Weak Values and Measurement of the Photon's Wavefunction

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Please feel free to photograph my visuals if you want to.

Princeton-TAMU Symposium on Quantum Physics and Engineering, Princeton University, June 15-17, 2017.

Weak Values and Measurement of the Photon's Wavefunction

1. Introduction to weak values

2. Weak value amplification

3. Direct measurement of the wavefunction using weak values
   - (a) Lundeen's method
   - (b) Measurement of a 27-dimensional quantum state
   - (c) Direct Measurement of a million-dimensional quantum state

4. Direct Measurement of the Wigner function

VOLUME 60, NUMBER 14 PHYSICAL REVIEW LETTERS 4 APRIL 1988

How the Result of a Measurement of a Component of the Spin of a Spin-$\frac{1}{2}$ Particle Can Turn Out to be 100

Yakir Aharonov, David Z. Albert, and Lev Vaidman
Physics Department, University of South Carolina, Columbia, South Carolina 29208, and School of Physics and Astronomy, Tel-Aviv University, Ramat Aviv 69978, Israel
(Received 30 June 1987)

We have found that the usual measuring procedure for preselected and postselected ensembles of quantum systems gives unusual results. Under some natural conditions of weakness of the measurement, its result consistently defines a new kind of value for a quantum variable, which we call the weak value. A description of the measurement of the weak value of a component of a spin for an ensemble of preselected and postselected spin-$\frac{1}{2}$ particles is presented.

PACS numbers: 03.65.Bz

standard expectation value: $< A >=< \Psi|\hat{A}|\Psi >$

weak value: $A_w \equiv \langle \psi_f \mid A \mid \psi_{\text{in}} \rangle / \langle \psi_f \mid \psi_{\text{in}} \rangle.$

Why are weak values important?

can lead to amplification of small signals

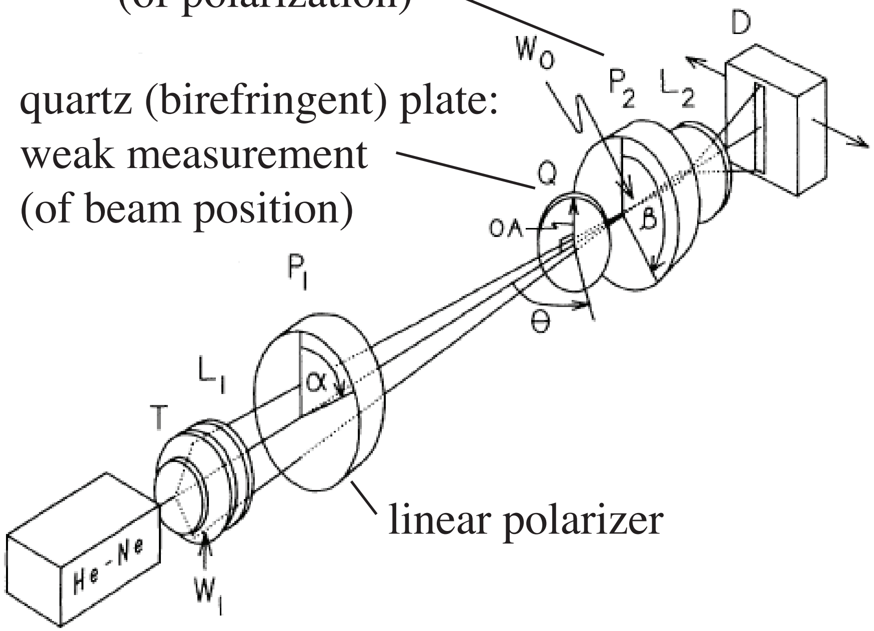
can lead to direct measurement of the quantum wavefunction

Realization of a Measurement of a "Weak Value"

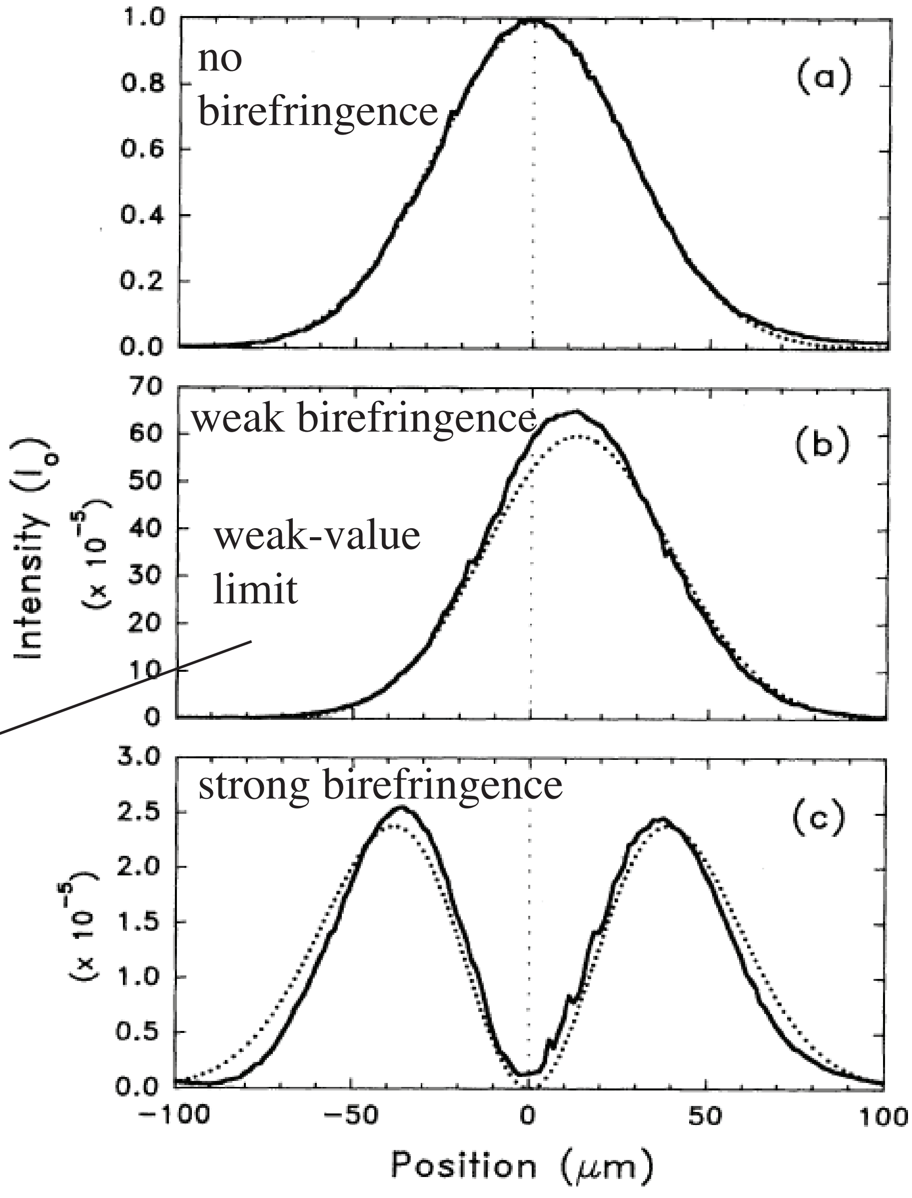
N. W. M. Ritchie, J. G. Story, and Randall G. Hulet
Department of Physics and Rice Quantum Institute, Rice University, Houston, Texas 77251-1892
(Received 7 December 1990)

linear polarizer:
strong measurement
(of polarization)

quartz (birefringent) plate:
weak measurement
(of beam position)



linear polarizer



Birefringence separates polarized beams by 0.64 μm, but gaussian in (b) is displaced by 12 μm.

PRL 1991

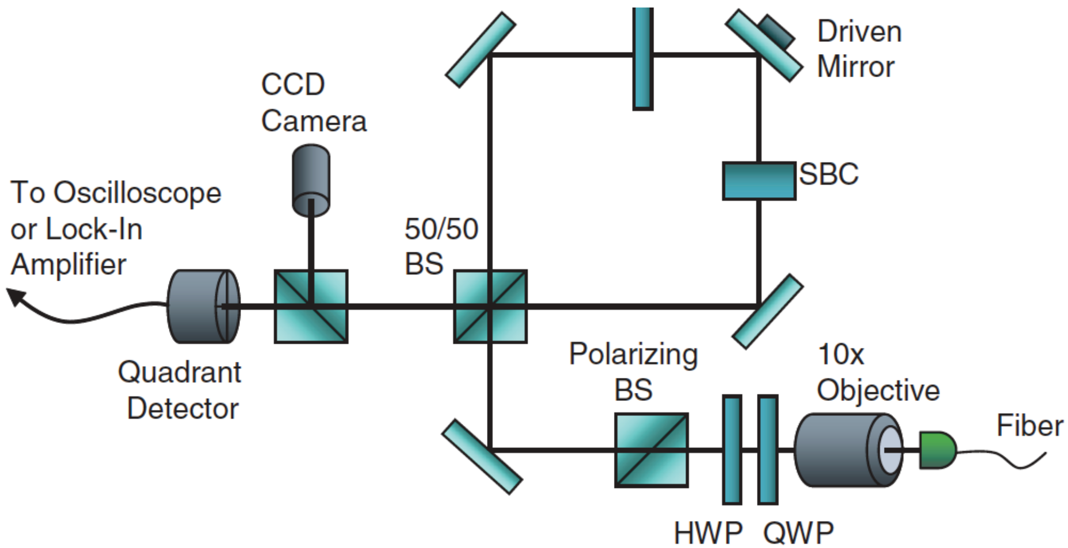
Selected for a Viewpoint in *Physics*
PRL **102**, 173601 (2009) PHYSICAL REVIEW LETTERS week ending 1 MAY 2009

Ultrasensitive Beam Deflection Measurement via Interferometric Weak Value Amplification

P. Ben Dixon, David J. Starling, Andrew N. Jordan, and John C. Howell
Department of Physics and Astronomy, University of Rochester, Rochester, New York 14627, USA
(Received 12 January 2009; published 27 April 2009)

We report on the use of an interferometric weak value technique to amplify very small transverse deflections of an optical beam. By entangling the beam's transverse degrees of freedom with the which-path states of a Sagnac interferometer, it is possible to realize an optical amplifier for polarization independent deflections. The theory for the interferometric weak value amplification method is presented along with the experimental results, which are in good agreement. Of particular interest, we measured the angular deflection of a mirror down to 400 ± 200 frad and the linear travel of a piezo actuator down to 14 ± 7 fm.

(400 frad corresponds to 150 micrometers at the distance to the moon.)



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doi:10.1038/nature10120

Direct measurement of the quantum wavefunction

Jeff S. Lundeen[1], Brandon Sutherland[1], Aabid Patel[1], Corey Stewart[1] & Charles Bamber[1]

$$\langle A\rangle_W = \frac{\langle c|A|\Psi\rangle}{\langle c|\Psi\rangle}$$

Returning to our example of a single particle, consider the weak measurement of position ($A = \pi_x \equiv |x\rangle\langle x|$) followed by a strong measurement of momentum giving $P = p$. In this case, the weak value is:

$$\langle \pi_x\rangle_W = \frac{\langle p|x\rangle\ \langle x|\Psi\rangle}{\langle p|\Psi\rangle} \quad (2)$$

$$= \frac{e^{ipx/\hbar}\,\Psi(x)}{\Phi(p)} \quad (3)$$

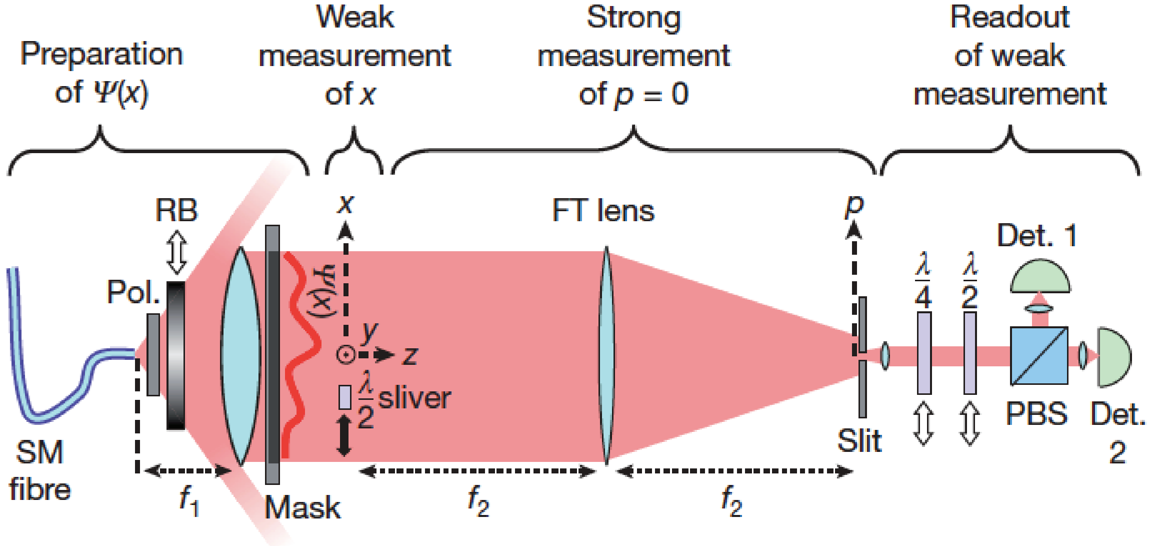
In the case $p = 0$, this simplifies to

$$\langle \pi_x\rangle_W = k\Psi(x) \quad (4)$$

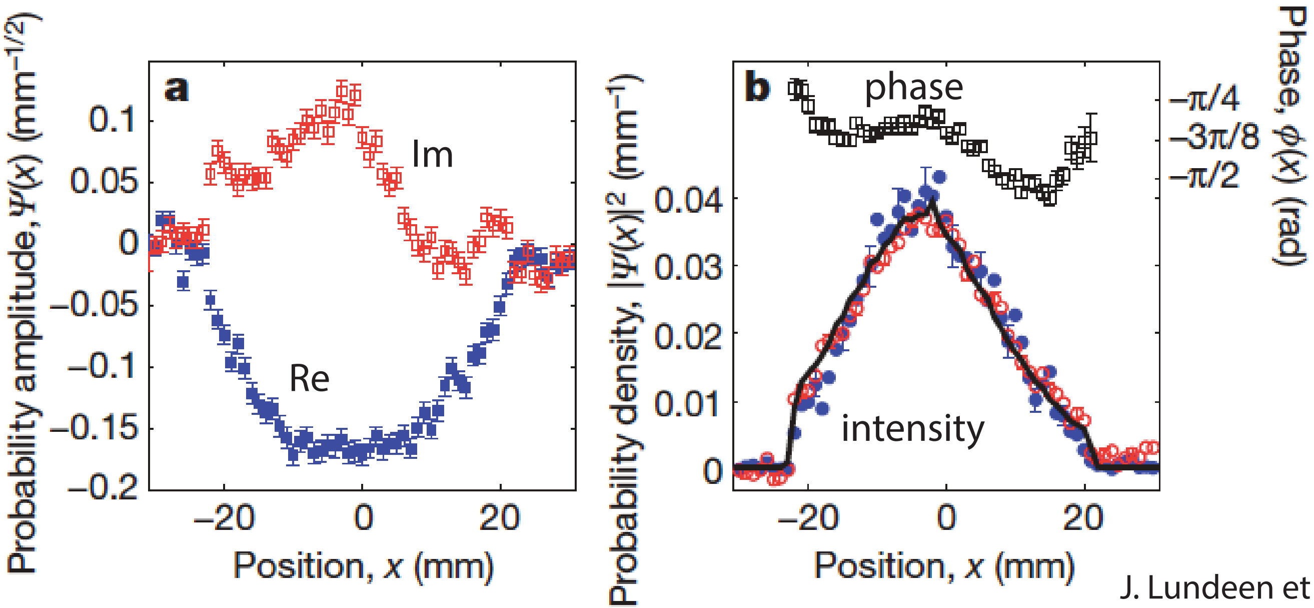
where $k = 1/\Phi(0)$ is a constant (which can be eliminated later by normalizing the wavefunction). The average result of the weak mea-

Direct Measurement of the Photon "Wavefunction"

Measurement setup



Typical results



J. Lundeen et al., Nature 474, 188 (2011)

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High Dimensional QKD Protocol

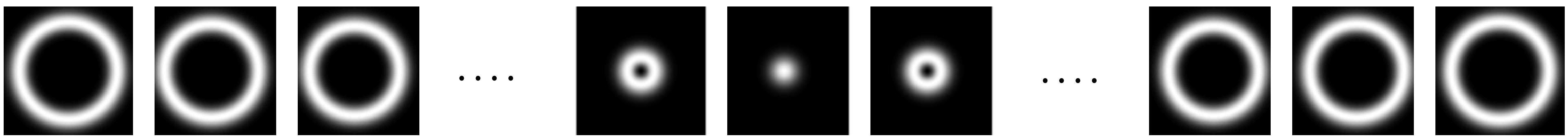
We are constructing a QKD system in which each photon carries many bits of information

We encode in states that carry OAM such as the Laguerre-Gauss states

As a diagnostic, we need to be able to measure the statevector of OAM states

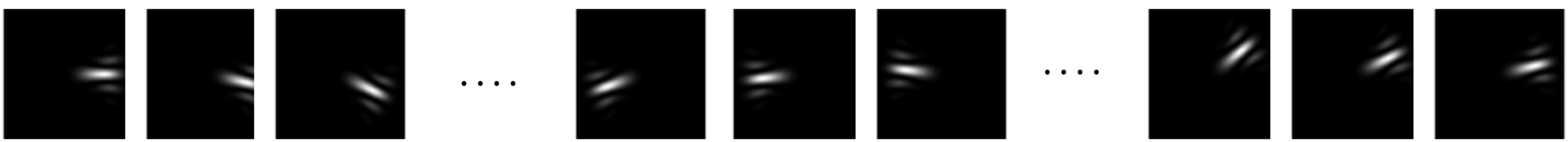
Single Photon States

Laguerre-Gaussian Basis $\ell = -13, \ldots, 13$



0 1 2 12 13 14 25 26 27

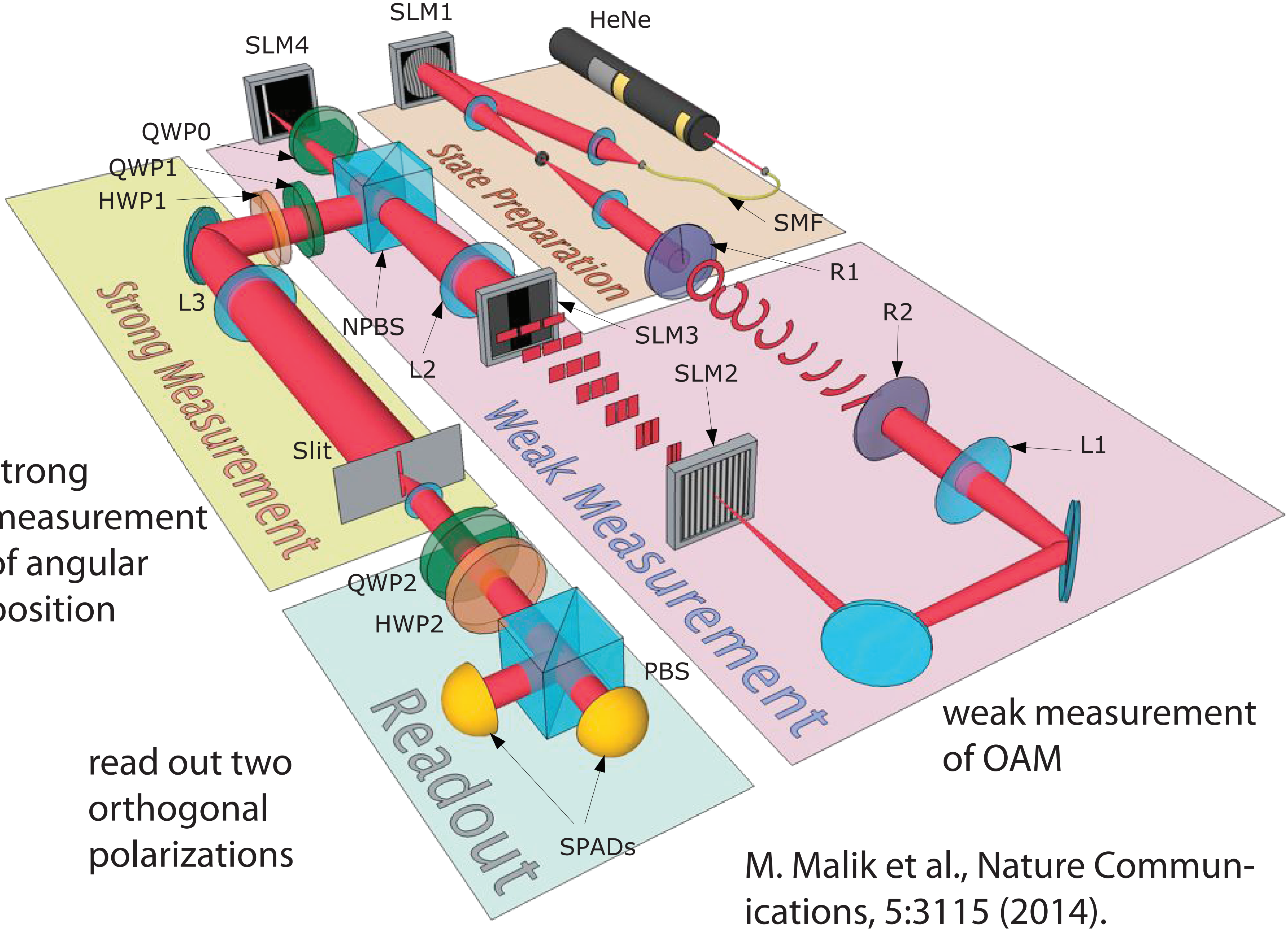
"Angular" Basis (mutually unbiased with respect to LG)



0 1 2 12 13 14 25 26 27

$$\Psi^N_{\mathrm{AB}} = \frac{1}{\sqrt{27}} \sum_{l=-13}^{13} \mathrm{LG}_{l,0} \exp\left(i2\pi N l/27\right)$$

Direct Measurement of a High-Dimensional OAM State



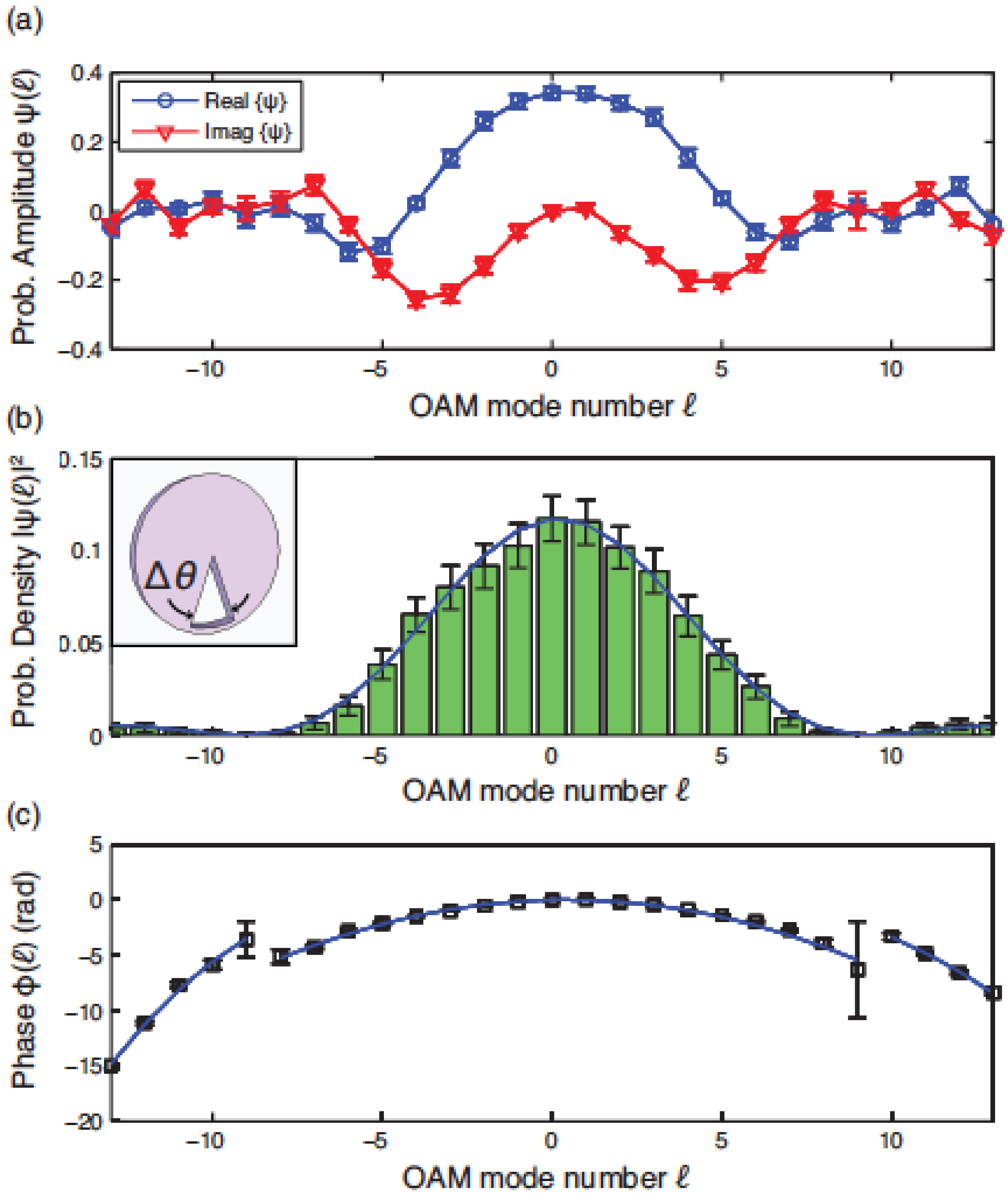
strong measurement of angular position

read out two orthogonal polarizations

weak measurement of OAM

M. Malik et al., Nature Communications, 5:3115 (2014).

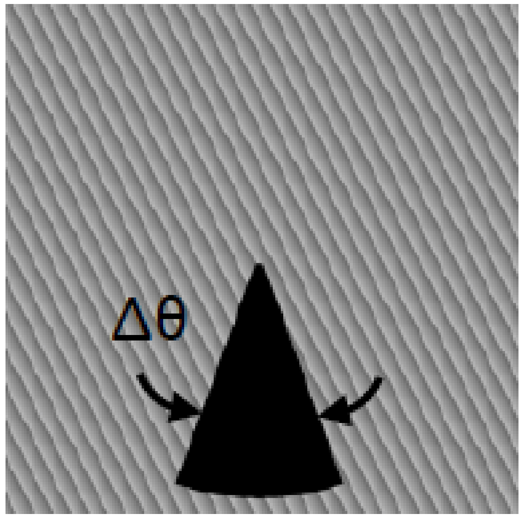
Direct Measurement of the OAM State Vector



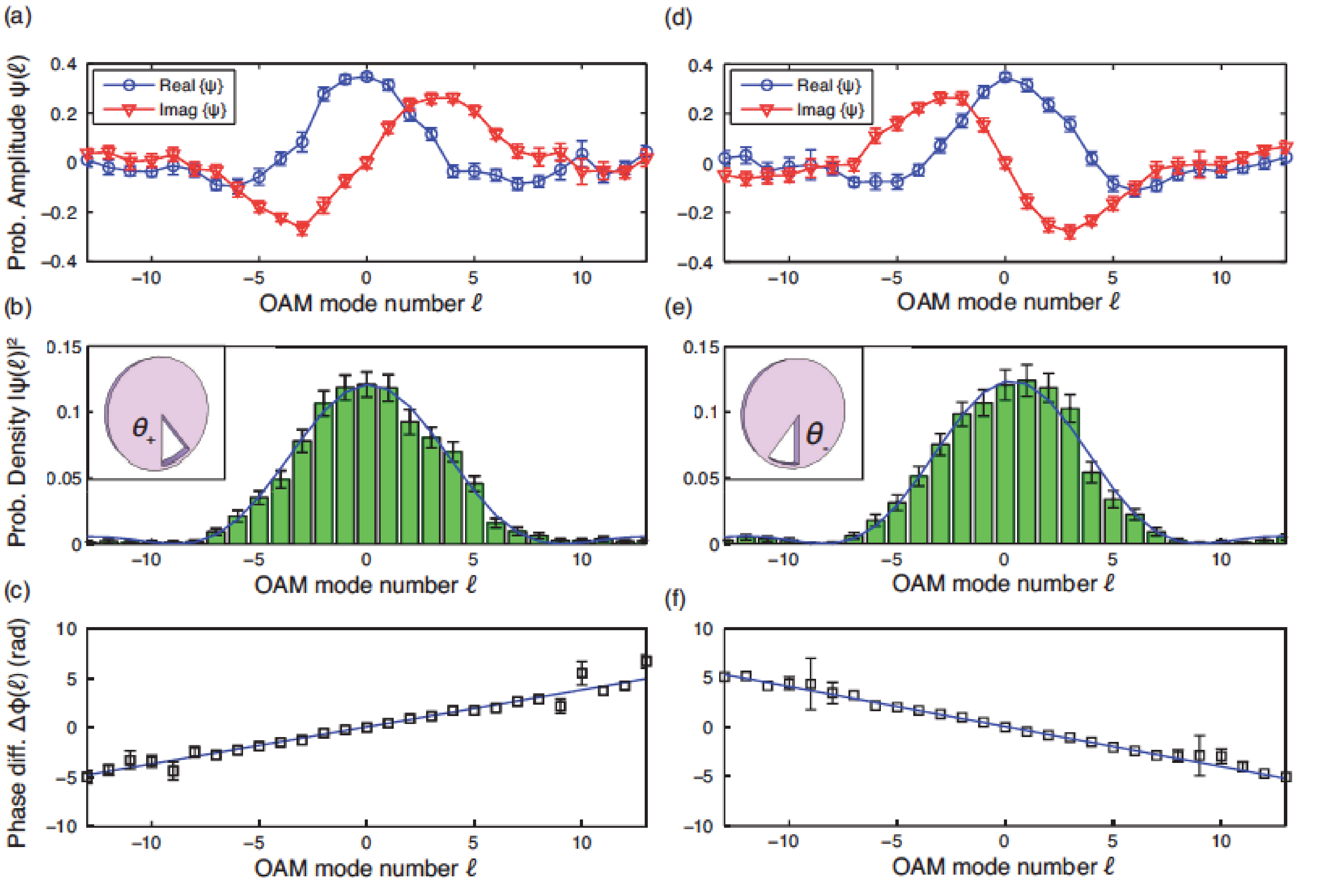
- We prepare a test wave function shaped as an angular wedged such that

$$|\Psi\rangle = \sum_{\ell} a_\ell |\ell\rangle$$

$$a_\ell = \frac{\sqrt{2\pi}}{d}\,\mathrm{sinc}\left(\frac{\ell\pi}{d}\right)$$



Direct Measurement Procedure Properly Measures Phase



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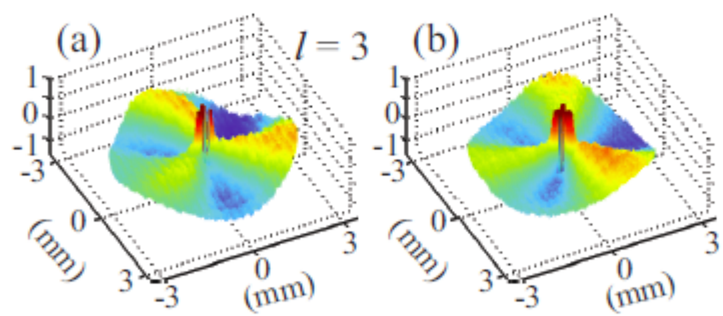
Direct Measurement of a one-million-dimensional photonic states



Zhimin Shi[1], Mohammad Mirhosseini[2], Jessica Margiewicz[1], Mehul Malik[2;3], Freida Rivera[1], Ziyi Zhu[1] and Robert W. Boyd[2;4]

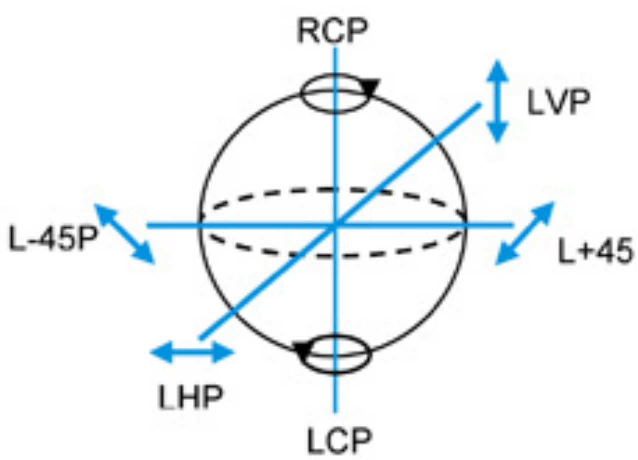
Email: zhiminshi@usf.edu

[1]*Department of Physics, University of South Florida, Tampa, Florida 33620, USA*
[2]*The Institute of Optics, University of Rochester, Rochester, New York 14627 USA*
[3]*IQOQI, Austrian Academy of Sciences, Boltzmanngasse 3, A-1090 Vienna, Austria*
[4]*Department of Physics, University of Ottawa, Ottawa, ON K1N 6N5 Canada*

PIERS 2016, Shanghai China

Conventional approach: quantum state tomography

- for a *N*-dimensional quantum state
  1. design N^2 -1 projection measurement.
  2. take the measurements in sequence.
  3. post-process the data to reconstruct the quantum state.



Example: polarization state

$$|\psi\rangle = \alpha|H\rangle + \beta|V\rangle$$

Needs to measure S_1, S_2 and S_3

$$S_1 = |\alpha|^2 - |\beta|^2$$

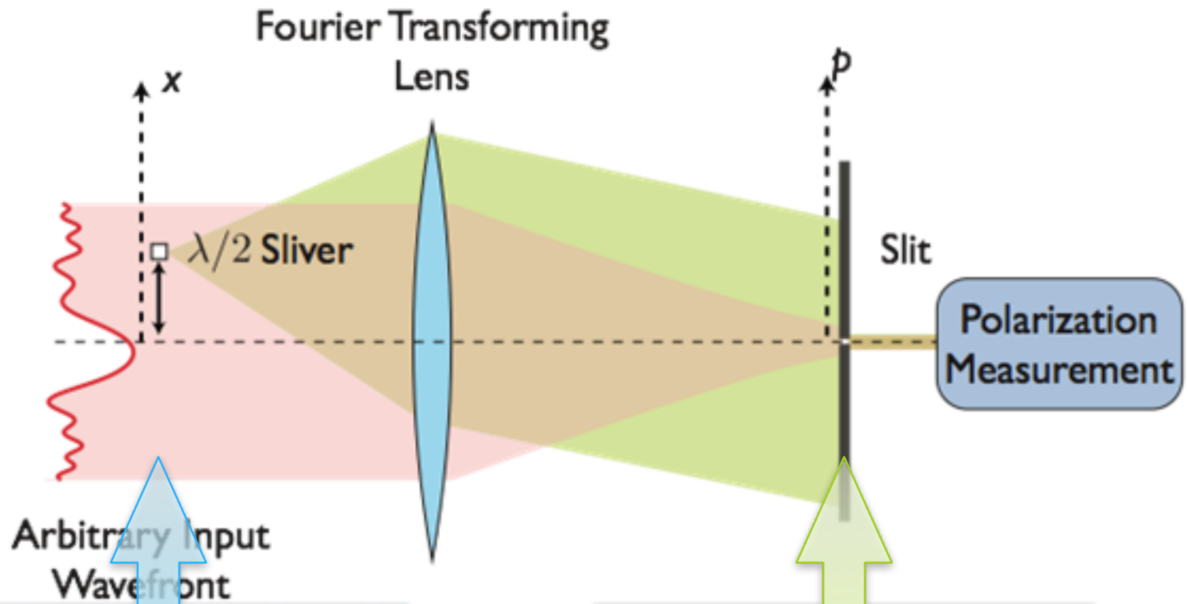
$$S_2 = 2\Re\left(\alpha\beta^*\right)$$

$$S_3 = -2\Im\left(\alpha\beta^*\right)$$

⇨ Reconstruct α and β

Complexity | **Time** | **Efficiency** | **scales badly with *N***

Conventional Direct Measurement



To measure

$$\langle x|\psi\rangle$$

weak measurement in position space

$$\hat{A} = |x\rangle\langle x|$$

strong measurement in momentum space

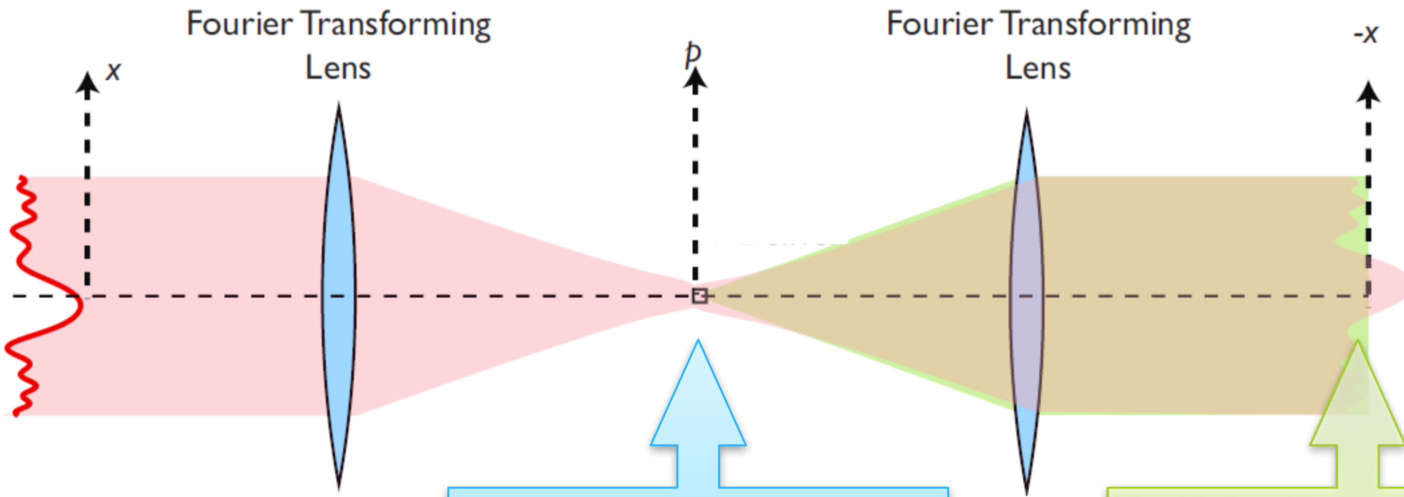
$$|\Psi_f\rangle = |p_0\rangle$$

Final measurement result (weak value)

$$A_{\mathrm{w}}(x) = \frac{\langle p_0|x\rangle\langle x|\psi\rangle}{\langle p_0|\psi\rangle} \propto \psi(x)$$

- Lundeen *et al.* , *Nature* **474**, 188–191 (2011).

Our approach: scan-free direct measurement



To measure

$$\langle x|\Psi\rangle$$

weak measurement in momentum space

$$\hat{A} = |p\rangle\langle p|$$

strong measurement in position space

$$|\Psi_f\rangle = |x\rangle$$

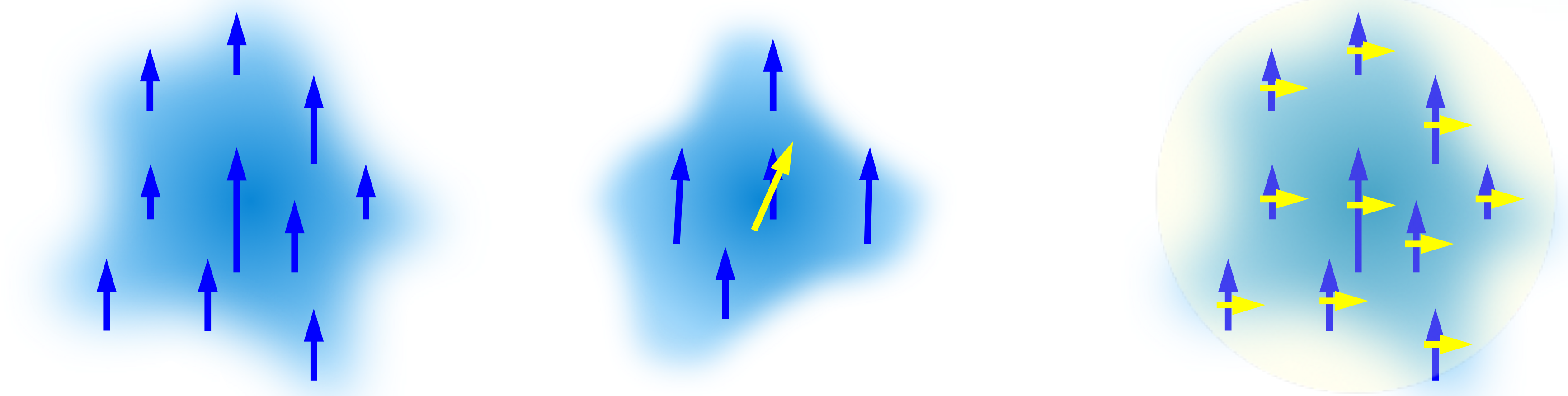
Final measurement result (weak value)

$$\langle \pi_p \rangle_x^{\mathrm{w}} = \frac{\langle x|p\rangle\langle p|\Psi\rangle}{\langle x|\Psi\rangle} = \frac{e^{i\hbar p x}\tilde{\Psi}(p)}{\Psi(x)}$$

- Shi *et al.*, *Optica*, 2(4), 388-392 (2015)

Weak measurement using polarization pointer state

x space | ***p*** space | ***x*** space



$$|\Psi\rangle|s_\mathrm{i}\rangle$$

$$\hat{H} = \alpha\hat{p}_0 \cdot \hat{\sigma}_2$$

$$|\psi\rangle|s_\mathrm{i}\rangle - \frac{it\sin\alpha}{\hbar}\hat{p}_0|\psi\rangle\hat{\sigma}_2|s_\mathrm{i}\rangle$$

- Final strong, polarization projection measurement

$$\langle s_\mathrm{f}|\hat{\sigma}_1|s_\mathrm{f}\rangle = \frac{-2t\sin\alpha}{\hbar}\Re\left[\langle\pi_{p_0}\rangle_x^\mathrm{W}\right]$$

$$\langle s_\mathrm{f}|\hat{\sigma}_2|s_\mathrm{f}\rangle = \frac{2t\sin\alpha}{\hbar}\Im\left[\langle\pi_{p_0}\rangle_x^\mathrm{W}\right]$$

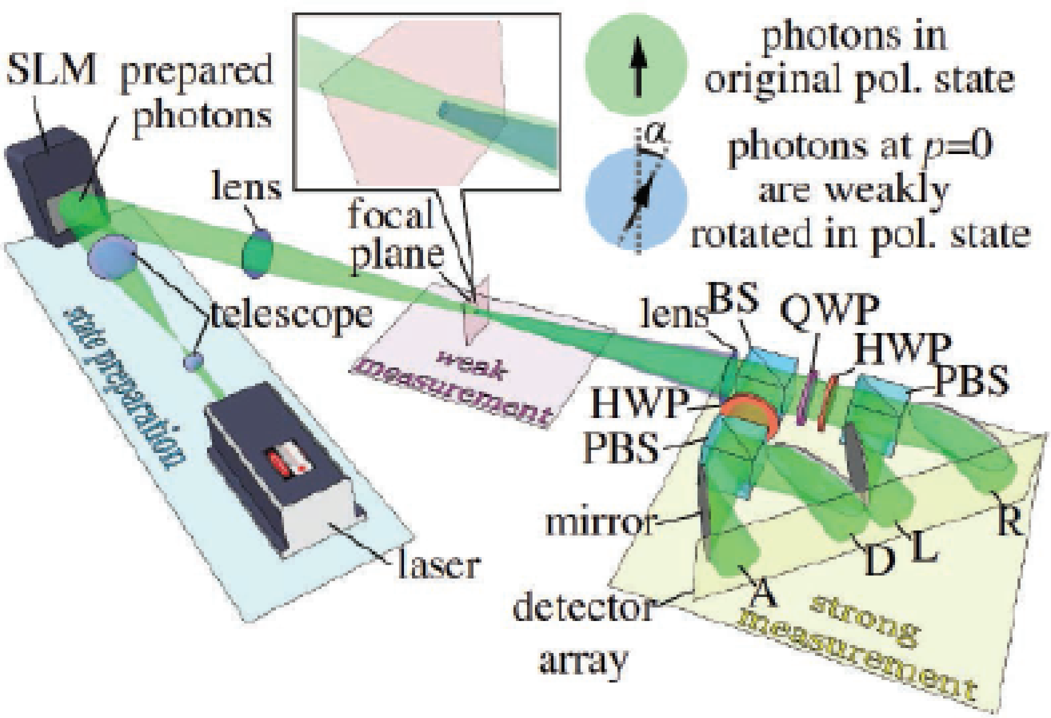
Research Article Vol. 2, No. 4 / April 2015 / Optica 388

optica

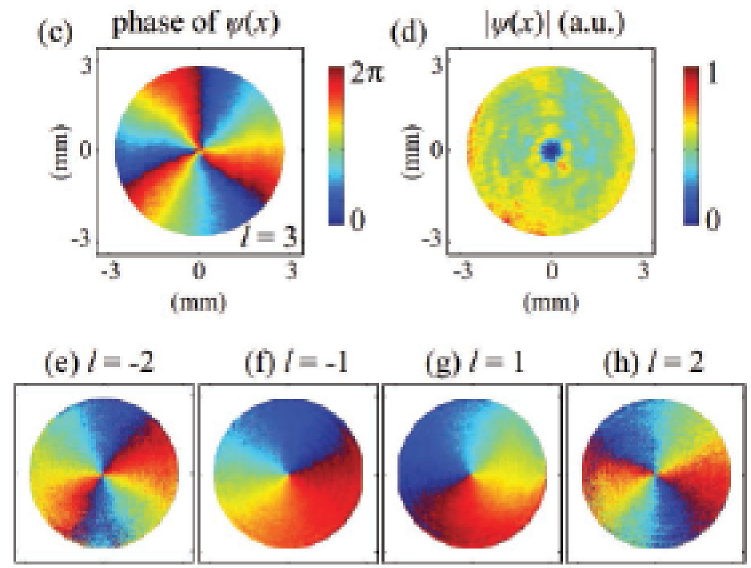
Scan-free direct measurement of an extremely high-dimensional photonic state

Zhimin Shi,[1,*] Mohammad Mirhosseini,[2] Jessica Margiewicz,[1] Mehul Malik,[2,3] Freida Rivera,[1] Ziyi Zhu,[1] and Robert W. Boyd[2,4]

Laboratory setup



Laboratory results for OAM beams



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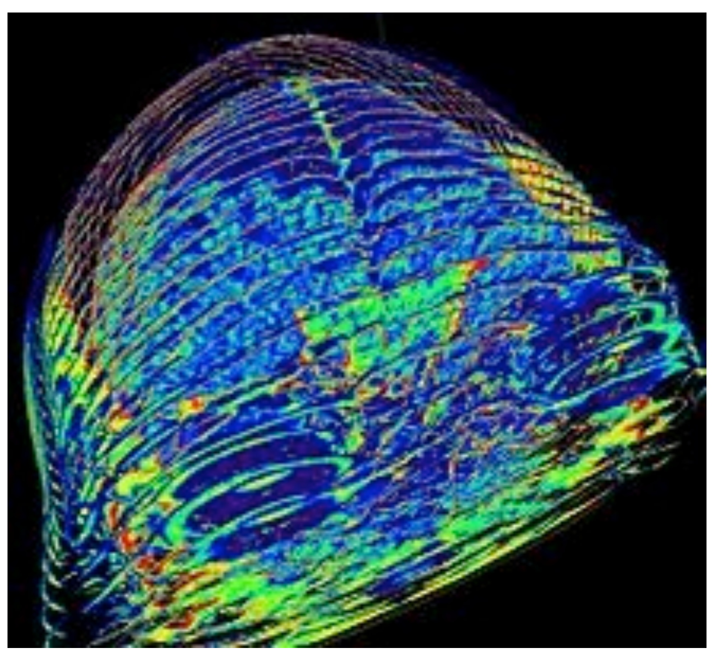
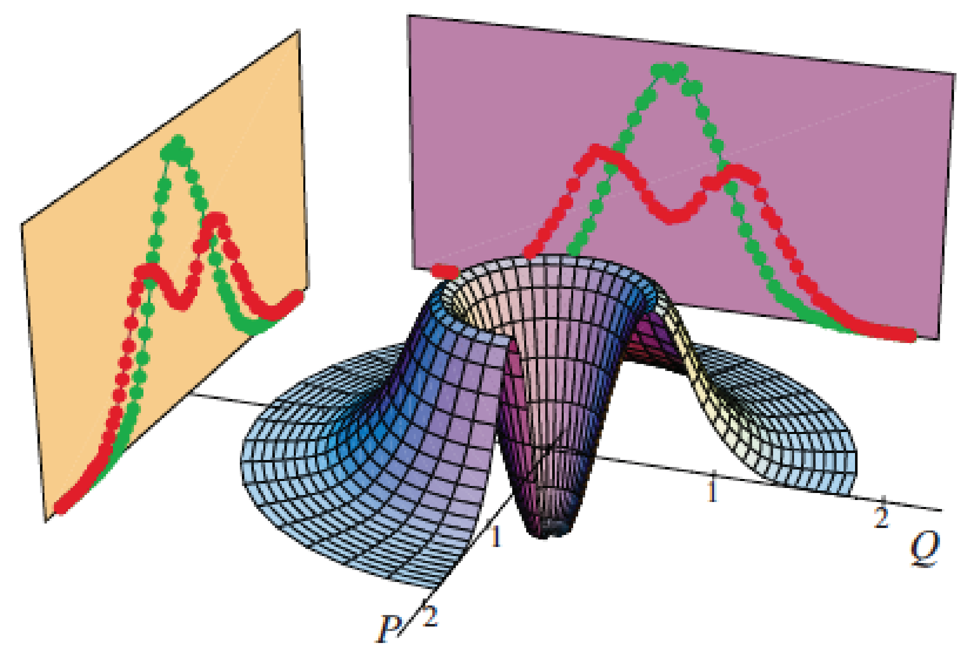
Characterization of partially coherent states

- The knowledge of amplitude and phase is not adequate for partially coherent states.
- The quantum state is described using more complicated mathematical functions such as density matrix or Wigner distribution function.
- Quantum state tomography is the standard way of finding the Wigner function in $\hat{a}$ and $\hat{a}^\dagger$ basis.





Lvovsky and Raymer. *Reviews of Modern Physics, 2009*

Wigner distribution of twisted photons

- Wigner distribution simultaneously store position and momentum representations

$$P(x,p) = \frac{1}{\pi\hbar}\int_{-\infty}^{\infty}\langle x+y|\hat{\rho}|x-y\rangle e^{-2ipy/\hbar}\,dy,$$

Wigner, E. (1932). Physical Review, 40(5), 749–759.

Alonso, M. A. (2011). Advances in Optics and Photonics, 3(4), 272–365.

- The conjugate of OAM is azimuthal angle. Angle is a tricky coordinate.

$$[\hat{\theta},\hat{\ell}] = ?$$

Barnett, S. M., & Pegg, D. T. (1990). Physical Review A, 41(7), 3427–3435.

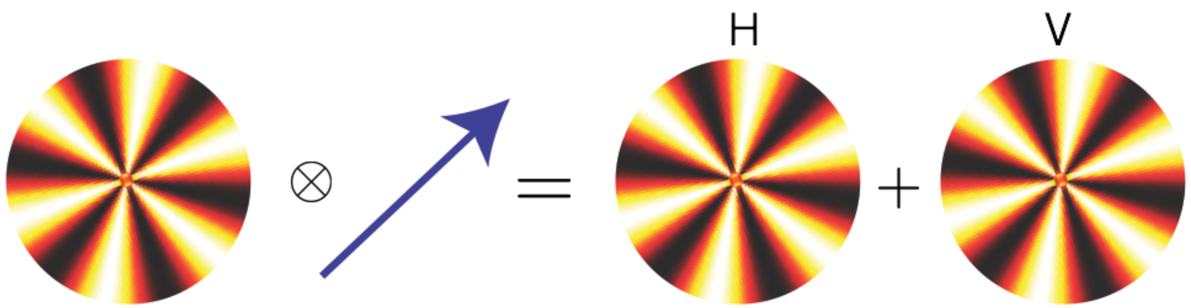
$$W(\theta,\ell) = \frac{1}{d}\sum_{\tau=-N}^{N}\exp\left(-\frac{4\pi i}{d}\ell\tau\right)\langle\theta-\tau|\hat{\rho}|\theta+\tau\rangle.$$

Leonhardt, U. (1995). Physical Review Letters, 74(21), 4101–4105.

Wigner distribution of twisted photons

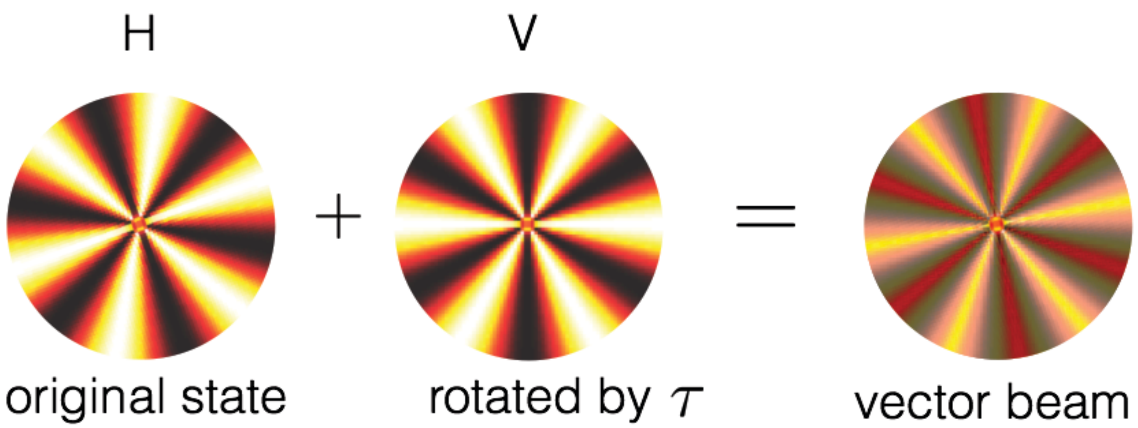
$$W(\theta,\ell) = \frac{1}{d}\sum_{\tau=-N}^{N} \exp\left(-\frac{4\pi i}{d}\ell\tau\right)\langle\theta-\tau|\hat{\rho}|\theta+\tau\rangle.$$

- We prepare the initial state (polarization is the pointer)



$$\hat{\rho}\times|D\rangle\langle D|$$

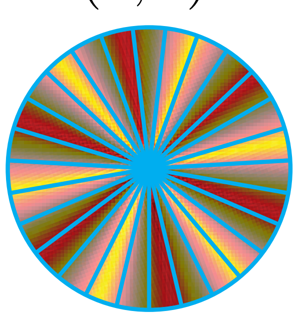
- We perform a polarization-sensitive rotation.



$$\hat{U}(\tau) = \exp\left(-\frac{2\pi i}{d}\tau\hat{L}\otimes\hat{\sigma}_z\right)$$

- We post-select on an angle eigenstate.

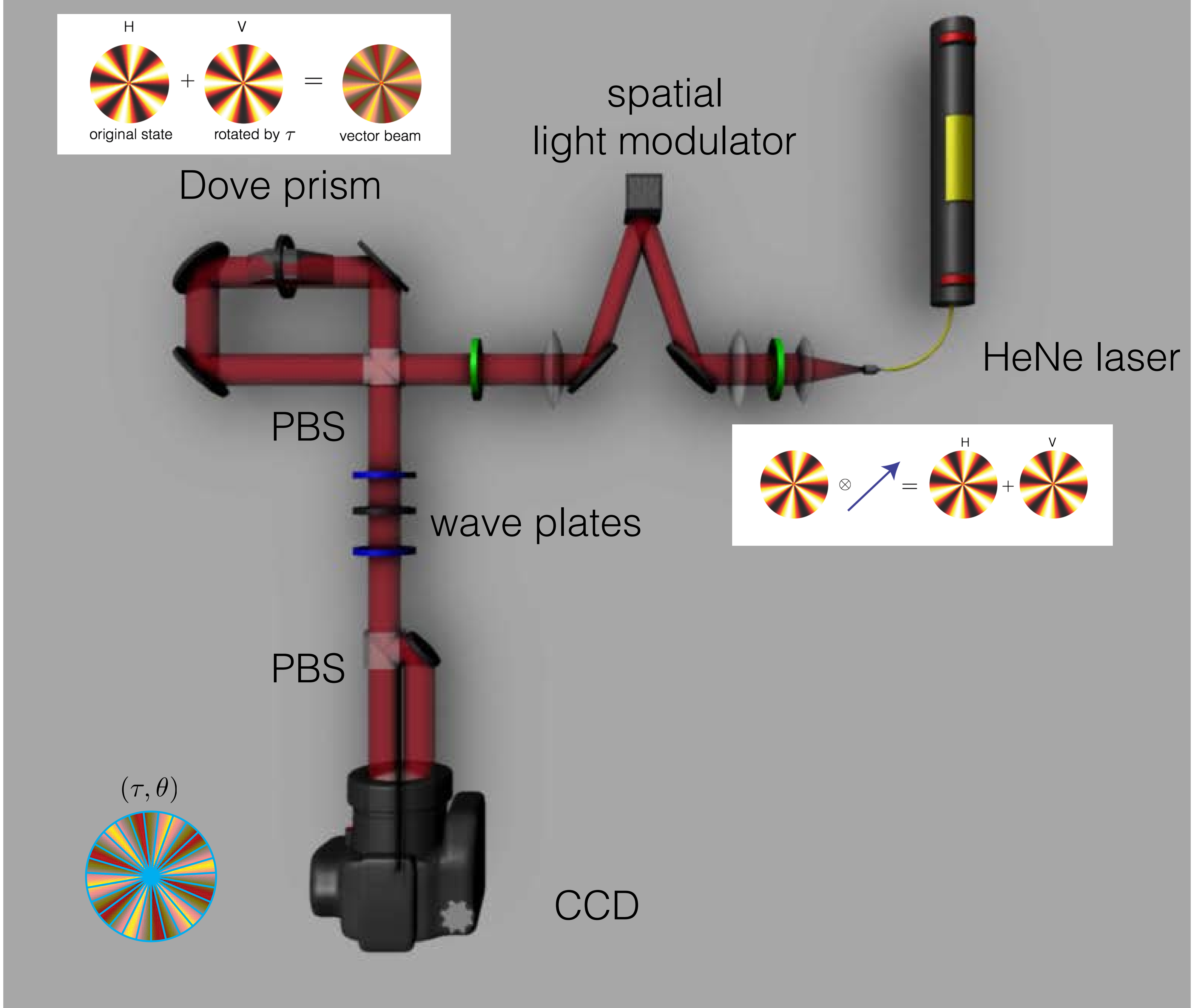
(τ,θ)



$$\langle\hat{\sigma}_x(\theta,\tau)\rangle = \frac{2}{N(\theta,\tau)}\mathrm{Re}\left[\langle\theta-\tau|\hat{\rho}|\theta+\tau\rangle\right]$$

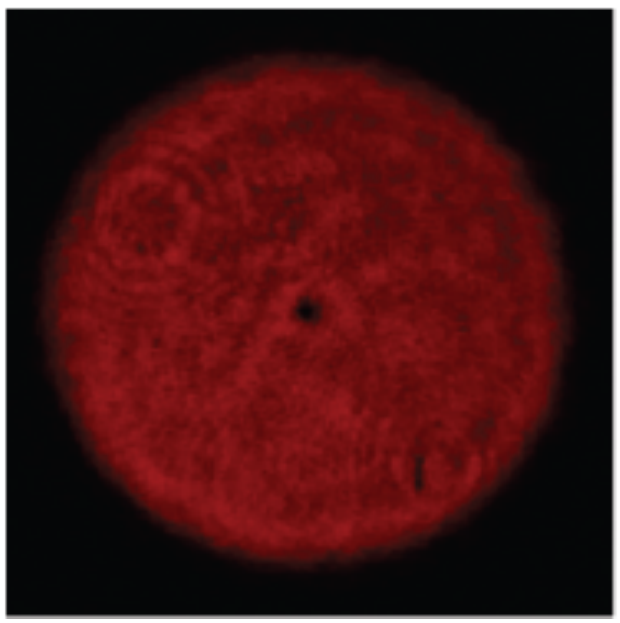
$$\langle\hat{\sigma}_y(\theta,\tau)\rangle = \frac{2}{N(\theta,\tau)}\mathrm{Im}\left[\langle\theta-\tau|\hat{\rho}|\theta+\tau\rangle\right]$$

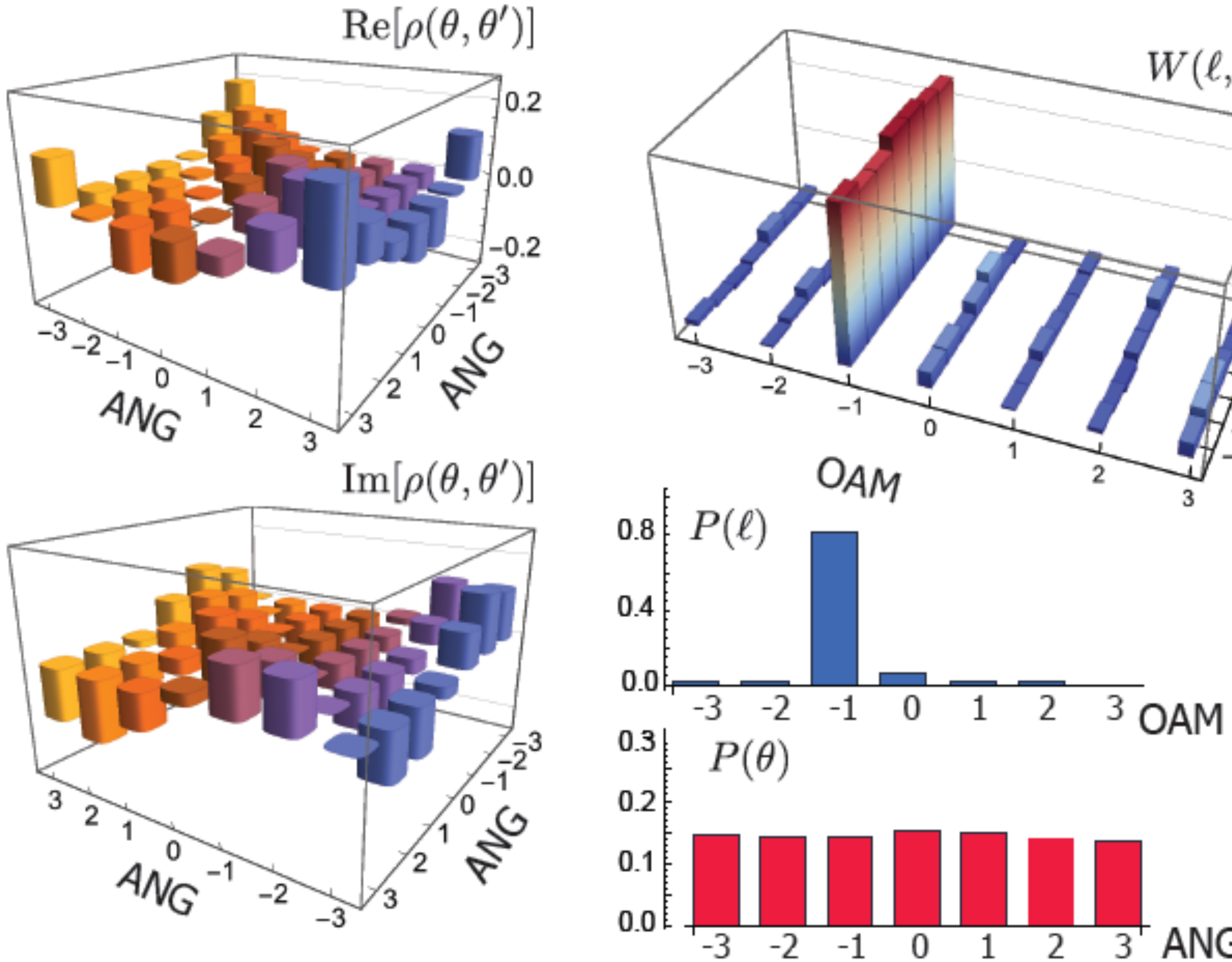
Experimental setup



Measurement of the density matrix, the Wigner function, and its marginals

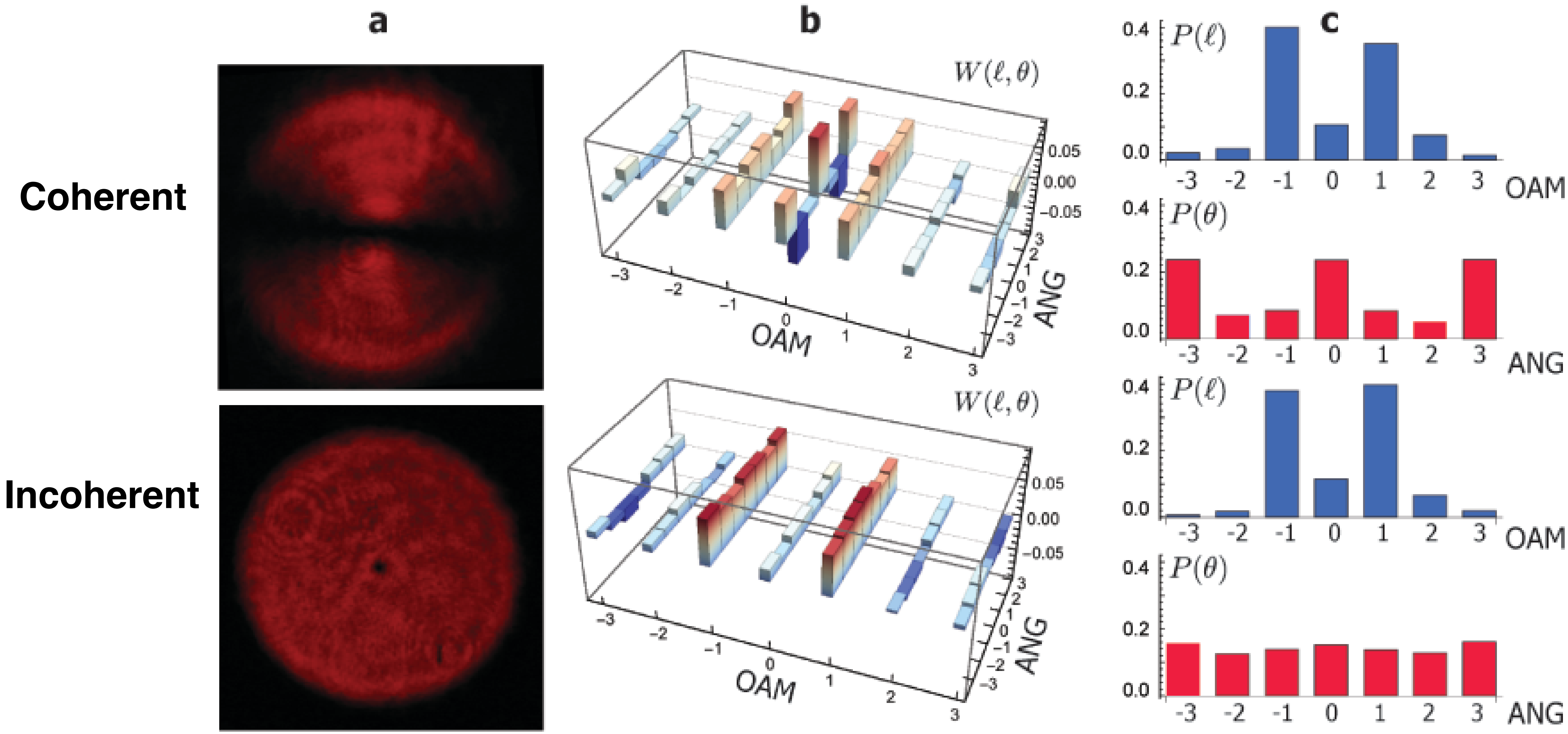
Classical light field. l = -1,





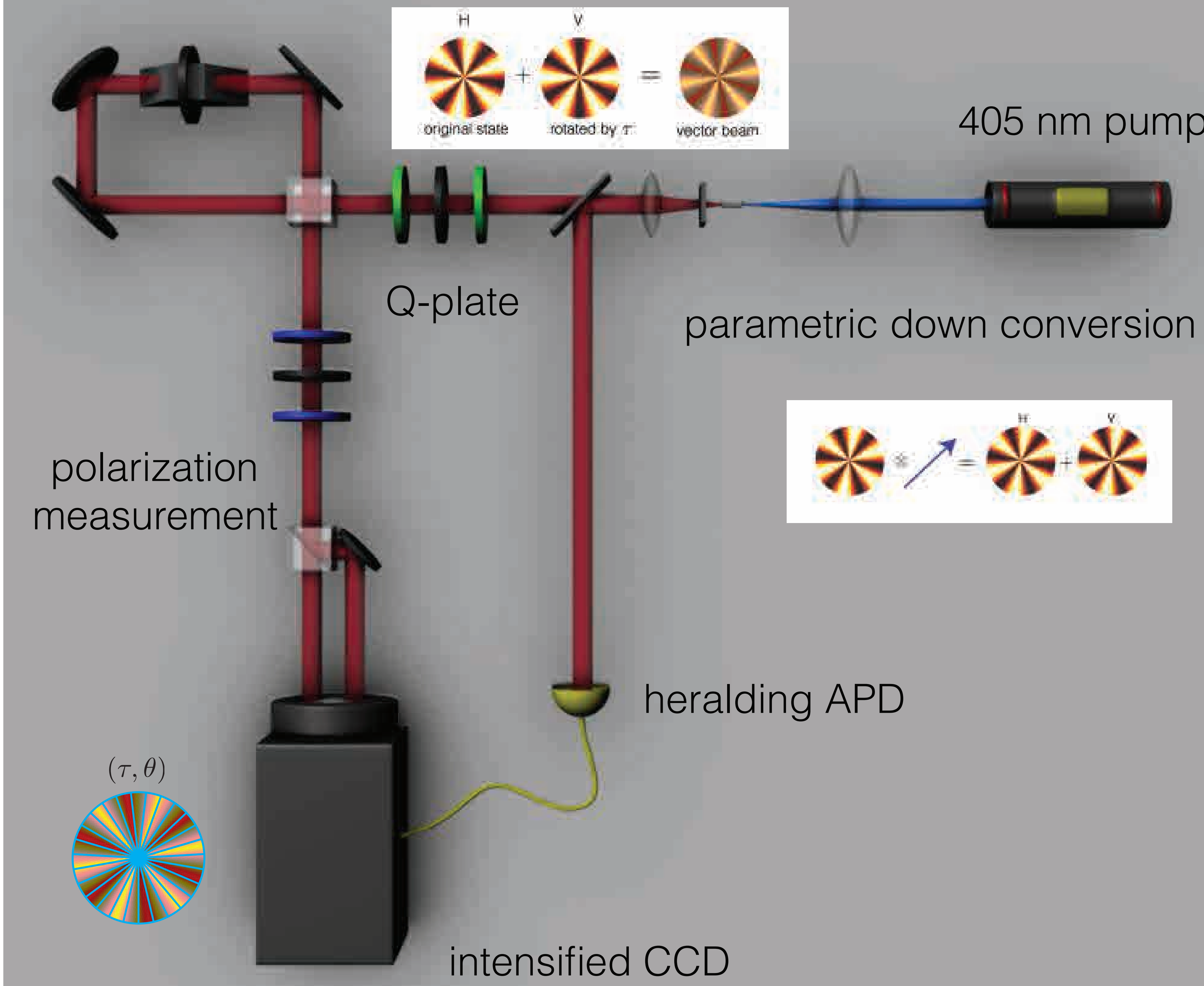
Coherent superposition vs incoherent mixture

- Full characterization requires understanding of the state's behavior in both OAM and ANG bases.

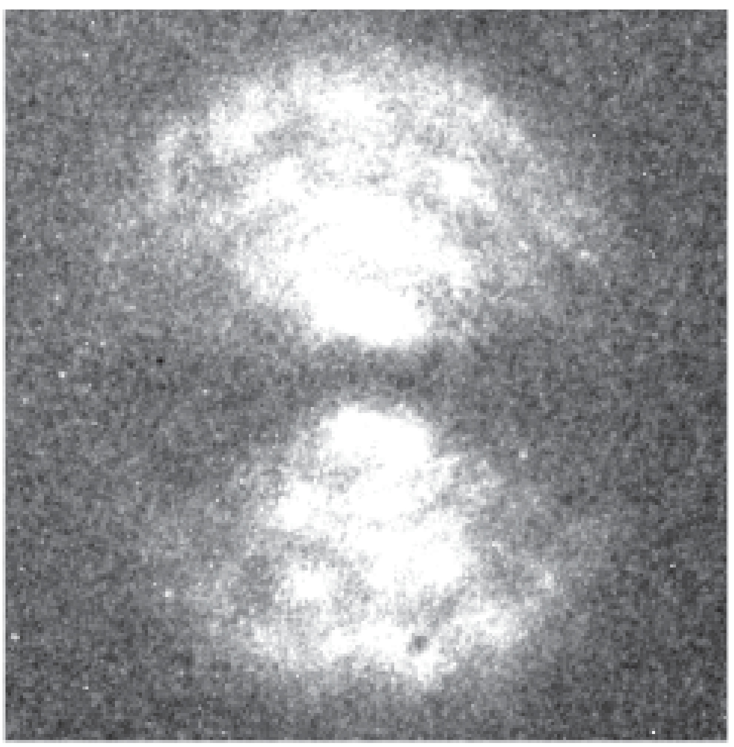


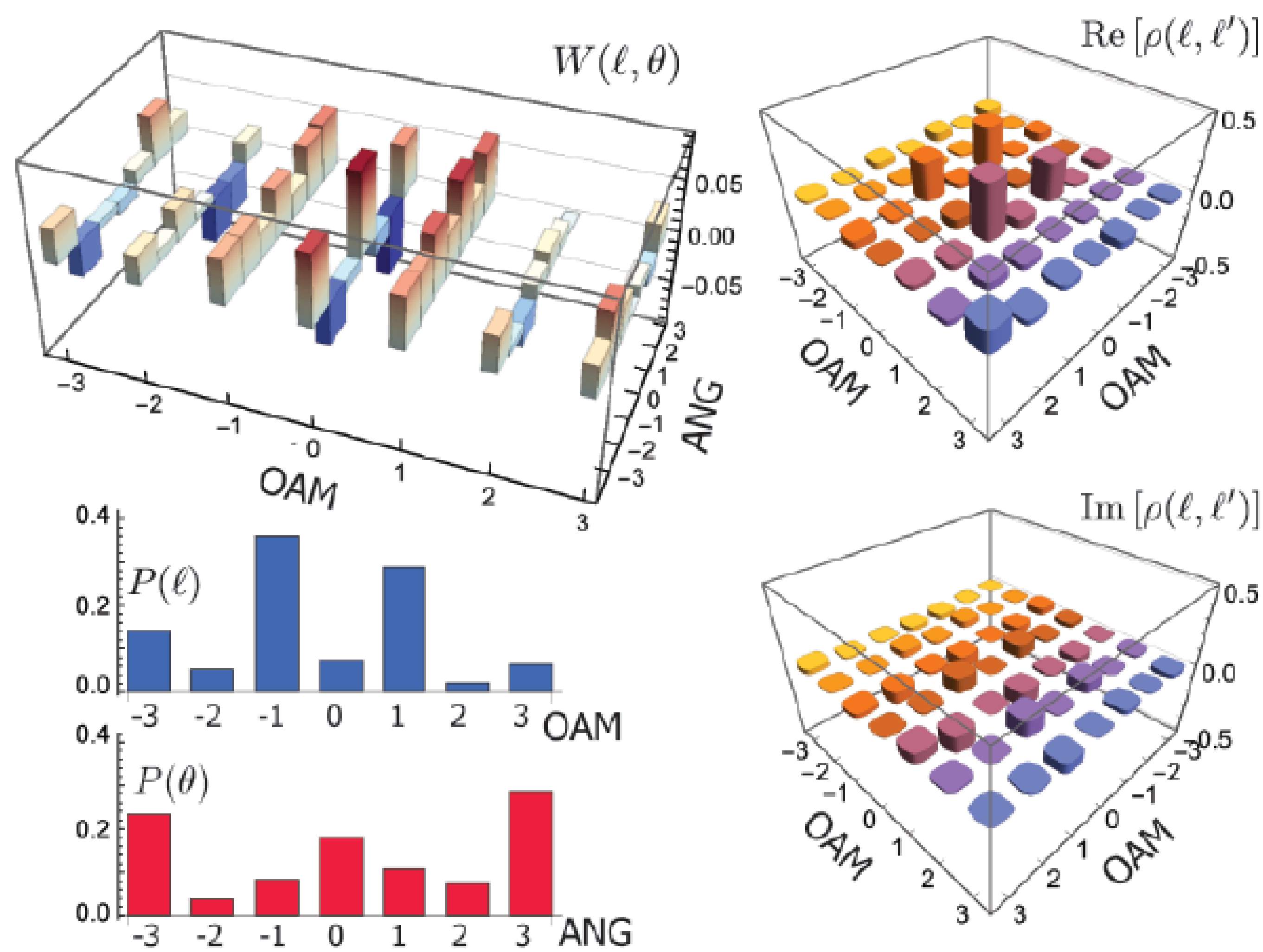
Mirhosseini et al. Physical Review Letters, (2016).

Experimental setup for quantum light fields



Wigner function of a "single" photon





Mirhosseini et al. Physical Review Letters, (2016).

Happy Birthday Wolfgang!