026

Bob

063<u>0</u>3360<u>1</u>1320<u>21</u>444456141026645
5450<u>22</u>3<u>5</u>3603025261643215524164
2301466025<u>1</u><u>5</u>40<u>3</u>613<u>4</u>22451551026

Error Correction

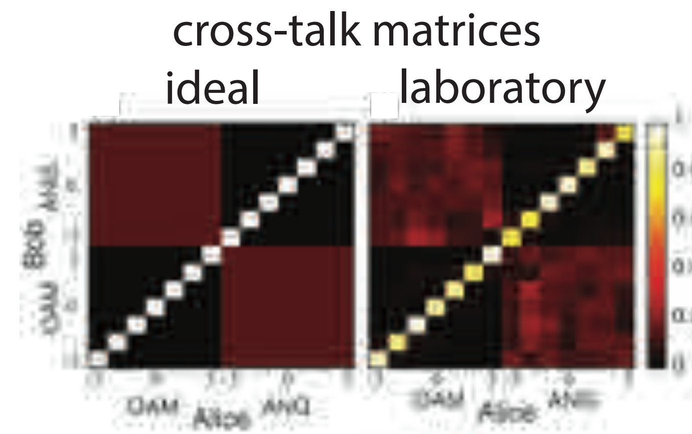
0000 0100 0111 0100 0101 0001 1100 0010 1100 1001 0010 0100 1101 0001 0000
0001 0011 0100 0000 0100 0010 0001 0001 0011 0000 0101 0010 0111 1000 0101
1101 0011 0001 1001 1011 0101 0000 0100 0111 0100 0011 0110 0101 0000 1101
0101 0010 1111 1010 1111 0010 1000

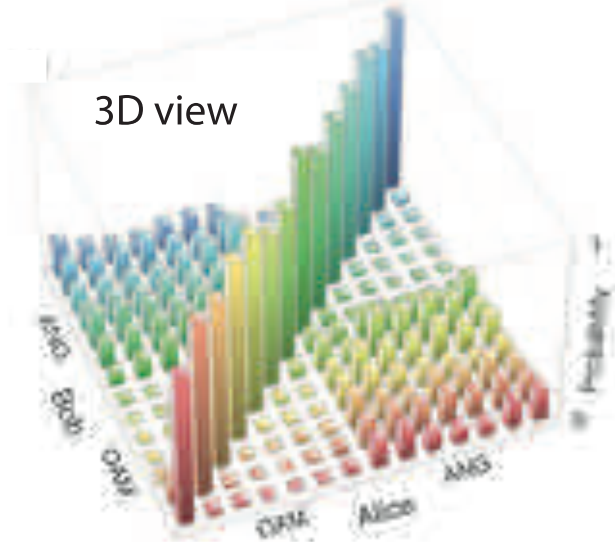
Privacy Amplification

0001 0100 1011 1111 1111 1010 1010 0111 1010 1011 1111 0001 0000 1101 1111
1110 1110 1101 1110 0001 0011 1000 0101 1101 1100 0011 0111 1101 0001 1001
0100 0110 1011 0000 0110 1010 1

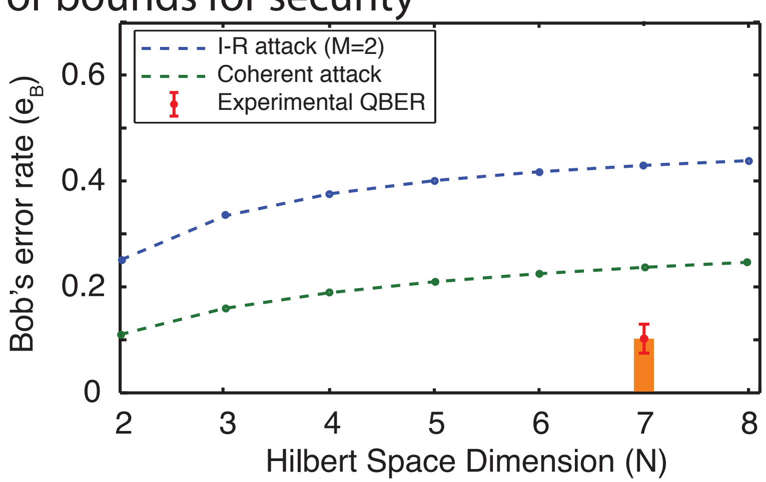
Alice Encrypts

Bob Decrypts





- error bounds for security



We use a 7-letter alphabet, and achieve a channel capacity of 2.1 bits per sifted photon.

We do not reach the full 2.8 bits per photon for a variety of reasons, including dark counts in our detectors and cross-talk among channels resulting from imperfections in our sorter.

Nonetheless, our error rate is adequately low to provide full security,

ARTICLES
PUBLISHED ONLINE: 24 JUNE 2012 | DOI: 10.1038/NPHOTON.2012.138

nature photonics

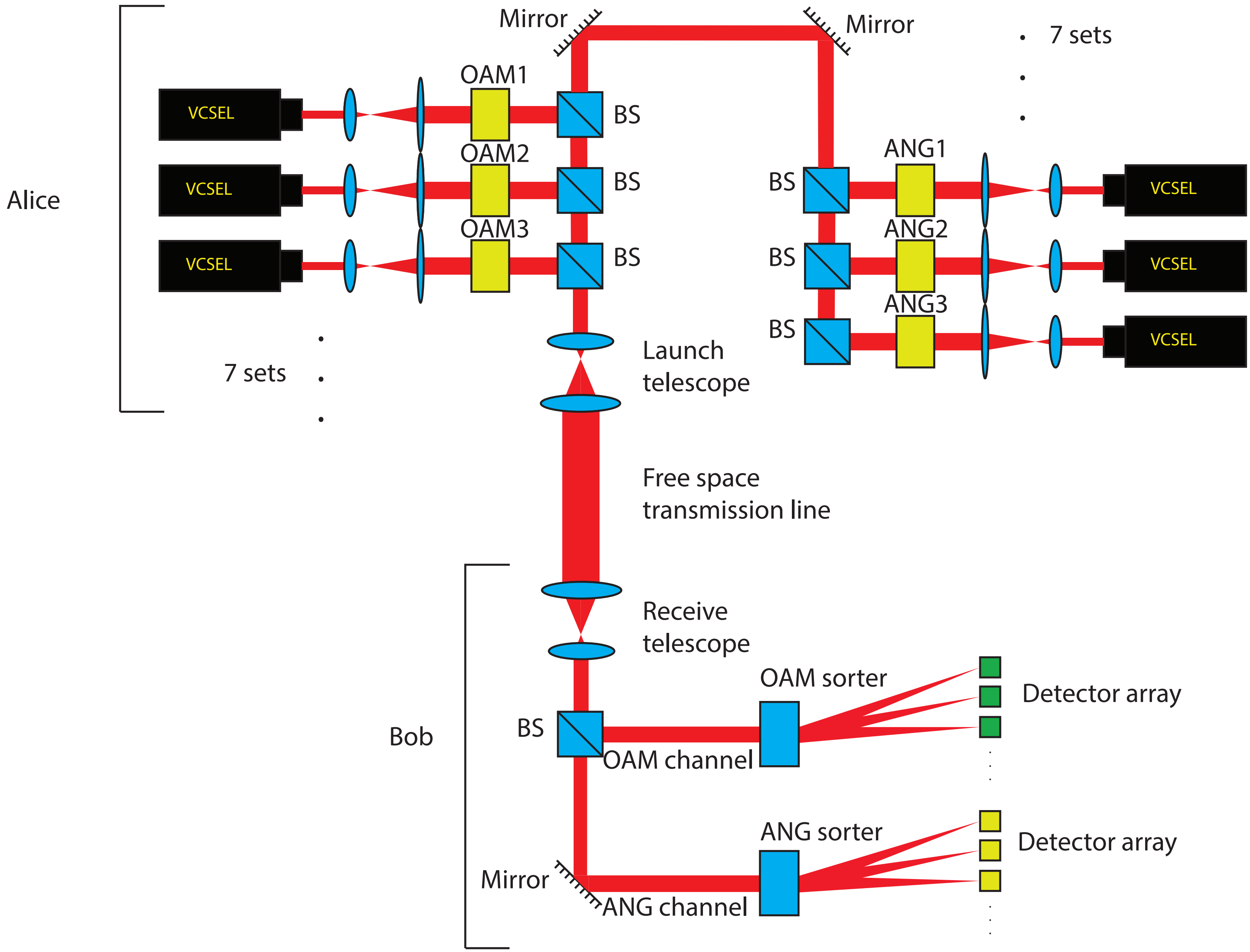
Terabit free-space data transmission employing orbital angular momentum multiplexing

Jian Wang[1,2]*, Jeng-Yuan Yang[1], Irfan M. Fazal[1], Nisar Ahmed[1], Yan Yan[1], Hao Huang[1], Yongxiong Ren[1], Yang Yue[1], Samuel Dolinar[3], Moshe Tur[4] and Alan E. Willner[1]*

The recognition in the 1990s that light beams with a helical phase front have orbital angular momentum has benefited applications ranging from optical manipulation to quantum information processing. Recently, attention has been directed towards the opportunities for harnessing such beams in communications. Here, we demonstrate that four light beams with different values of orbital angular momentum and encoded with 42.8×4 Gbit s^{-1} quadrature amplitude modulation (16-QAM) signals can be multiplexed and demultiplexed, allowing a 1.37 Tbit s^{-1} aggregated rate and 25.6 bit s^{-1} Hz^{-1} spectral efficiency when combined with polarization multiplexing. Moreover, we show scalability in the spatial domain using two groups of concentric rings of eight polarization-multiplexed 20×4 Gbit s^{-1} 16-QAM-carrying orbital angular momentum beams, achieving a capacity of 2.56 Tbit s^{-1} and spectral efficiency of 95.7 bit s^{-1} Hz^{-1}. We also report data exchange between orbital angular momentum beams encoded with 100 Gbit s^{-1} differential quadrature phase-shift keying signals. These demonstrations suggest that orbital angular momentum could be a useful degree of freedom for increasing the capacity of free-space communications.

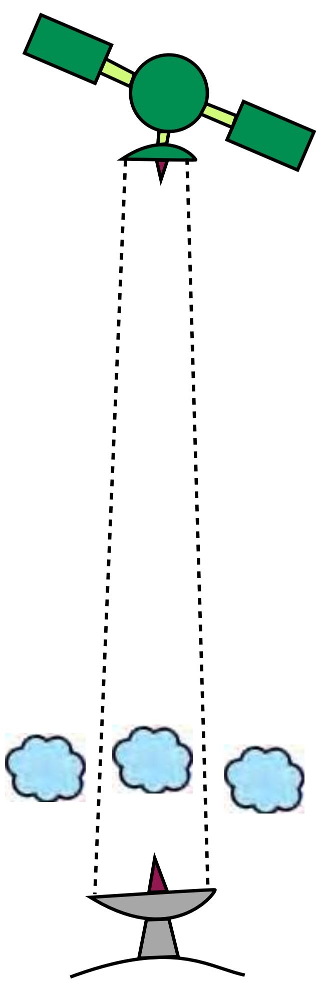
Next Step: gigabit-per-second OAM-based QKD system

- Use direct modulation of laser diode to encode at gigabits per sec.

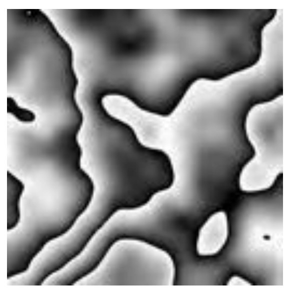


Turbulence and Adaptive Optics

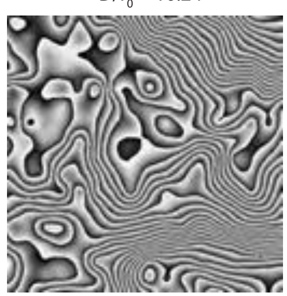
Atmospheric Turbulence Model



$D/r_0 = 5.12$

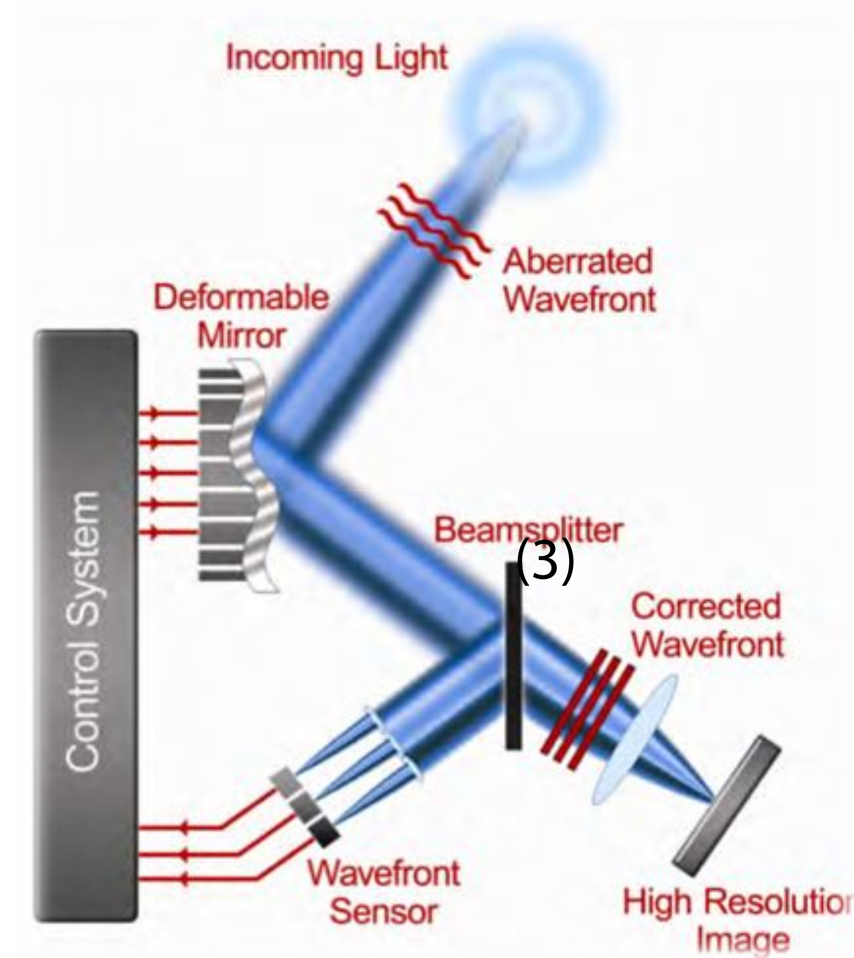


$D/r_0 = 10.24$



$D/r_0 = 102.4$

Our Adaptive Optics System



bp

We're bringing oil to American shores.