Quantum Properties of Twisted Light

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Quantum Information and Orbital Angular Momentum (OAM) of Light

• Utilize the transverse degree of freedom of light
  - Spatial division multiplexing

• In particular, encode in angular position and its conjugate variable, orbital angular momentum (OAM)

• Motivation: Encode more information per photon
What Are the OAM States of Light?

• Light can carry spin angular momentum (SAM) by means of its circular polarization.

• Light can also carry orbital angular momentum (OAM) by means of the phase winding of the optical wavefront.

• A well-known example are the Laguerre-Gauss modes. These modes contain a phase factor of \( \exp(i l \phi) \) and carry angular momentum of \( l \hbar \) per photon. (Here \( \phi \) is the azimuthal coordinate.)
Laguerre-Gauss Modes

The paraxial approximation to the Helmholtz equation $(\nabla^2 + k^2)E(k) = 0$ gives the paraxial wave equation which is written in the cartesian coordinate system as

$$\left( \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + 2ik \frac{\partial}{\partial z} \right) E(x, y, z) = 0.$$  \hspace{1cm} (1)

The paraxial wave equation is satisfied by the Laguerre-Gaussian modes, a family of orthogonal modes that have a well defined orbital angular momentum. The field amplitude $LG^l_p(\rho, \phi, z)$ of a normalized Laguerre-Gaussian modes is given by

$$LG^l_p(\rho, \phi, z) = \sqrt{\frac{2p!}{\pi(|l| + p)!w(z)}} \left[ \frac{\sqrt{2\rho}}{w(z)} \right]^{(|l|)} L^l_p \left[ \frac{2\rho^2}{w^2(z)} \right]$$

$$\times \exp \left[ -\frac{\rho^2}{w^2(z)} \right] \exp \left[ -\frac{i k^2 \rho^2 z}{2(z^2 + z_R^2)} \right] \exp \left[ i(2p + |l| + 1)\tan^{-1} \left( \frac{z}{z_R} \right) \right] e^{-il\phi},  \hspace{1cm} (2)$$

where $k$ is the wave-vector magnitude of the field, $z_R$ the Rayleigh range, $w(z)$ the radius of the beam at $z$, $l$ is the azimuthal quantum number, and $p$ is the radial quantum number. $L^l_p$ is the associated Laguerre polynomial.
How to create a beam carrying orbital angular momentum?

- Pass beam through a spiral phase plate

- Use a spatial light modulator acting as a computer generated hologram (more versatile)

Spin angular momentum can be transferred to OAM through use of a Q-plate

Ability to change basis of encoding useful for quantum information processing

Q-plate. Usually a carefully constructed liquid-crystal cell

Marrucci et al., PRL 96, 163905 (2006)
Karimi et al., APL 98, 231124 (2009)
Fabrication of a Nano Plasmonic q-Plate

- A q-plate is a device that converts spin angular momentum into orbital angular momentum.
- It functions as a quantum interface.
- Fabricated device is only 30-nm thick and thus suitable for use in integrated quantum circuits.
A beam of light with helicity of the phase front given by the azimuthal phase dependence of $e^{il\phi}$ carries orbital angular momentum (OAM).
Background

Entanglement of the orbital angular momentum states of photons

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Angular Two-Photon Interference and Angular Two-Qubit States

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Background
Quantum Hilbert Hotel

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Use of Quantum States for Secure Optical Communication

• The celebrated BB84 protocol for quantum key distribution (QKD) transmits one bit of information per received photon.

• We have built a QKD system that can carry more than one bit per photon.
  – Note that in traditional telecom, one uses many photons per bit!

• Our procedure is to encode using beams that carry orbital angular momentum (OAM), such as the Laguerre-Gauss states, which reside in an infinite dimensional Hilbert space.
We are constructing a QKD system in which each photon carries many bits of information.

We encode in states that carry OAM such as the Laguerre-Gauss states.

As a diagnostic, we need to be able to measure the statevector of OAM states.

**Single Photon States**

\[ Laguerre-Gaussian \text{ Basis} \quad \ell = -13, \ldots, 13 \]

\[ \Psi_{AB}^N = \frac{1}{\sqrt{27}} \sum_{\ell=-13}^{13} \text{LG}_{\ell,0} \exp(i2\pi \ell l/27) \]
Is there an eavesdropper?

(Alice (Alison) and Bob)
In any real system, Bob’s key will have errors due to system imperfections.

1. Error Correction (Cascade Protocol)
2. Privacy Amplification

Under many conditions, these protocols can be successfully implemented if Alice/Bob share more bits of information than Alice and Eve.
Spatially Based QKD System

Source
Weak Coherent Light (Using decoy states)

Protocol
Modified BB84 as discussed

Challenges
1. State Preparation
2. State Detection
3. Turbulence
Mode Sorting

A mode sorter

single photon with transverse structure

multiplexed hologram (quantum state sorter)

each output beam represents a different quantum eigenstate
Optically implement the transformation \( \phi \rightarrow x \)

\[
e^{i\ell \phi} e^{i\ell \phi} \rightarrow \frac{y\phi + x \log r - x}{-\exp(-x) \cos(y)}
\]

Experimental Results (CCD images in output plane)

- Can also sort angular position states.
- Limited by the overlap of neighboring states.

Our Laboratory Setup

We use a seven-dimensional state space.


We use a seven-dimensional state space.
Laboratory Results - OAM-Based QKD

We use a 7-letter alphabet, and achieve a channel capacity of 2.1 bits per sifted photon.

We do not reach the full 2.8 bits per photon for a variety of reasons, including dark counts in our detectors and cross-talk among channels resulting from imperfections in our sorter.

Nonetheless, our error rate is adequately low to provide full security,
Terabit free-space data transmission employing orbital angular momentum multiplexing

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The recognition in the 1990s that light beams with a helical phase front have orbital angular momentum has benefited applications ranging from optical manipulation to quantum information processing. Recently, attention has been directed towards the opportunities for harnessing such beams in communications. Here, we demonstrate that four light beams with different values of orbital angular momentum and encoded with $42.8 \times 4$ Gbit s$^{-1}$ quadrature amplitude modulation (16-QAM) signals can be multiplexed and demultiplexed, allowing a 1.37 Tbit s$^{-1}$ aggregated rate and 25.6 bit s$^{-1}$ Hz$^{-1}$ spectral efficiency when combined with polarization multiplexing. Moreover, we show scalability in the spatial domain using two groups of concentric rings of eight polarization-multiplexed $20 \times 4$ Gbit s$^{-1}$ 16-QAM-carrying orbital angular momentum beams, achieving a capacity of 2.56 Tbit s$^{-1}$ and spectral efficiency of 95.7 bit s$^{-1}$ Hz$^{-1}$. We also report data exchange between orbital angular momentum beams encoded with 100 Gbit s$^{-1}$ differential quadrature phase-shift keying signals. These demonstrations suggest that orbital angular momentum could be a useful degree of freedom for increasing the capacity of free-space communications.

Angular momentum, sometimes described as the rotational analogue of linear momentum, is one of the most fundamental physical quantities in both classical and quantum mechanics1. Angular momentum can be divided into spin angular momentum (SAM) and orbital angular momentum (OAM) in paraxial beams2-3. SAM is associated with photon spin and manifested as circular polarization, as anticipated by Poynting in 19094 and demonstrated by Beth in 19365. In contrast, OAM is linked to the spatial distribution6. It was shown by Allen in 19927 that helically phased beams comprising an azimuthal phase term $\exp(i \ell \phi)$, have an OAM of $\pm \ell h$ per photon (where $\ell$ is topological charge, $\phi$ is azimuthal angle, and $h$ is Plank’s constant $h$ divided by 2$\pi$). OAM is a natural property of various types of helically phased beams, ranging from electron beams to radio waves8-15. It has given rise to many developments in optical manipulation, optical trapping, optical tweezers, optical vortex knots, imaging, astronomy and quantum information processing16-27.

In addition to these established areas, OAM has recently seen applications in free-space information transfer and communications28. In contrast to SAM, which has only two possible values of $\pm \ell h$, the theoretically unlimited values of $\ell$, in principle, provide an infinite range of possibly achievable OAM states. OAM therefore has the potential to tremendously increase the capacity of communication systems, either by encoding information as OAM states of the beam or by using OAM beams as information carriers for multiplexing28-36. In this Article, we consider the latter option of using OAM beams for multiplexing, which can be regarded as the analogue of various other multiplexing technologies in optical fibre communications, such as wavelength-division multiplexing (WDM)37-39, optical time-division multiplexing (OTDM)40, polarization-division multiplexing (PDM)37-41, spatial-division multiplexing (SDM)41 and mode-division multiplexing (MDM)42.

Note that recent advances in optical communication systems in relation to multilevel amplitude/phase modulation formats, coherent detection and electronic digital signal processing have facilitated dramatic increases in capacity and spectral efficiency37-45. Hence, a valuable goal would be to use OAM beams to carry information with multilevel amplitude/phase modulation formats, resulting in yet another increase of capacity and spectral efficiency, gained by the multiplexing of OAM beams. Moreover, when using OAM beams to carry different data information, a potentially desirable operation for flexible data processing would be data exchange between OAM beams.

Here, we demonstrate multiplexing/demultiplexing of four polarization-multiplexed (pol-muxed) OAM beams, each carrying a $42.8 \times 4$ Gbit s$^{-1}$ (4 bits per symbol) quadrature amplitude modulation (16-QAM) signal, thereby achieving a capacity of 1,369.6 Gbit s$^{-1}$ (4 bits per symbol for the 16-QAM, with 4 OAM beams and 2 polarization states) with a spectral efficiency of 25.6 bit s$^{-1}$ Hz$^{-1}$ (50 GHz grid). Moreover, we show scalability in the spatial domain using two groups of concentric rings of eight pol-muxed OAM beams, each carrying a 20 $\times$ 4 Gbit s$^{-1}$ 16-QAM signal, for which a capacity of 2,560 (20 $\times$ 4 $\times$ 8 $\times$ 2 $\times$ 2) Gbit s$^{-1}$ (4 bits per symbol for the 16-QAM, with 8 OAM beams, 2 polarization states and 2 groups of concentric rings) is achieved together with a spectral efficiency of 95.7 bit s$^{-1}$ Hz$^{-1}$ (25 GHz grid). Finally, we demonstrate data exchange between two OAM beams, each carrying a 100 Gbit s$^{-1}$ differential quadrature phase-shift keying (DQPSK) signal.

Multiplexing of information-carrying OAM beams

Figure 1a,b presents a schematic representation of the generation and back-conversion of an information-carrying OAM beam, where a light beam with OAM serves as a carrier of information, which...
Next Step: gigabit-per-second OAM-based QKD system

- Use direct modulation of laser diode to encode at gigabits per sec.
Turbulence and Adaptive Optics

Atmospheric Turbulence Model

Our Adaptive Optics System

\[ \frac{D}{r_0} = 5.12 \]

\[ \frac{D}{r_0} = 10.24 \]

\[ \frac{D}{r_0} = 102.4 \]