

# Material slow light and structural slow light: similarities and differences for nonlinear optics [Invited]

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There are two standard methods for controlling the group velocity of light. One makes use of the dispersive properties associated with the resonance structure of a material medium. The other makes use of structural resonances, such as those that occur in photonic crystals. Both procedures have proved useful in a variety of situations. In this work we contrast these two approaches, especially in terms of issues such as the kinematics of energy flow through the system and the resulting implications for the behavior of nonlinear optical processes in these situations. Stated differently, this paper addresses the question of when nonlinear optical processes are enhanced through use of slow-light interactions and when they are not. © 2011 Optical Society of America

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## 1. INTRODUCTION

For the past 10 years or more, the optical physics community has been fascinated by the opportunities afforded by nonlinear optics to exercise dramatic control over the velocity with which light pulses pass through material systems. One speaks of light being “slow” under circumstances in which the group velocity of light  $v_g$  is much smaller than the velocity of light in vacuum  $c$ . Even more quixotically, there are circumstances in which the group velocity can exceed  $c$ ; this occurrence is referred to as “fast light.” Most counterintuitive is the case in which the group velocity is negative, implying that the peak of a pulse travels in a direction opposite to that of phase velocity and to that of the energy flow; this circumstance is known as “backwards light.” Since all of these phenomena share a common origin, it is usual to refer to them collectively as “slow light,” a convention that we follow in the present article.

It is not our intention to review the entire field of slow light in the present article. The field has been reviewed extensively in recent years, including the publication of review articles [1,2], dedicated books [3,4], and special issues of scientific journals [5,6]. Rather, we will focus on one particular aspect of slow-light research, that of determining how properties of a light field such as intensity, energy density, and electric field strength become modified when the light enters a slow (or fast) light medium. This question has the practical importance of helping to determine how nonlinear optical processes become modified when they occur within a slow-light medium. We shall see that the nature of these modifications are very different for slow light based on the intrinsic optical properties of a material system than for slow light based on structured optical materials, such as photonic crystals (PhCs) and fiber Bragg gratings (FBGs).

## 2. MATERIAL SLOW LIGHT

By material slow light, we mean situations in which the velocity of light pulses can be described fully in terms of a spatially

uniform but frequency-dependent refractive index  $n$  of a material. Under these circumstances, we can define the group velocity of a light pulse as

$$v_g = c/n_g, \quad (1)$$

where we have introduced the group index given by

$$n_g = n + \omega \frac{dn}{d\omega}. \quad (2)$$

We note that, because  $dn/d\omega$  can be either positive or negative, the group index can be either larger or smaller than unity. Also, if  $dn/d\omega$  is negative and sufficiently large, the group index can itself become negative. Because slow light depends so crucially on the frequency dependence of the refractive index, one expects slow light effects to be particularly strong in the vicinity of an absorption or gain resonance of a material system. Behavior of this sort is shown in Figs. 1 and 2. In Fig. 1, the two top panels show an absorption resonance and a gain resonance. The two middle panels show the frequency dependence of the refractive index associated with these resonances, as required by the Kramers-Kronig relations. The two lower panels show the resulting values of the group index as calculated from Eq. (2). One sees that either fast or slow light can occur in either case, depending on the detuning of the optical wave from resonance.

In many applications of slow light, it is desirable to be able to use nonlinear optical methods to control the group velocity. Such a possibility is shown in Fig. 2. A typical sort of nonlinear optical response leads to the establishment of a sharp dip in an absorption or gain feature. Simple saturation effects can lead to such behavior, as well as more complicated effects such as electromagnetically induced transparency (EIT) and coherent population oscillations (CPO), which are described in more detail below. We see from Fig. 2 that both

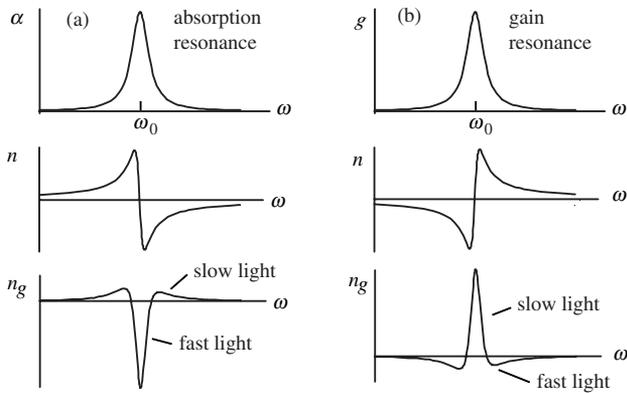


Fig. 1. Origin of slow and fast light for an isolated absorption resonance (a) and gain resonance (b).

slow and fast light can occur for a dip in either a gain or absorption feature. Moreover, the value of the group index can be controlled in this case by means of the intensity of the optical field, which governs the extent of the dip.

We now turn our attention to a consideration of the electromagnetic energy relations appropriate to slow-light media. It turns out that it is possible to derive Poynting's theorem for the case of a dispersive optical medium, that is, for a medium with a refractive index that is frequency dependent. The details are somewhat subtle and we will not provide a derivation here. These relations are derived in textbooks [7,8] and in Section 2.7 of [3]. For the case of a material without appreciable gain or loss at the frequencies of interest, one finds that the energy density is given by

$$u = \frac{1}{2} n n_g \epsilon_0 |E|^2 \quad (3)$$

and that the intensity (magnitude of the Poynting vector) is given by

$$S = \frac{1}{2} n c \epsilon_0 |E|^2 \quad (4)$$

One sees that the intuitively expected relation

$$S = u v_g = u(c/n_g) \quad (5)$$

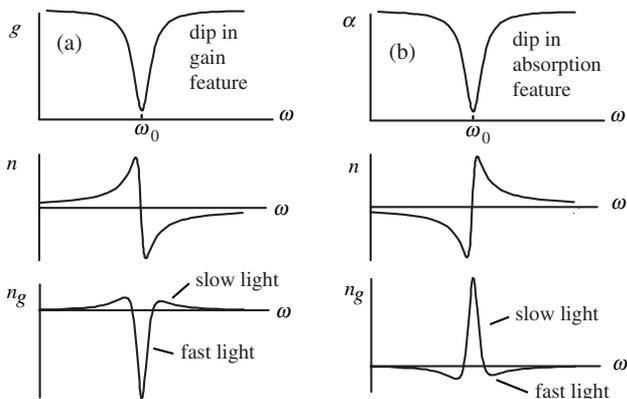


Fig. 2. Origin of slow and fast light for a dip in a gain feature (a) and an absorption feature (b).

is obeyed [9]. These properties are illustrated heuristically in Fig. 3. In the top panel we see that when a pulse enters a slow-light medium it becomes spatially compressed by a factor equal to the group index, and its peak energy increases by the same factor. It is crucial to note that the pulse becomes compressed in space but not in time. Thus, the pulse intensity remains constant, as it must for reasons of energy conservation. The power flow of an optical wave through each transverse plane cannot increase for propagation through a passive medium. For  $n$  approximately equal to unity, which is the case for many material slow-light media, there is no increase in field strength within the medium even though the energy density has increased. This conclusion follows from the relation expressed in Eq. (3). In the bottom panel of Fig. 3 we see that when a pulse enters a fast-light medium it becomes spatially expanded, and its peak energy decreases.

There have been many experimental studies of slow and fast light based on the dispersion associated with material resonances. One early study was that of Chu and Wong [10], who observed both slow and fast light effects in the propagation of ps optical pulses through a crystal of GaP:N. Another early study was that of Wang *et al.* [11], who demonstrated distortion-free fast light propagation by tuning the central frequency of their pulse to the midpoint between two gain features.

Many studies of slow-light effects have been based on the use of EIT. This process makes use of quantum coherence to produce a sharp dip in the absorption profile of an otherwise opaque atomic transition [12]. The EIT process was used as the slow-light mechanism in the celebrated demonstration of Hau *et al.* of light speeds as low as 17 m/s in an ultra-cold gas of atoms [13]. Another approach to slow light is to make use of CPOs. This process utilizes the response of the ground-state population to the beating between pump and probe waves in a saturable absorbing medium. These oscillations lead to a pronounced dip in the probe absorption spectrum for low modulation frequencies. This process is largely immune to the presence of dephasing collisions, and thus it can lead to slow-light effects even in room-temperature, solid-state materials. Slow light [14], fast light [15], backwards light [16], and rotary image drag [17] based on the process of CPO have all been observed.

Even though the electric field strength is not increased in an EIT slow light interaction, very large optical nonlinearities are nonetheless predicted. This point has been emphasized in a

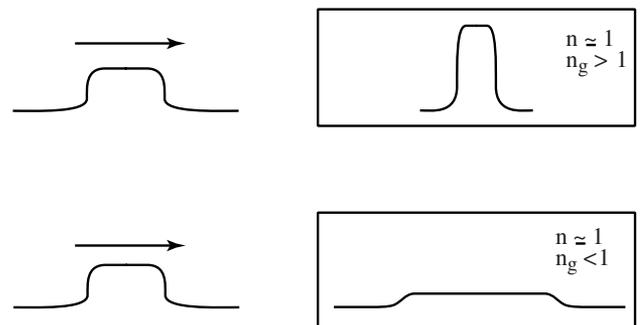


Fig. 3. (top) Schematic illustration of the pulse compression that occurs when light enters a slow-light medium. (bottom) Pulse expansion occurs for the case of a fast-light medium. In each case the pulse is represented in terms of its energy density  $u$ .

theoretical work of Harris and Hau [18] who show that efficient nonlinear coupling can occur with pulse energies as small as a few photons per atomic cross section [18]. This prediction is borne out in the experimental results of Kash *et al.* [19]. In the opinion of the present author, these results should not be taken to imply that the nonlinear response is large *because* the light travels slowly. Rather, it is that both slow light and enhanced nonlinearity are a consequence of the strong resonant coupling that occurs under conditions of EIT.

### 3. STRUCTURAL SLOW LIGHT

Qualitatively different behavior occurs for the case of structural slow light. By structural slow light, one means that the propagation of light pulses is significantly modified by the (typically periodic on distances scales of the order an optical wavelength) spatial modulation of the optical properties of a material system. A well-known example of such behavior is that of PhCs [20–22]. PhCs are formed by a periodic modulation of the local dielectric constant in either one, two, or three dimensions. By inducing defects in the periodic structure, devices such as resonators and waveguides can be formed. A representative PhC structure is shown in Fig. 4, and a typical dispersion diagram is shown in Fig. 5. It should be noted that the band diagram becomes nearly horizontal near the right-hand side of the plot, that is, near the edge of the Brillouin zone. This situation corresponds to slow light, because the group velocity given by  $d\omega/dk$  is very small.

Considerable research has been conducted leading to the design of PhC structures that are tailored to produce a strong slow light effect [25–30]. Group indices as large as 300 (corresponding to a slow-down factor  $S \equiv n_g/n$  of nearly 100) have been observed in such structures [26], although in many cases scattering losses resulting from fabrication errors become prohibitive for group indices larger than approximately 30–100 [31,32].

Unlike the case of material slow light, there is a true increase of the electric field strength within a structural slow-light medium. This increase can lead to direct enhancement of nonlinear optical interactions, as we describe in detail below. Because for a PhC structure the dielectric properties

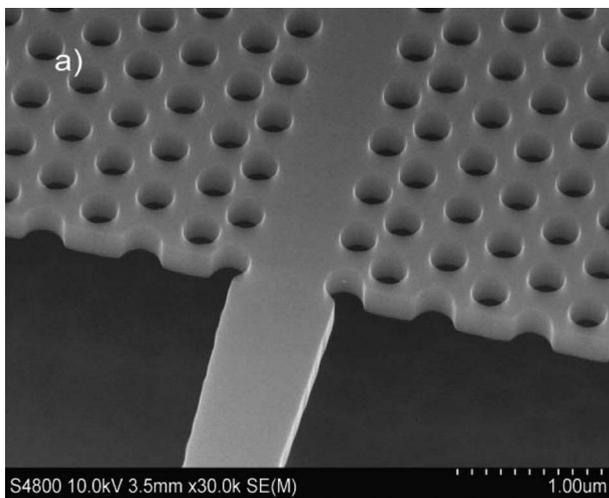


Fig. 4. A representative slow-light structure in the form of a line-defect photonic-crystal structure. Reproduced with permission from [23].

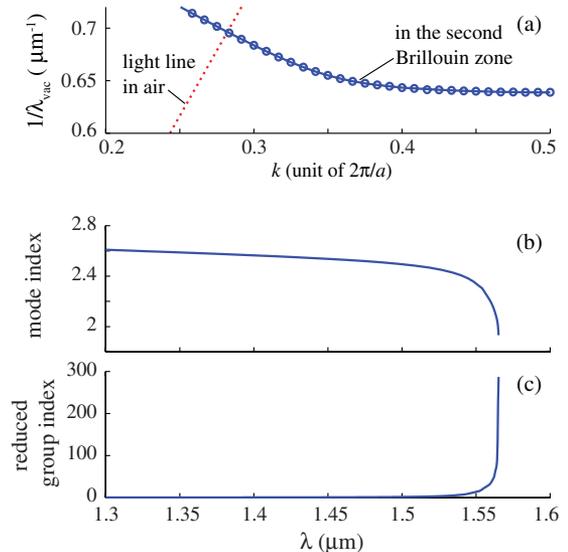


Fig. 5. (Color online) (a) Dispersion diagram of a typical line-defect photonic-crystal waveguide. (b) Mode index determined from the dispersion diagram of part (a). (c) Reduced group index ( $n_g - n$ ) for this structure. Note that while the group index is very large, it changes rapidly with wavelength, which is undesirable for many applications. Much current work is aimed at developing dispersion-engineered PhC waveguides that minimize this effect. Reproduced with permission from [24].

are modulated on a subwavelength scale, a precise calculation of the buildup of local electric field within the structure requires detailed numerical modeling, for instance using FDTD or plane-wave expansion methods [21]. Nonetheless, much insight into the nature of the enhancement of the electric field strength can be obtained by considering the case of a FBG, which can be thought of as a one-dimensional PhC.

The reason why a FBG structure leads to slow light and to enhanced nonlinear response can be understood heuristically in terms of the diagram shown in Fig. 6. This diagram is intended to show that the light bounces back and forth many times in its passage through the structure. Also, because there are both forward- and backward-going waves within the structure, each of which carries power, the total energy stored within the structure is larger than the energy stored in a wave propagating freely through a medium of the same mean refractive index. There will thus be an increase in electric field strength within the structure, which can lead to enhanced nonlinear optical effects.

We now turn to a more formal treatment of the FBG. We use the notation and methodology of Winful [34–36]. We consider a one-dimensional medium with a refractive index structure given by  $n(z) = n_0 + n_1 \cos(2k_B z)$ , where  $n_1$  (which is assumed to be much smaller than the background index  $n_0$ ) is the amplitude of the index modulation. In addition, the quantity  $k_B$  is given by  $\pi/\Lambda$ , where  $\Lambda$  is the spatial period of the refractive index variation that constitutes the grating.



Fig. 6. Schematic illustration of the origin of slow light in a Bragg grating structure. Reproduced with permission from [33].

$k_B$  is thus the wavenumber of a lightwave that satisfies the Bragg condition. The frequency of such a lightwave is given by  $\omega_B = k_B c / n_0$  and is known as the Bragg frequency. Under the assumption of the slowly varying envelope approximation, the forward- and backward-going waves within the structure are coupled according to

$$\frac{\partial E_F}{\partial z} + \frac{1}{v} \frac{\partial E_F}{\partial t} = i\kappa E_B e^{2i\Delta\beta z} \quad (6)$$

$$\frac{\partial E_B}{\partial z} - \frac{1}{v} \frac{\partial E_B}{\partial t} = -i\kappa E_F e^{-2i\Delta\beta z}, \quad (7)$$

where  $\kappa = n_0 n_1 \omega_B / 2c$  is the coupling strength,  $\Delta\beta = n_0 \Omega / c$  is the wavenumber mismatch,  $\Omega = \omega - \omega_B$  is the frequency detuning from the Bragg resonance, and  $v$  is the group velocity of the background material. These equations can be solved analytically. The steady state solution for an incident wave of field strength  $E_0$  is given by

$$E_F(z) = E_0 [\gamma \cosh \gamma(z-L) + i(\Omega/v) \sinh \gamma(z-L)] / g \quad (8)$$

$$E_B(z) = -iE_0 [\kappa \sinh \gamma(z-L)] / g \quad (9)$$

with  $\gamma = [\kappa^2 - (\Omega/v)^2]^{1/2}$  and  $g = \gamma \cosh \gamma L - i(\Omega/v) \sinh \gamma L$ . These equations are written in terms of hyperbolic functions because Winful's primary interest was in barrier tunneling. In such a case,  $\gamma$  is a real quantity. However, Eqs. (8) and (9) remain valid even for larger values of  $\Omega$  that lie outside of the photonic bandgap, where  $\gamma$  as defined above becomes imaginary and the spatial dependences of  $E_F$  and  $E_B$  become oscillatory. In this case, it is useful to define a real quantity  $q = i\gamma$  in terms of which the dispersion relation for the FBG can be written as [37]

$$(\Omega/v)^2 = q^2 + \kappa^2. \quad (10)$$

This relation is plotted in Fig. 7 in terms of the physical quantities  $\omega = \omega_B + \Omega$  and  $k = k_B + q$ . We see that a band gap centered on the Bragg frequency  $\omega_B$  with a frequency width given by  $2\kappa v$  is created. Incident light within this range of frequencies cannot propagate through the structure and experience exponential decay with a decay constant given by the quantity  $\gamma$ .

From the results of Eqs. (8) and (9), we see that both forward- and backward-going waves of comparable amplitudes

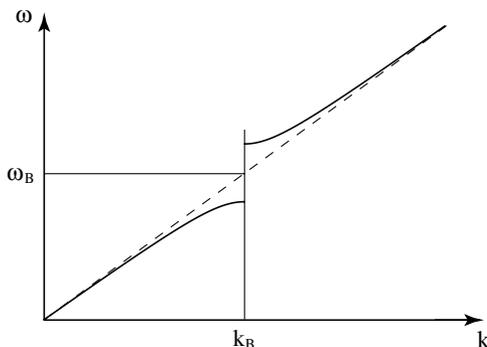


Fig. 7. Dispersion relation of a FBG as given by Eq. (10).

exist within the interaction region. Energy is stored in each of these waves, and thus the energy stored within the FBG region exceeds that of a wave of amplitude  $E_0$  propagating through a uniform medium. The stored energy can be calculated as the volume average of the time-averaged energy density  $u = \frac{1}{2} n^2 \epsilon_0 (|E_F|^2 + |E_B|^2)$  and is given by [36].

$$U = U_0 \left[ \frac{(\kappa/\gamma)^2 (\tanh \gamma L) / \gamma L - (\Omega/\gamma v)^2 \operatorname{sech}^2 \gamma L}{1 + (\Omega/\gamma v)^2 \tanh^2 \gamma L} \right], \quad (11)$$

where  $U_0 = \frac{1}{2} \epsilon_0 n_0^2 |E_0|^2 AL$  and  $A$  is the cross-sectional area of the waveguide.

In his paper [36], Winful goes on to show that the group delay  $\tau_g$  (roughly, the time difference between the entrance and exit of the peak of the pulse) through the structure can be identified with the dwell time  $\tau_d$  (roughly the average amount of time that light spends within the structure). These two quantities are defined mathematically by  $\tau_g = d\phi/d\Omega$  where  $\phi$  is the phase of the transmitted, forward-going field and by  $\tau_d = U/P_i$  where  $P_i = \frac{1}{2} \epsilon_0 n_0 c |E_0|^2 A$  is the incident power. Thus the normalized group delay (that is,  $\tau_g/\tau_0$ , where  $\tau_0 = L/v$  is known as the "equal time") and the normalized stored energy (that is,  $U/U_0$ ) are identical quantities. The dependence of these quantities on the frequency detuning  $\Omega$  is shown in Fig. 8 for the case  $\kappa L = 4$ .

There have been many studies of slow- and fast-light effects involving FBGs. Some of these studies have been summarized in a review article by Longhi *et al.* [38], which concentrates on superluminal pulse propagation effects. Some of their results are shown in Fig. 9. This figure shows measured values as solid lines and predicted curves as dashed lines, although the agreement is so good that the dashed lines almost cannot be discerned. We see that, in agreement with the predictions of Fig. 8, there is a region of slow light outside of the band gap and a region of fast light inside the band gap. In this same paper, the authors report that by carefully engineering of the FBG structure they were able to achieve a fast-light velocity of  $5c$ .

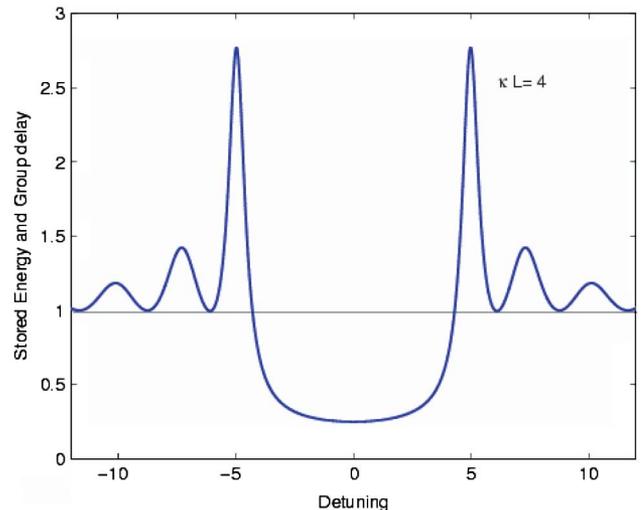


Fig. 8. Dependence of either the normalized group delay or the normalized stored energy of a FBG structure on the detuning of the incident light field from the Bragg frequency  $\omega_B$ . Reproduced with permission from [36].

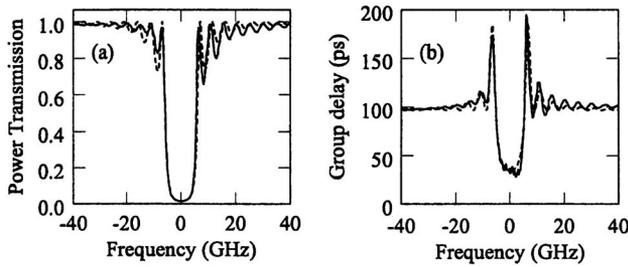


Fig. 9. (a) Power transmission through a FBG as a function of detuning from the Bragg resonance. (b) Group delay through this structure. Reproduced with permission from [38].

We now turn to the crucial issue of how the buildup of fields within a structural slow-light medium leads to a modification of its linear and nonlinear optical properties. These issues are in fact quite subtle, and we will not attempt to derive all of these results. Instead we will quote important results and refer the reader to the relevant literature. A particularly good summary is given in [39]. We quote results in terms of the “slow-down factor”  $S$ , which is the ratio of the effective group index of the structure to the mean refractive index of the structure. The effective group index can usually be interpreted by obvious analogy to the group index of material slow light given by Eq. (1). More generally it is defined by  $n_g^{(\text{eff})} = c/v_g^{(\text{eff})}$ , where  $v_g^{(\text{eff})} = L/\tau_g$ , where  $\tau_g$  is the group delay defined above.

Linear optical properties, such as attenuation and phase accumulation, tend to scale linearly with the slow-down factor  $S$ , reflecting the fact that the dominant influence of structuring a material is to increase the effective path length that light undergoes in passing through the material. Nonetheless, it has been pointed out [31] that for large values of  $S$  (typically values larger than about 30–50) there is a contribution to the attenuation that scales quadratically with  $S$ . This contribution arises from multiple scattering within the waveguide. Current work has emphasized means of engineering PhC waveguides to minimize these sources of loss [23]. We also note that careful experimental studies performed by Thevenaz and coworkers have confirmed that linear absorption scales with the group index for structural but not material slow light [40,41].

We next consider how the nonlinear optical process of the intensity-dependent phase accumulation (sometimes referred to as self-phase modulation) depends on the slow-down factor  $S$ . Here the dominant dependence is  $S^2$ , reflecting the fact that the light intensity is increased by one factor of  $S$ , while the effective pathlength is increased by another factor of  $S$ . But the details are much more subtle. The theory of this effect was developed by Bhat and Sipe [42] and is summarized in a recent review [39]. One finds that the nonlinear coefficient is given by [37]

$$\Gamma = \left( \frac{3 - S^{-2}}{2} \right) S^2 \gamma_0, \quad (12)$$

where  $\gamma_0$  is the nonlinear coefficient of the unstructured background material, which is proportional to the standard  $n_2$  coefficient. Here the prefactor results from the nonuniform field distribution within each unit cell of the FBG structure. Clearly, the prefactor reduced to unity for  $S = 1$  and takes on the value  $3/2$  for very large values of  $S$ . These predictions have been largely confirmed in a series of recent experiments [43–45].

Still different scaling laws are predicted for four-wave mixing processes. Here theory predicts [46] that the power efficiency should scale as  $S^4$ , and recent experiments have verified this dependence [47,48]. At first sight, it might seem surprising that the intensity-dependent phase accumulation scales as  $S^2$  whereas four-wave mixing scales as  $S^4$ , when both are in fact  $\chi^{(3)}$  processes. The reason for the different dependence is that one usually quotes the induced phase shift for the first case but the generated power in the second. Squaring the field amplitude necessarily entails squaring the functional dependence on  $S$ .

Higher-order nonlinear optical effects enhanced by structural slow-light have also been observed. For example, Corcoran *et al.* [49] have observed green light emission at the third-harmonic frequency through slow-light enhanced third-harmonic generation in silicon photonic-crystal waveguides pumped by a 1550 nm laser source.

#### 4. COMPARISON OF THE TWO CASES AND SUMMARY

In this article we have taken a close look at some of the similarities and some of the differences between using material resonances and structural resonances as a means to induce extreme values of the group velocity. In broad terms, one finds that there is extremely interesting physics contained in each approach. Also, both approaches offer good opportunities for the development of applications. In general terms, material slow light offers better opportunity to control the group velocity of light in real time, whereas structural slow light offers a more robust platform for developing applications.

Even at the level of simple kinematics, there are significant differences between the two approaches. For material slow light, the energy density  $u$ , intensity  $S$ , and group velocity  $v_g$  obey the intuitively pleasing relation [Eq. (5)]  $S = uv_g$  is obeyed. We stress that there is no such analogous relation for the case of structural slow light. The reason is that the energy density has contributions from both forward- and backward-going streams of radiation. However, the net flow of energy is constrained by energy conservation to be equal to that of the incident light field.

A consequence of this sort of behavior is that there is no enhancement of the electric field strength within a material slow light material, whereas there is an increase for the case of structural slow light. As a result, nonlinear optical processes are inherently enhanced through use of a structural slow light material, but not for a material slow light material. The (real) enhancements of nonlinear effects for the case of material slow light presumably result from the fact that both nonlinear response and dispersion of the refractive index are enhanced by working close to resonance.

Although this paper was written as a review article, an important conclusion is that not all of the subtleties of slow and fast light are currently understood. There are still enormously interesting questions to be addressed both at the conceptual level and especially in terms of developing applications of slow light.

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- Especially intriguing are the consequences of Eq. (5) for circumstances under which the group velocity  $v_g$  becomes negative, implying a negative value of the Poynting vector magnitude  $S$ . In such circumstances, it is considered more useful to describe the situation in terms of an energy velocity rather than a group velocity. Details can be found in [3]. A negative value of the energy velocity would imply the flow of energy back toward the source. Some authors have argued that negative values of the energy velocity are physically meaningful [50], whereas others do not [3]. At present, there seems to be no complete understanding of this issue.
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