Quantum information with structured light

Mohammad Mirhosseini

Robert W. Boyd's Quantum Photonics Group

The Institute of Optics, University of Rochester, Rochester, NY

Outline

- Quantum key distribution with orbital angular momentum of photons
- Direct measurement of quantum wave function
- Measurement of Wigner distribution of twisted photons

Orbital angular momentum of light

- Circularly polarized light carries angular momentum (Spin angular momentum).

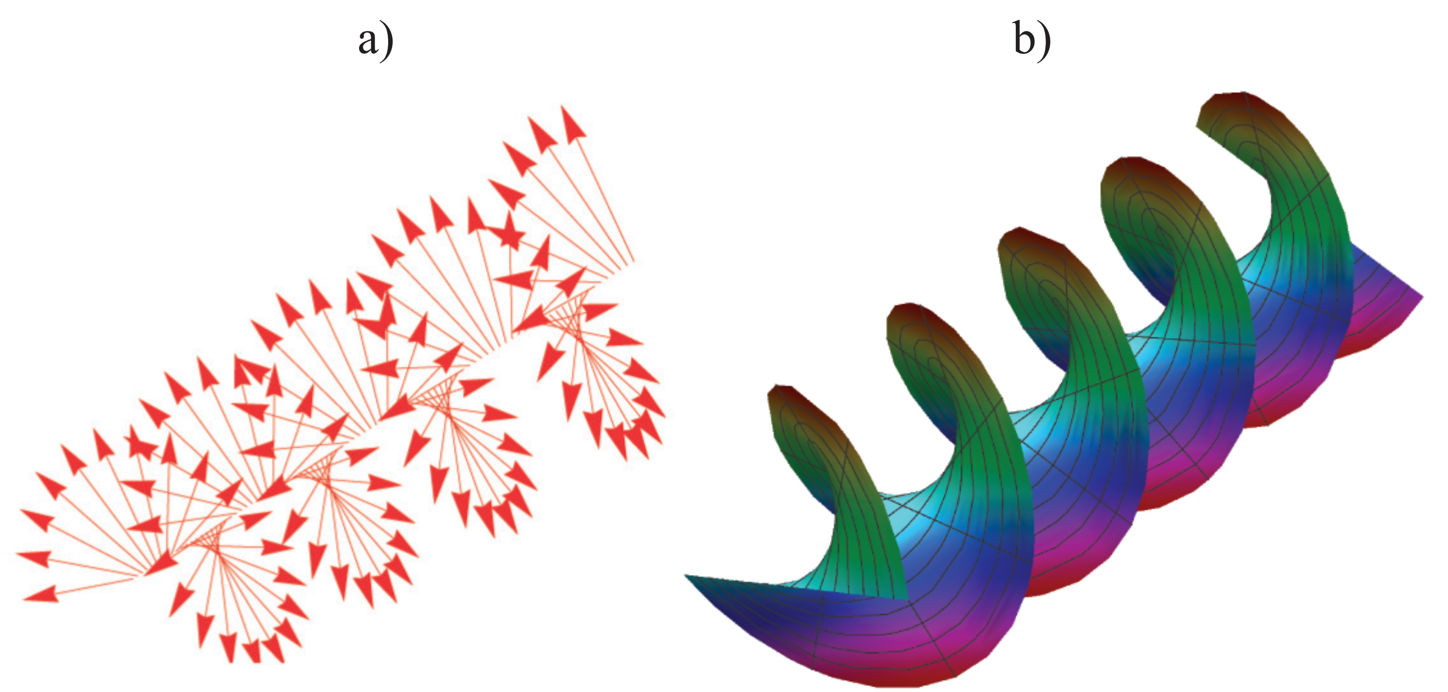
Poynting, Proc. R. Soc. London, Ser. A, (1909)

Beth, Phys. Rev., (1936)

- A light beam with helical phase also carries orbital angular momentum (OAM).

Allen et al., Phys. Rev. A, (1992)

$$e^{i\ell\varphi} \rightarrow \ell\hbar$$

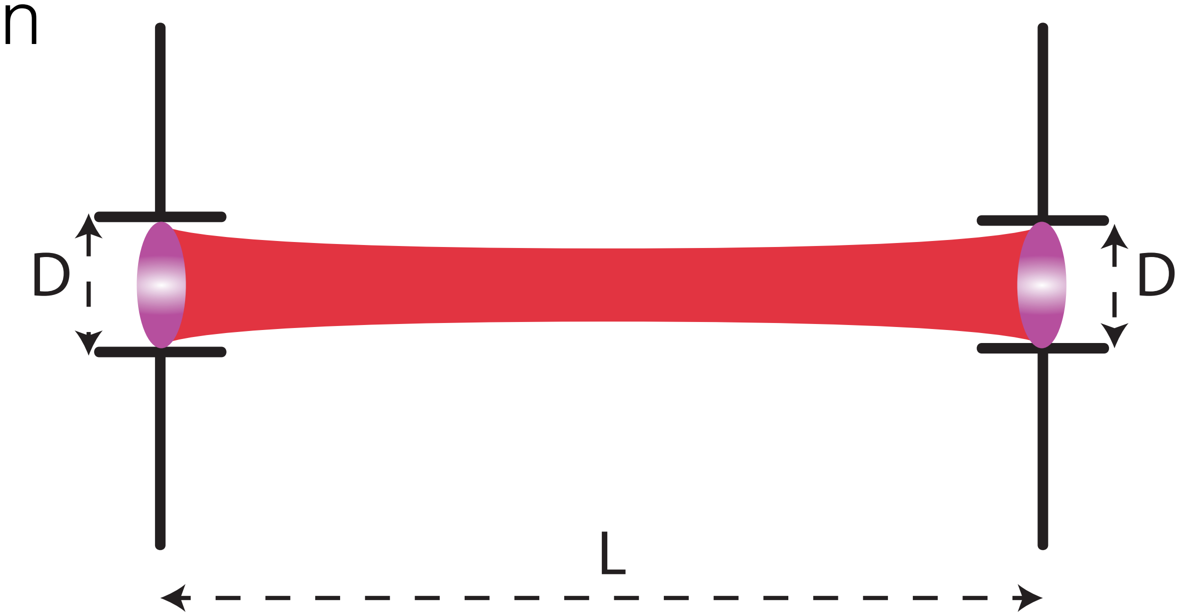


- The large Hilbert space of OAM is ideal for encoding quantum information

Mair et al., Nature, (2001)

Communication modes

- Singular value decomposition provides an optimal set of modes for free-space communication



$$\hat{G}|a_n\rangle = g_n|b_n\rangle$$

$$\sum_n |g_n|^2 = \frac{A_R A_T}{(\lambda L)^2}$$

$$\left\langle \mathbf{r} \middle| \hat{G} \middle| \mathbf{r}' \right\rangle \equiv G(\mathbf{r},\mathbf{r}') = \frac{P_R(\mathbf{r})P_T(\mathbf{r}')}{i\lambda z} \exp\left(ikz + ik\left|\mathbf{r}-\mathbf{r}'\right|^2/2z\right)$$

$$\hat{G} = \sum_n g_n\,|b_n\rangle\langle a_n|$$

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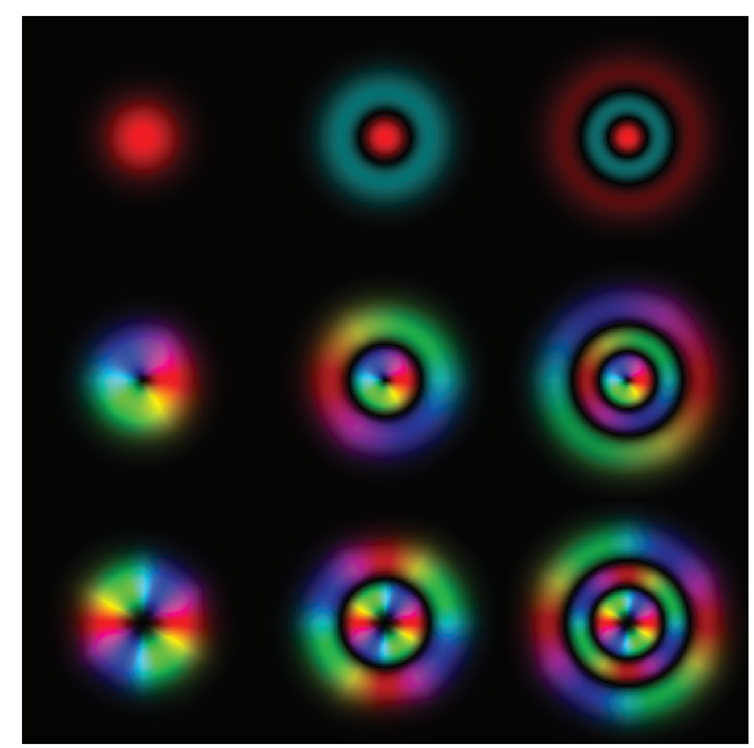

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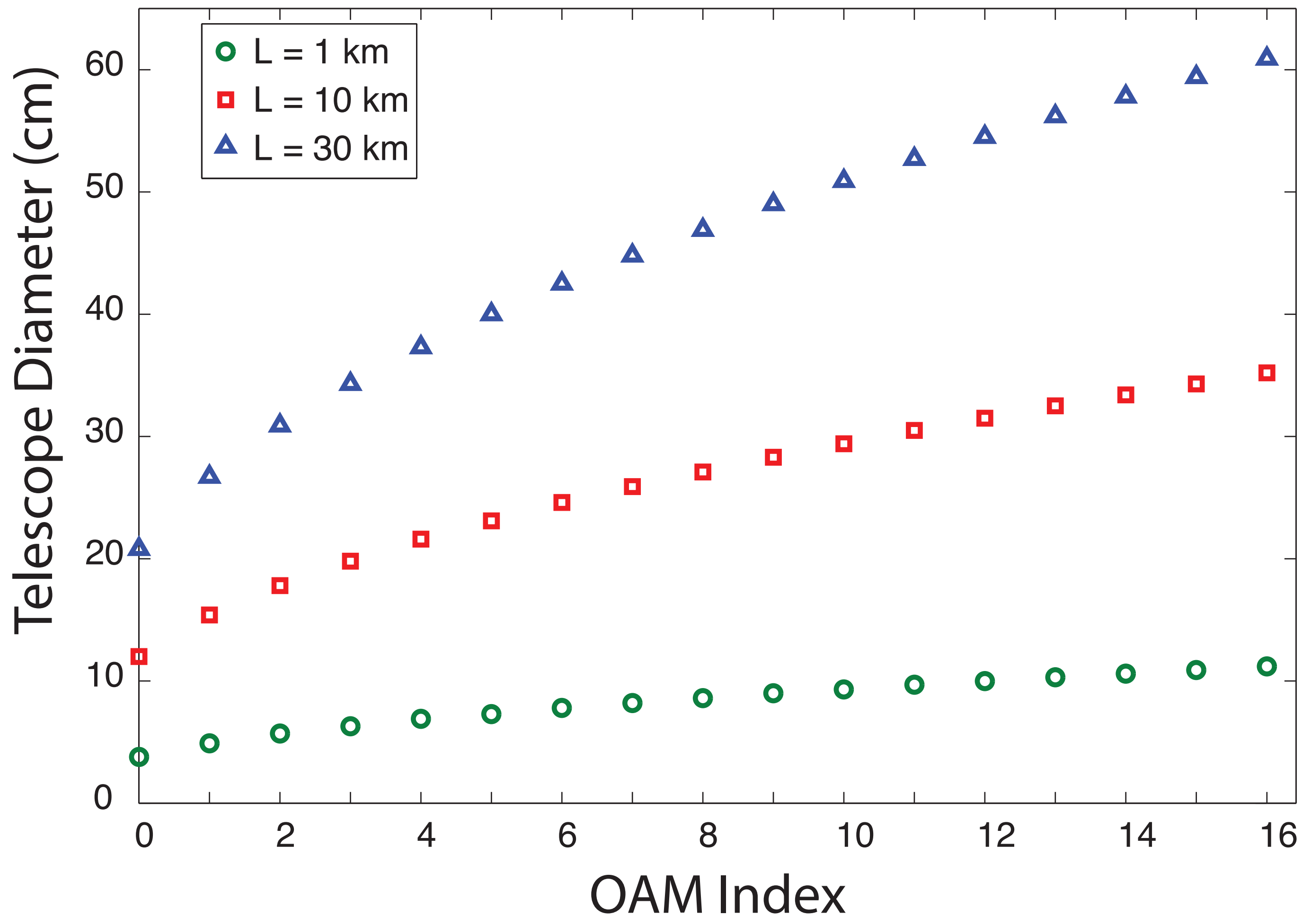
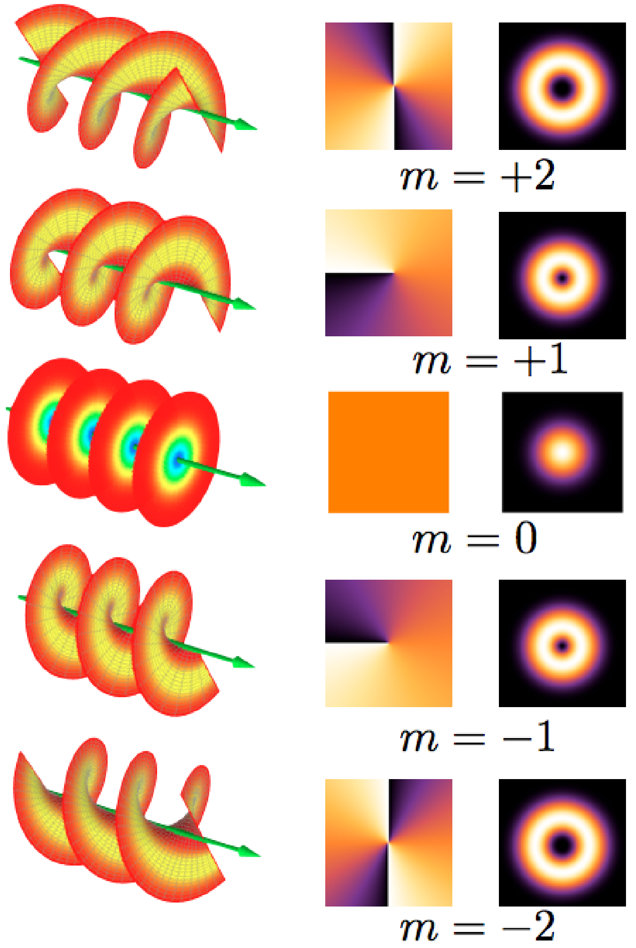
- The results of SVD gives us LG modes modes for a system with round apertures.

- LG modes are eigenstate of OAM

Communication with OAM modes

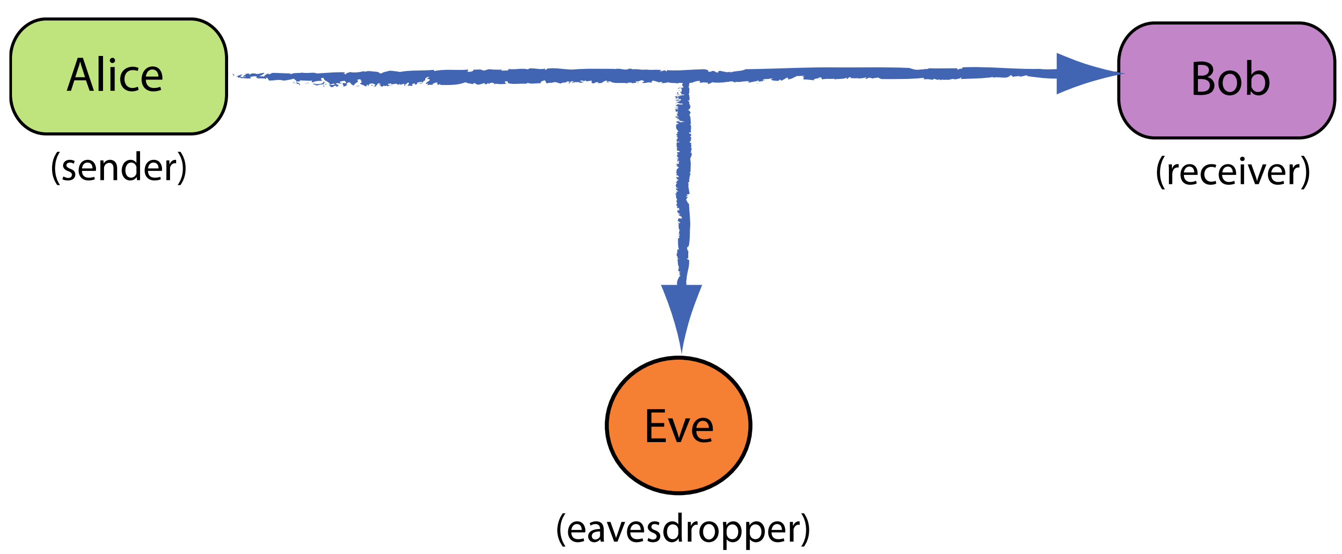
- OAM communication is feasible with available telescopes.





Quantum communication

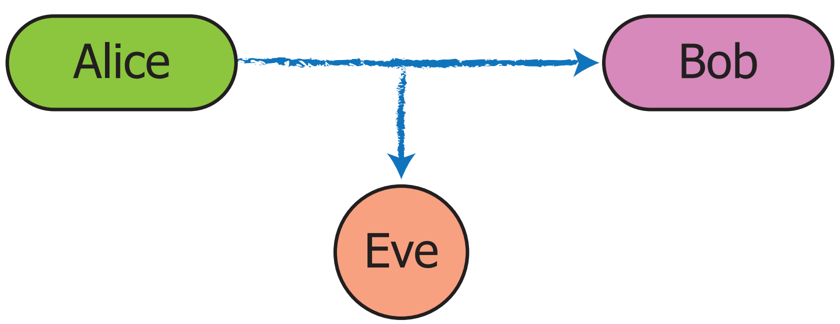
- Quantum key distribution (QKD) relies on fundamental properties of quantum physics for establishing a secure key.
- This is particularly exciting, since it has been shown that conventional public cryptography can be broken using quantum computers.



BB84 Protocol

- QKD schemes traditionally use the polarization of a photon for encoding information.

	Basis	1	0
HV	✛	↔	↕
DA	×	⤢	⤡

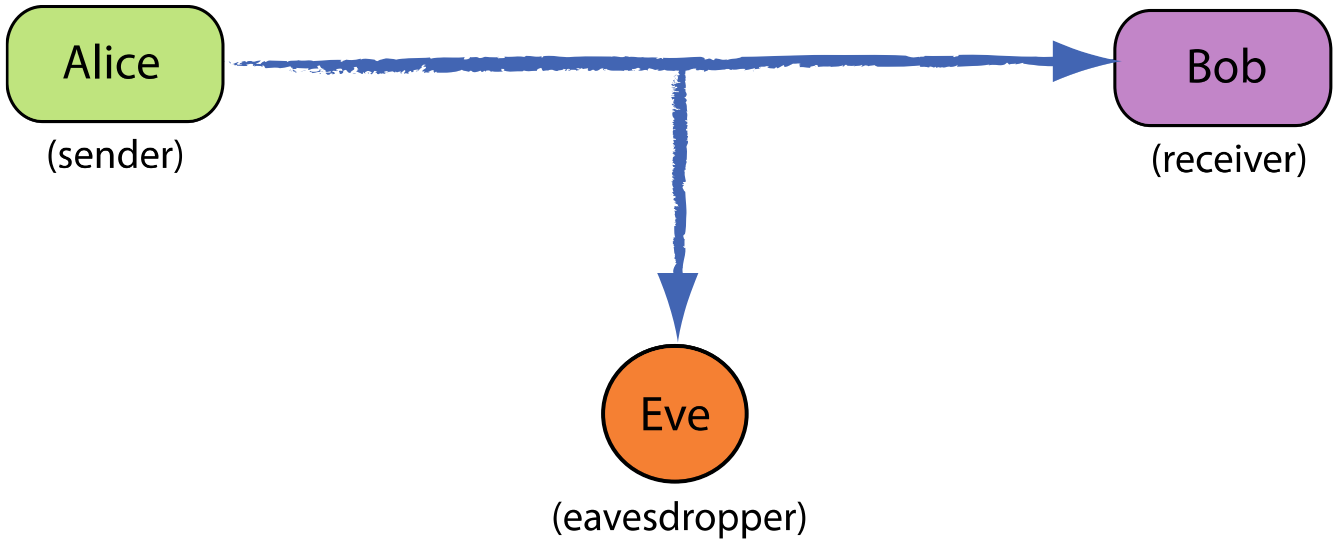


Alice's Random Bit	0	1	1	0	1	0	0	1
Alice's Random Basis	✛	✛	×	✛	×	×	×	✛
Alice's Sent Symbol	↕	↔	⤢	↕	⤢	⤡	⤡	↔
Bob's Random Basis	✛	×	×	×	✛	×	✛	✛
Bob's Measured Symbol	↕	⤡	⤢	⤡	↔	⤡	↔	↔
Bob's Measured Bit	0	0	1	0	1	0	1	1
Secret shared Key	0		1			0		1

Bennett & Brassard, *Proc. IEEE Int. Conf.*, Bangalore (1984)

QKD security

- Quantum mechanics dictates that the state of an unknown photon can not be determined without disturbing it.
- By checking for the disturbance in the transmitted states, Alice and Bob can establish an upper bound on the eavesdropping occurring in their communication channel.



QKD with OAM modes

- Multidimensional system offer a larger information capacity and a higher error threshold

Information capacity ~ log(d)

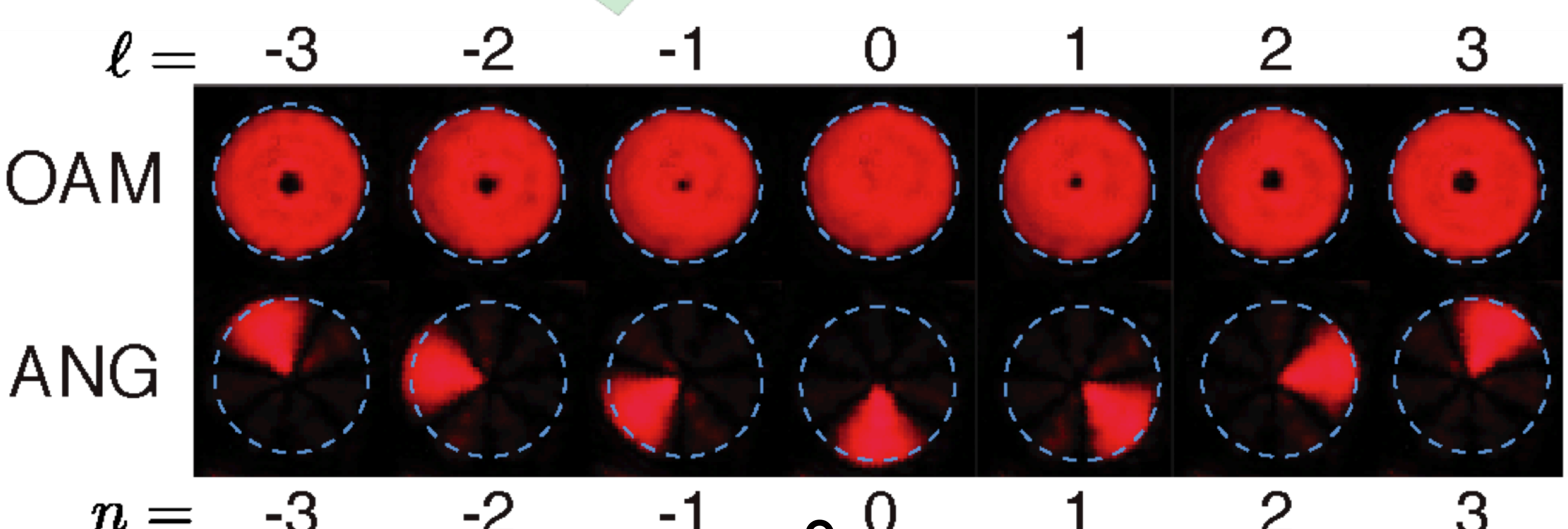
Cerf et al, *Physical Review Letters* (2002)

d	2	3	4	5	8
Error (%)	14.64	21.13	25	27.64	32.31
I (bits)	1	1.58	2	2.32	3

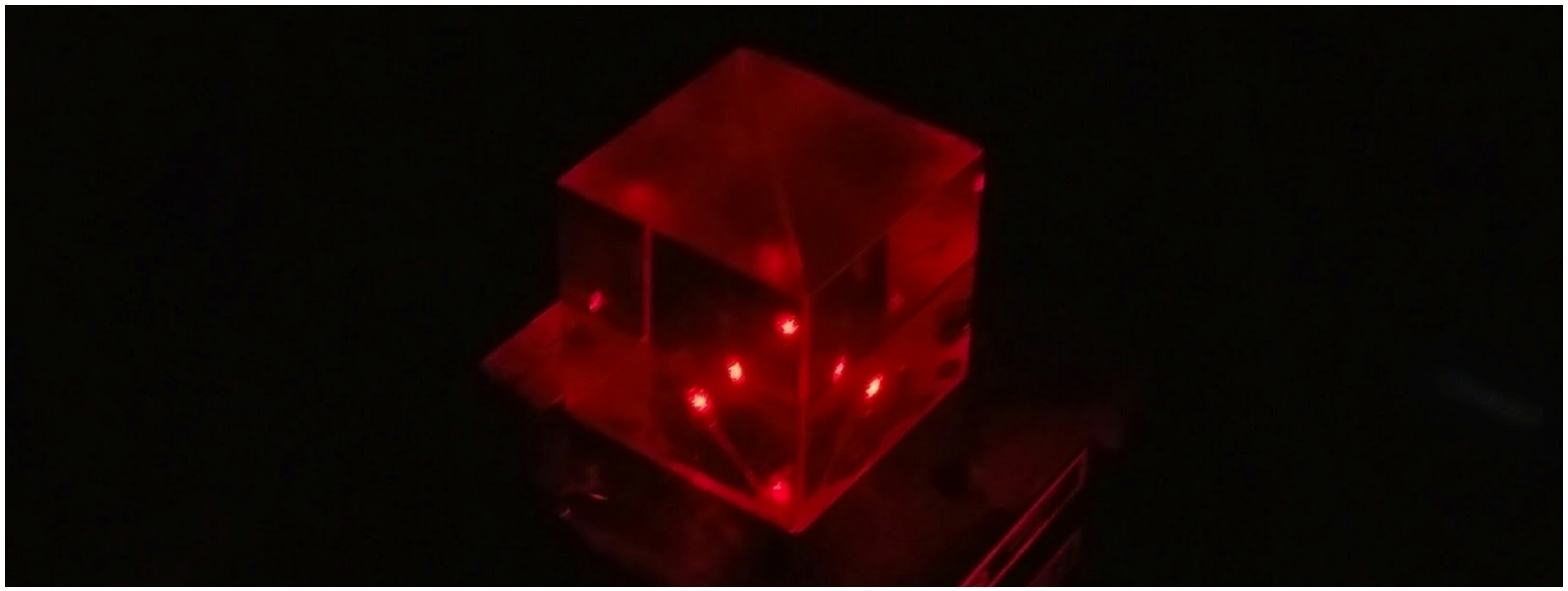
- ANG basis is unbiased with respect to the OAM basis.

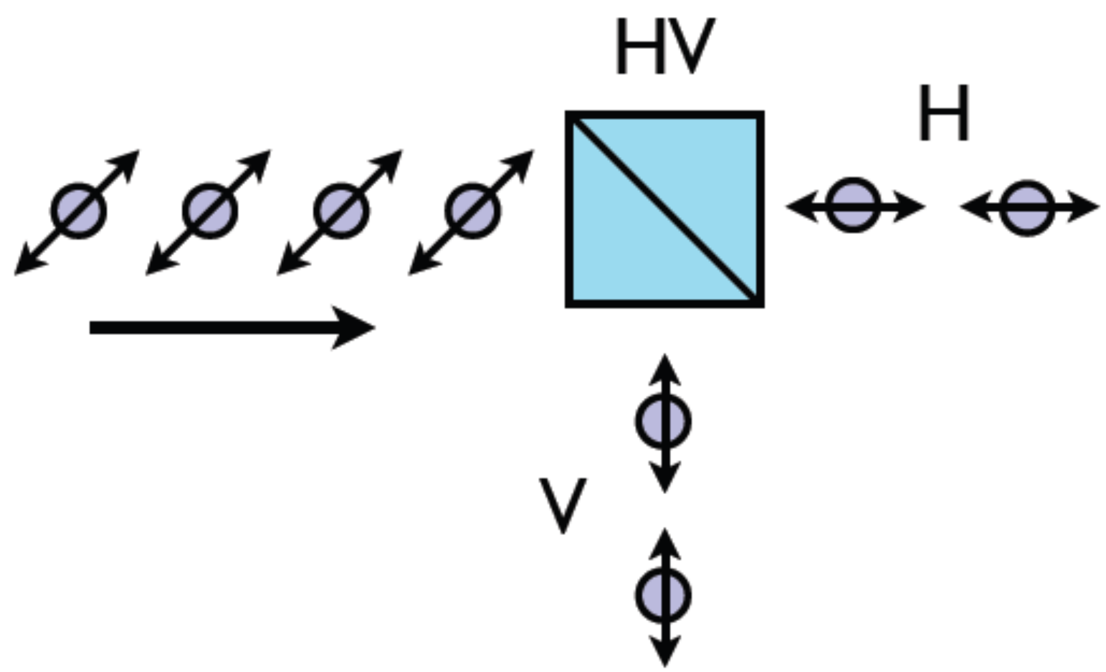
$$\Psi^n_{\mathrm{ANG}} = \frac{1}{\sqrt{d}} \sum_{\ell=-N}^{N} \Psi^\ell_{\mathrm{OAM}} \exp\left(\frac{i2\pi n\ell}{d}\right)$$

$$\langle \Psi^n_{\mathrm{ANG}} | \Psi^\ell_{\mathrm{OAM}} \rangle = \ 1/d \qquad \forall \{n, \ell\}$$



Single-photon measurement of polarization



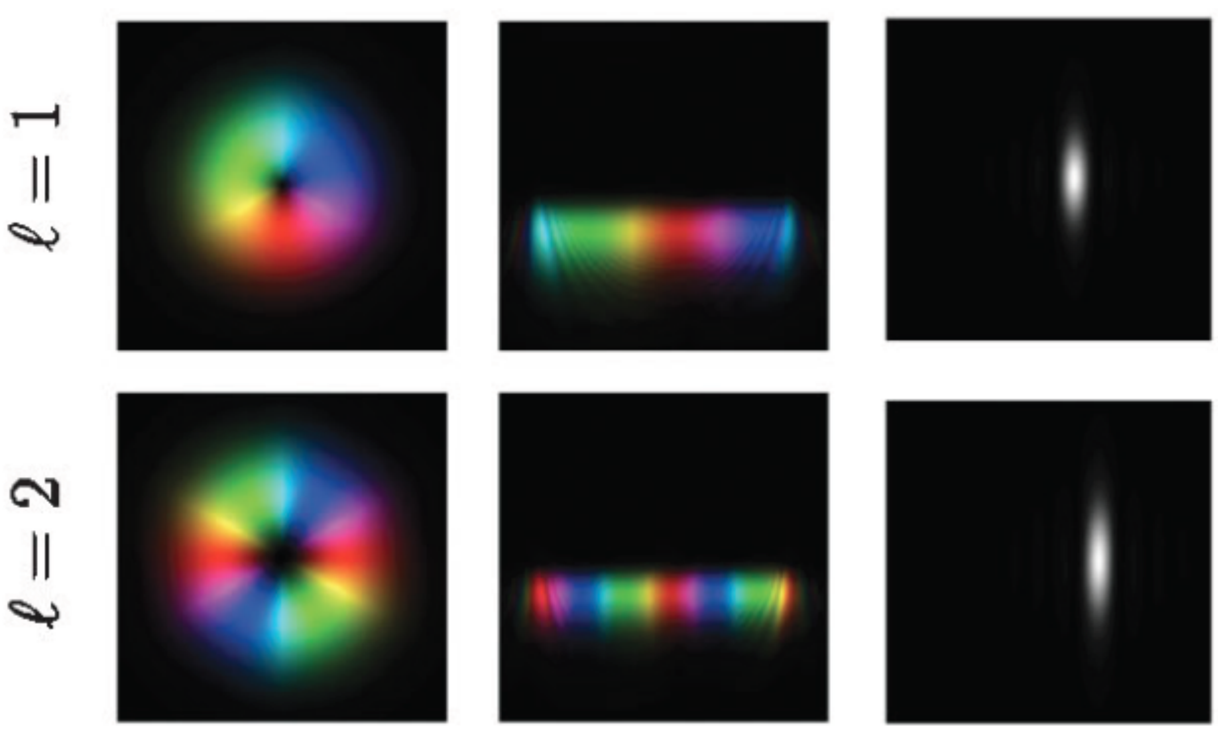


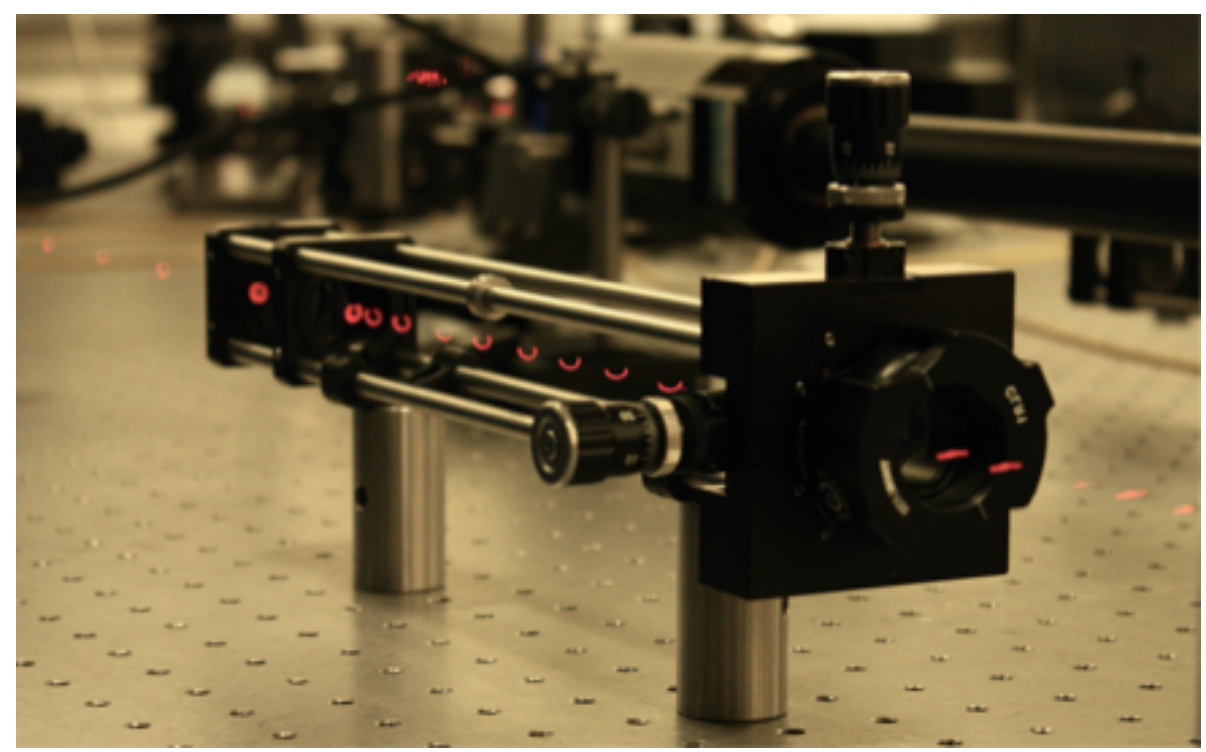
Mode sorting with coordinate transformation

- The polar coordinate is mapped to the Cartesian.
- OAM modes are mapped to plane waves.

$$v = a \arctan(y/x) \qquad u = -a \ln(\sqrt{x^2 + y^2}/b)$$

$$e^{i\ell\phi} \Rightarrow e^{i\ell v/a}$$



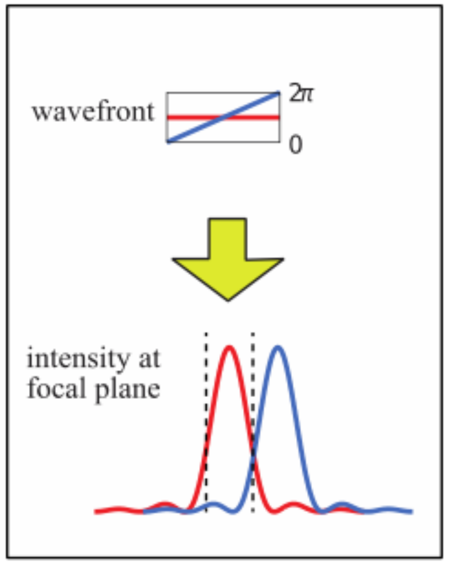


Berkhout et al., *Physical Review Letters* (2010)

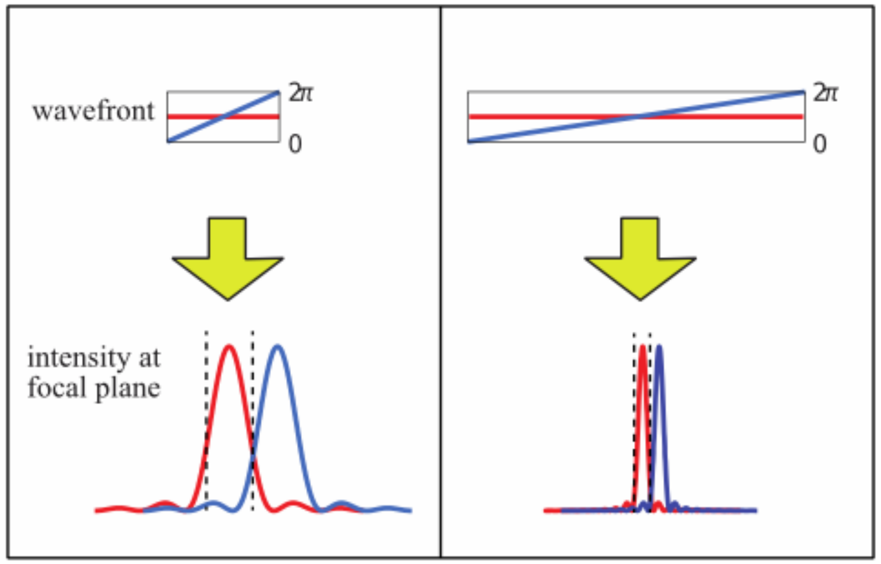
Lavery et al., *Optics Express* (2012)

Enhanced mode sorting with diffractive copying

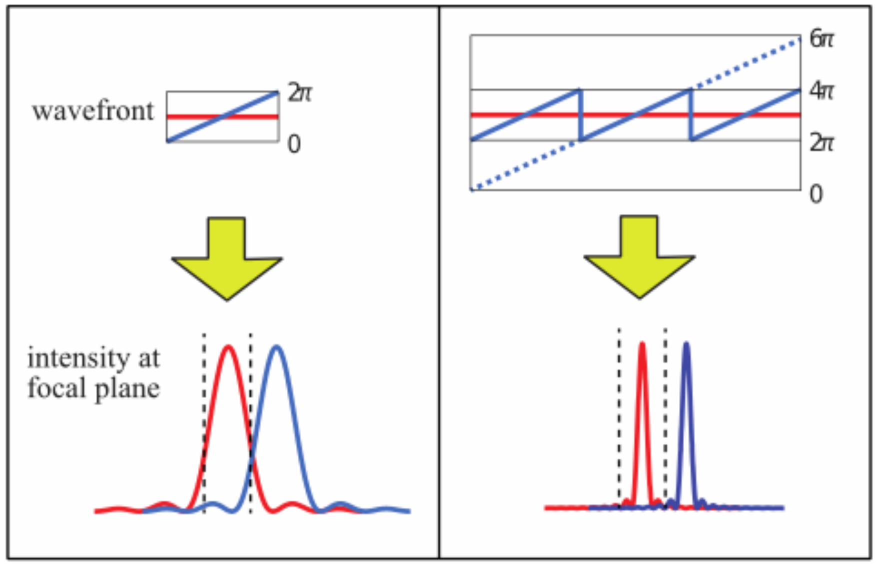
$$E(x) \propto \frac{\sin(\pi[\frac{ax}{\lambda f} - \ell])}{\frac{\pi x}{\lambda f}}$$

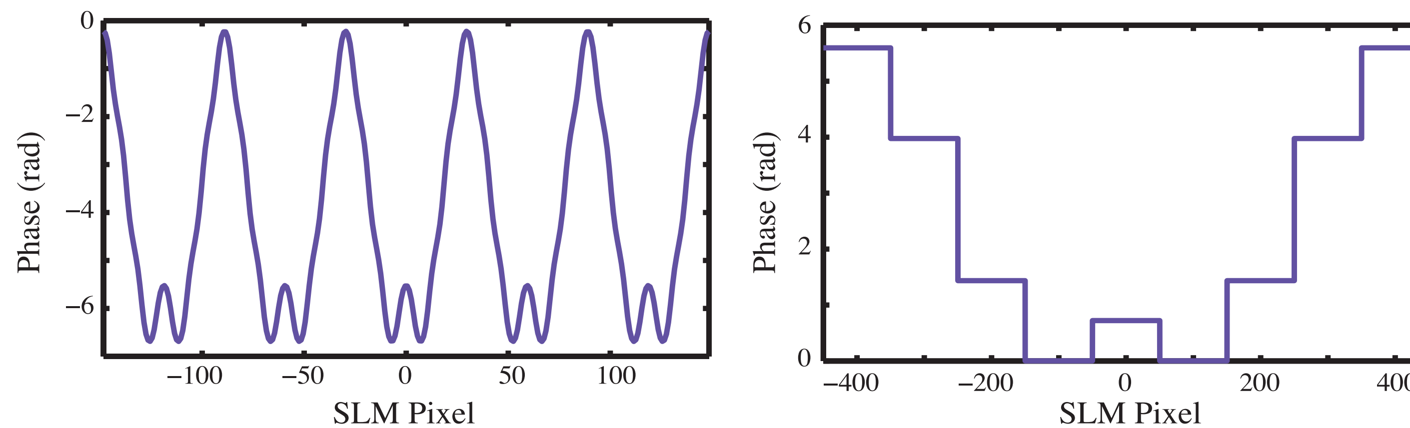


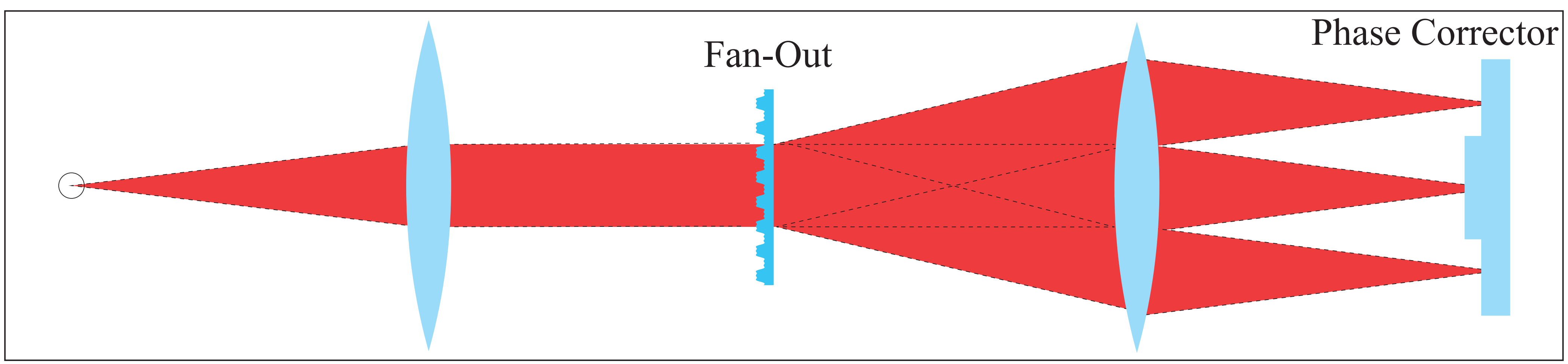
$$\frac{\int_{-1/2}^{1/2} \left|\frac{\sin(\pi x)}{\pi x}\right|^2 dx}{\int_{-\infty}^{\infty} \left|\frac{\sin(\pi x)}{\pi x}\right|^2 dx} = 77.4\%$$



Enhanced mode sorting with diffractive copying



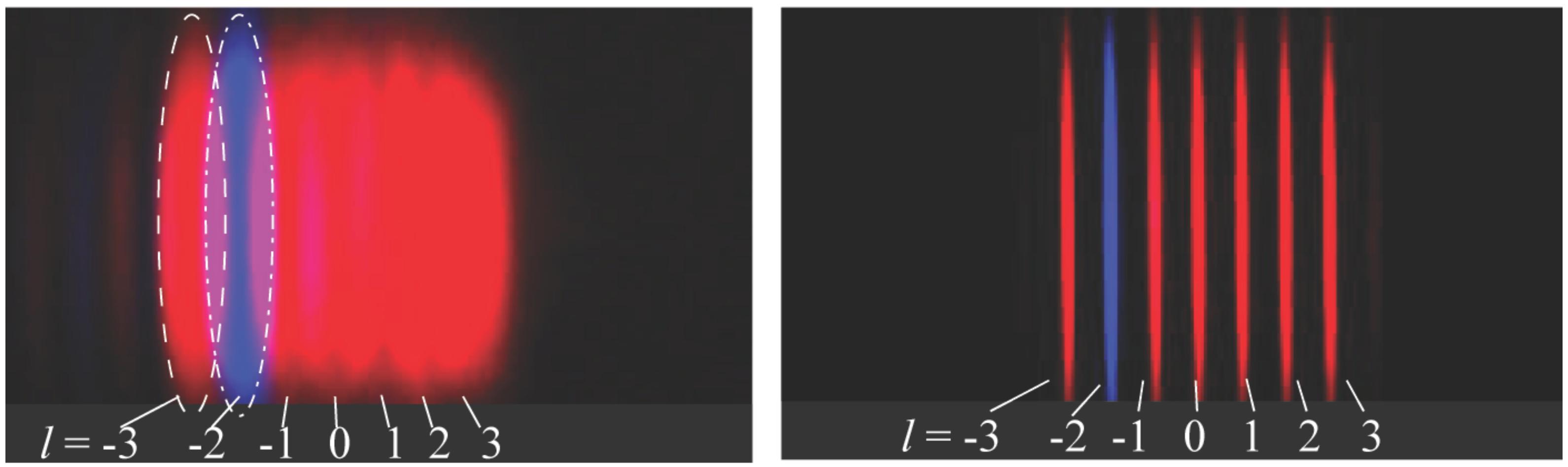


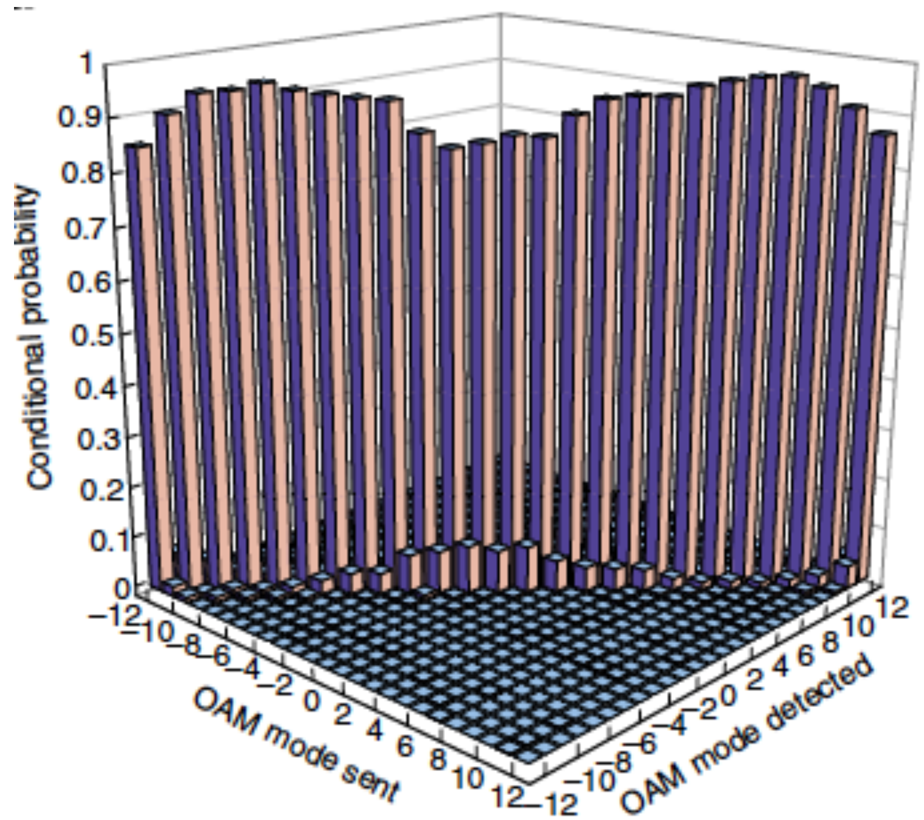


Prongue et al., *Applied Optics* (1992)

O'Sullivan et al., *Optics Express* (2012)

Experimental results

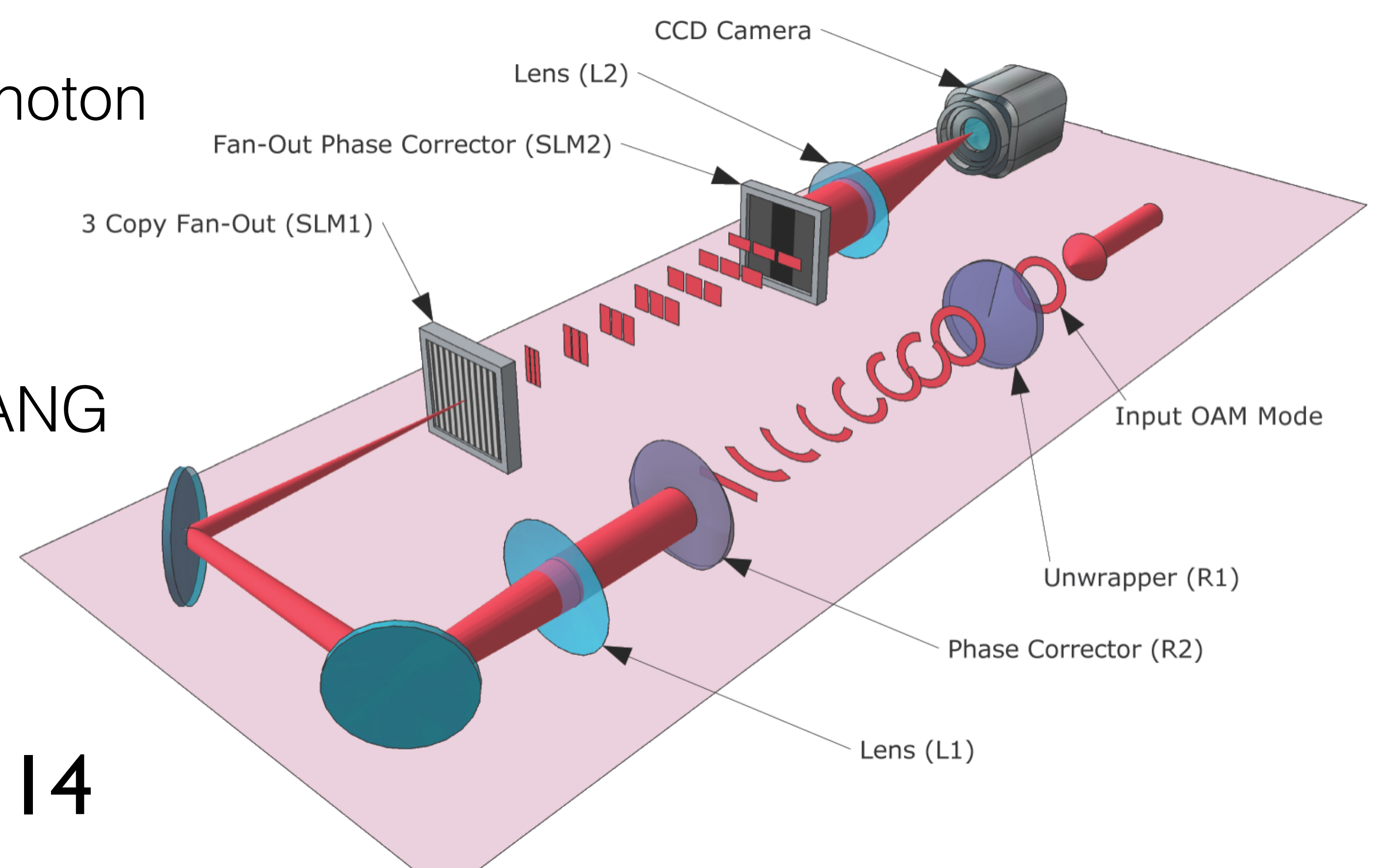




- Cross-talk reduces from 23% to 8%.

- For a set of 25 modes, we calculate information capacity of 4.18 bits per photon (4.64 is the upper bound for ideal separation).

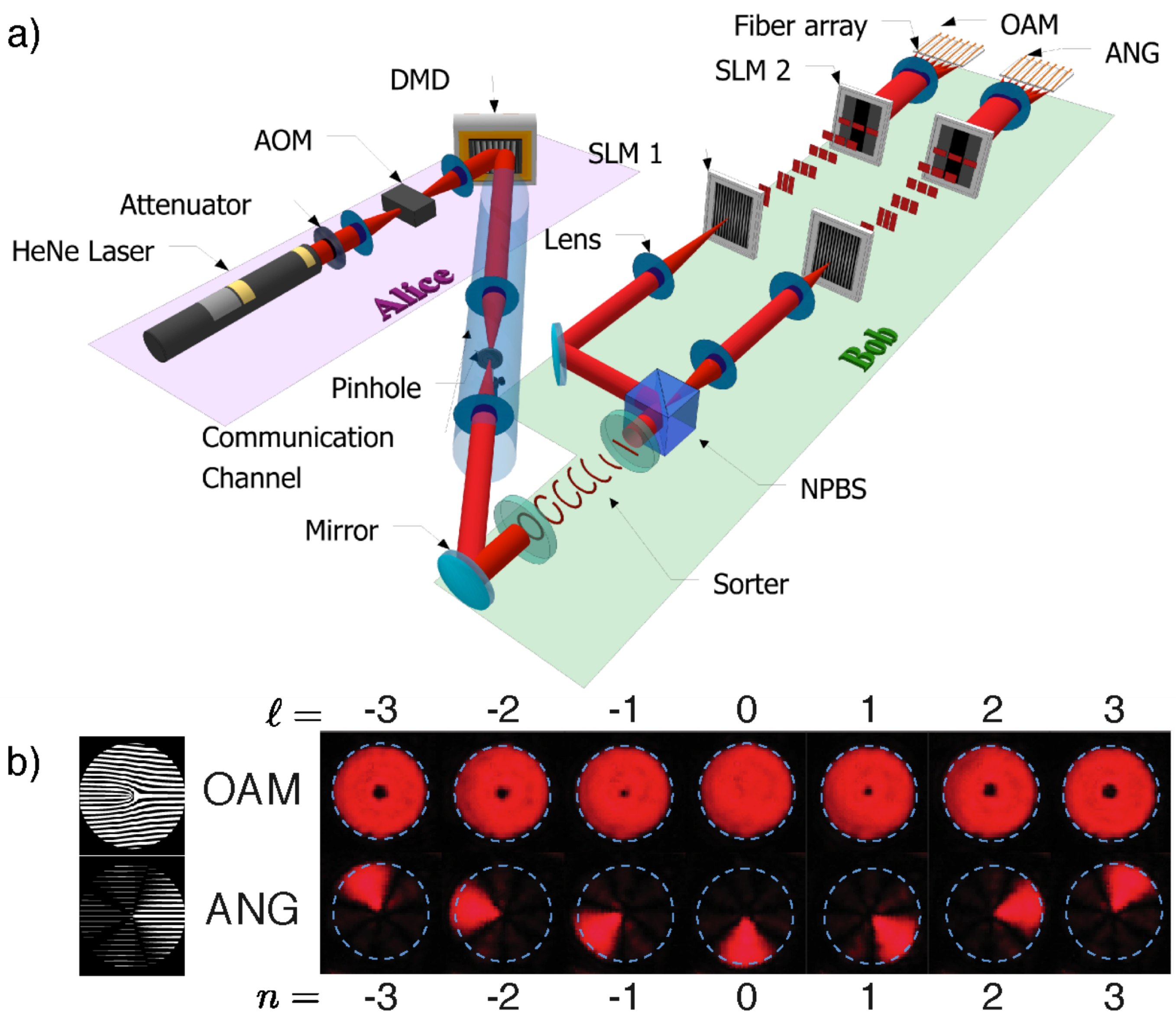
- A small variation of this method sorts ANG modes



Mirhosseini et al., *Nature Communications* (2013)

OAM QKD

- We use d = 7 modes.
- Single photons approximated as weak coherent pulses.
- 0.1 photons per each 125 ns pulse.
- Beam-splitter passively chooses a random basis.



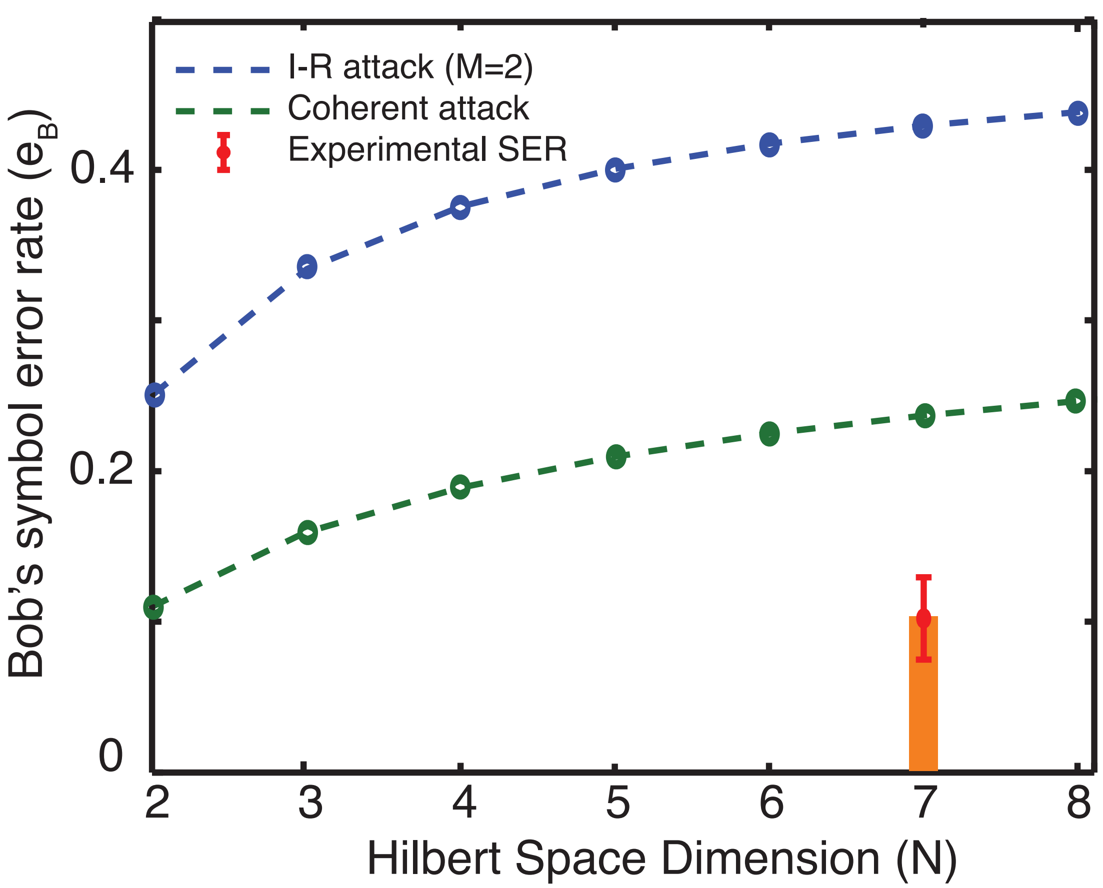
Experimental Results

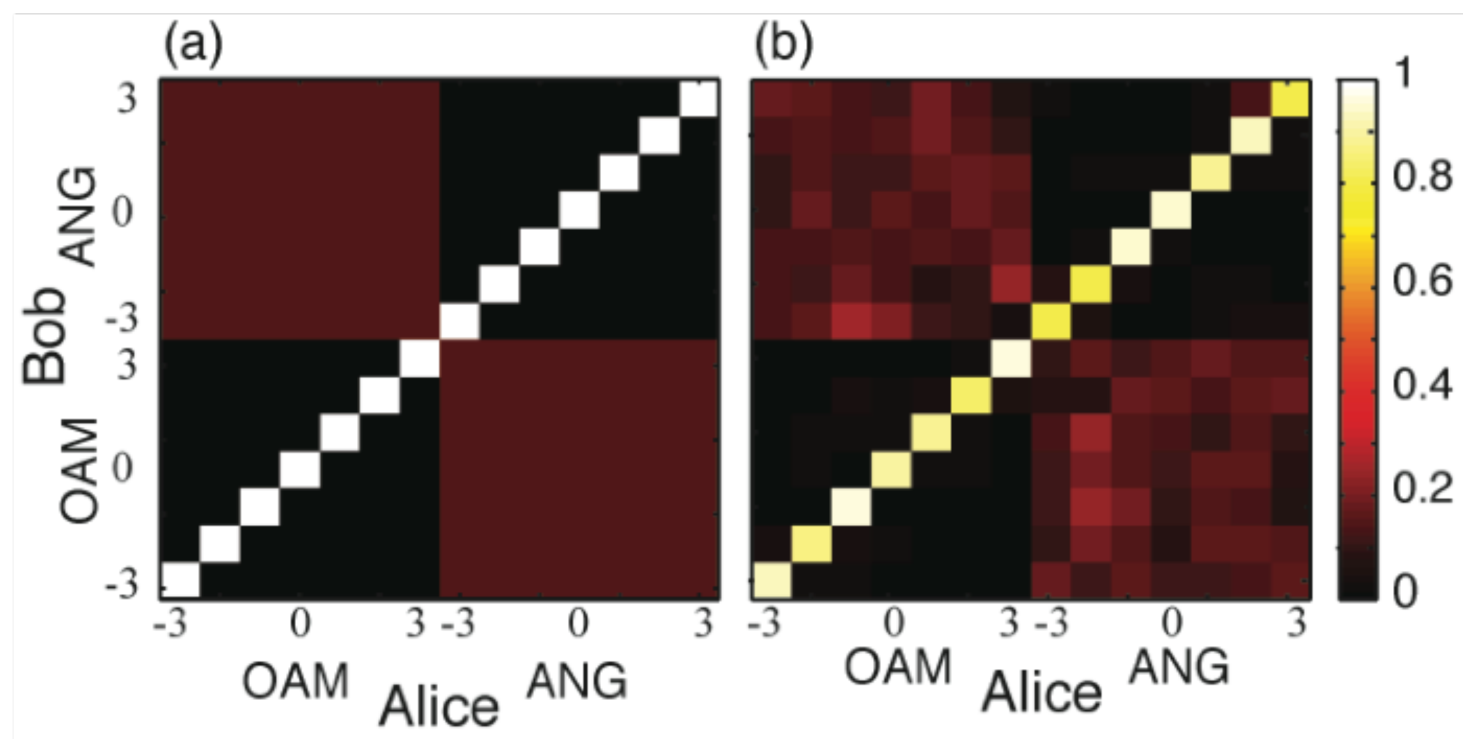
- Mutual information of 2.07 bits per sifted photon for 7 modes

  $I(X;Y) = H(Y) – H(Y|X)$

- 10.5% error rate in the sifted key is low enough to prove security against intercept resend attacks.

- $R_{net} = R_{sift}[I_{AB}\text{-}\max(I_{AE,}\ I_{AB})] = 1.7$



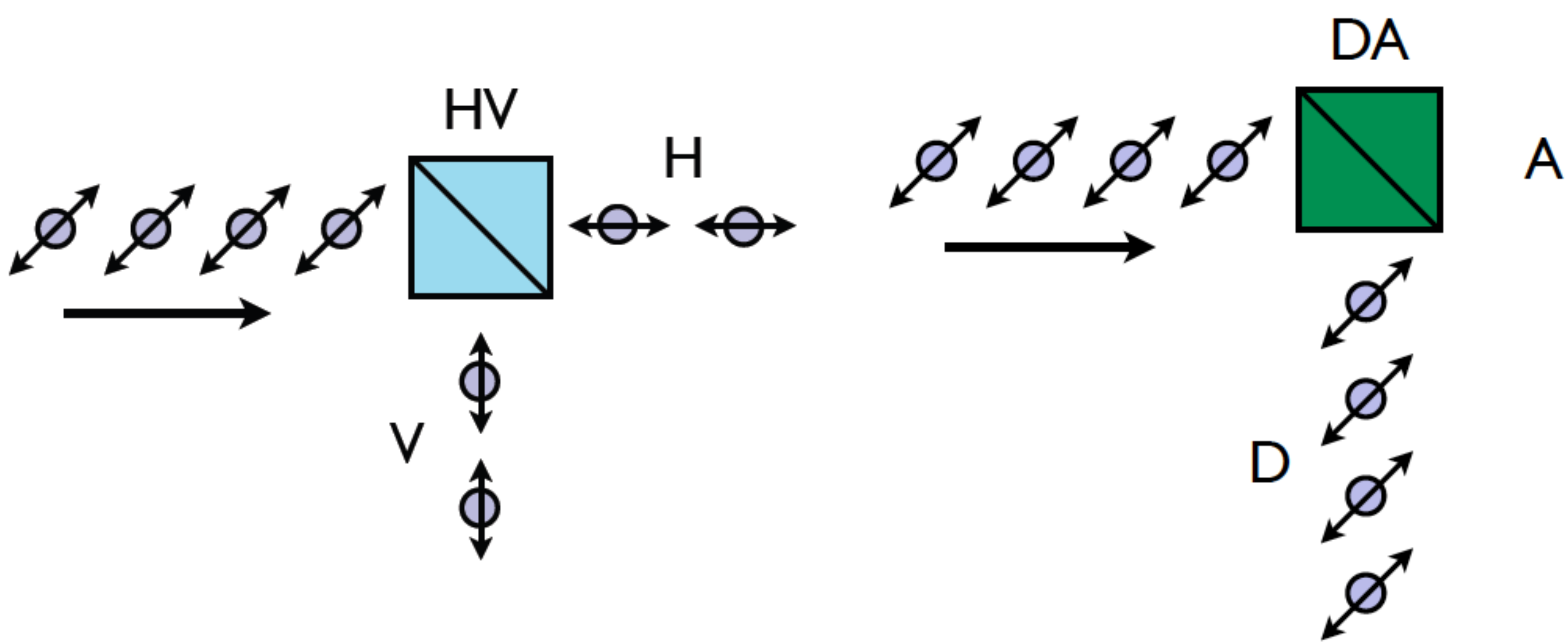


Mirhosseini et al. , *New Journal of Physics* (2015), selected by editors as the 2015 top highlighted paper in Quantum Physics

Limitations of projective measurements

- Characterizing the state is a crucial part of quantum protocols.
- Full characterization of the state requires measurements in multiple basis.

$$|\psi\rangle = a\,|H\rangle + b\,|V\rangle$$



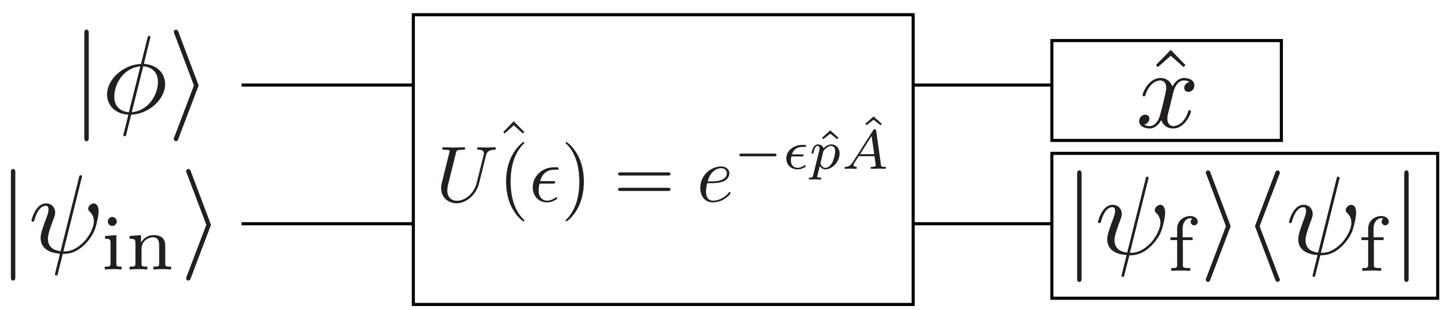
Weak measurements

- Weak unitary interaction of the quantum system with the pointer, that is followed by the post selection.

- It is possible to extract partial information about the system without completely disturbing it.

Aharonov, Albert,& Vaidman, PRL, 60(14), 1988

$$A_w = \frac{\langle \Psi_f | \hat{A} | \Psi_{in} \rangle}{\langle \Psi_f | \Psi_{in} \rangle}$$



- Weak values can be used to directly find the quantum state.

$$\langle \pi_x \rangle_{\mathrm{W}} = \frac{\langle p | x \rangle \; \langle x | \Psi \rangle}{\langle p | \Psi \rangle}$$

$$= \frac{e^{ipx/\hbar}\,\Psi(x)}{\Phi(p)}$$

J. S. Lundeen et al. *Nature*, *474*(7350), 2011.

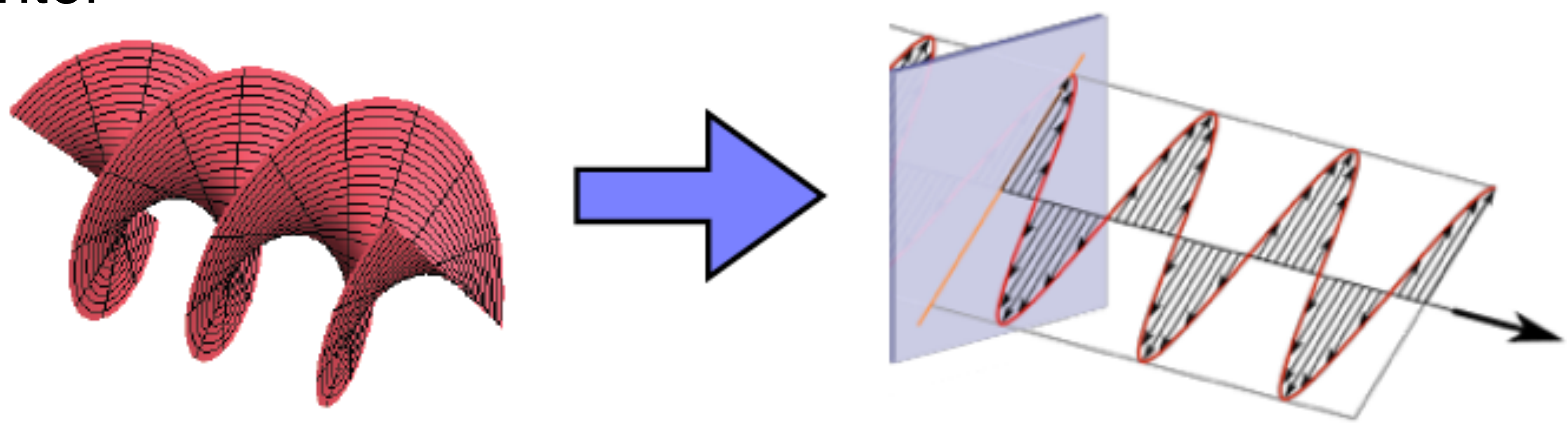
Direct measurement

$$A_w = \frac{\langle \Psi_f | \hat{A} | \Psi_{in} \rangle}{\langle \Psi_f | \Psi_{in} \rangle}$$

- How do we measure the OAM state vector?

$$|\Psi\rangle = \sum_\ell a_\ell \, |\ell\rangle \qquad c\,|\Psi\rangle = c\sum_\ell |\ell\rangle\langle\ell|\Psi\rangle = \sum_\ell |\ell\rangle \boxed{\frac{\langle\theta_0|\ell\rangle\langle\ell|\Psi\rangle}{\langle\theta_0|\Psi\rangle}} = \sum_\ell \langle\pi_\ell\rangle_{\mathrm{w}} \, |\ell\rangle$$

- Couple photon's OAM to their polarization —> measurement pointer

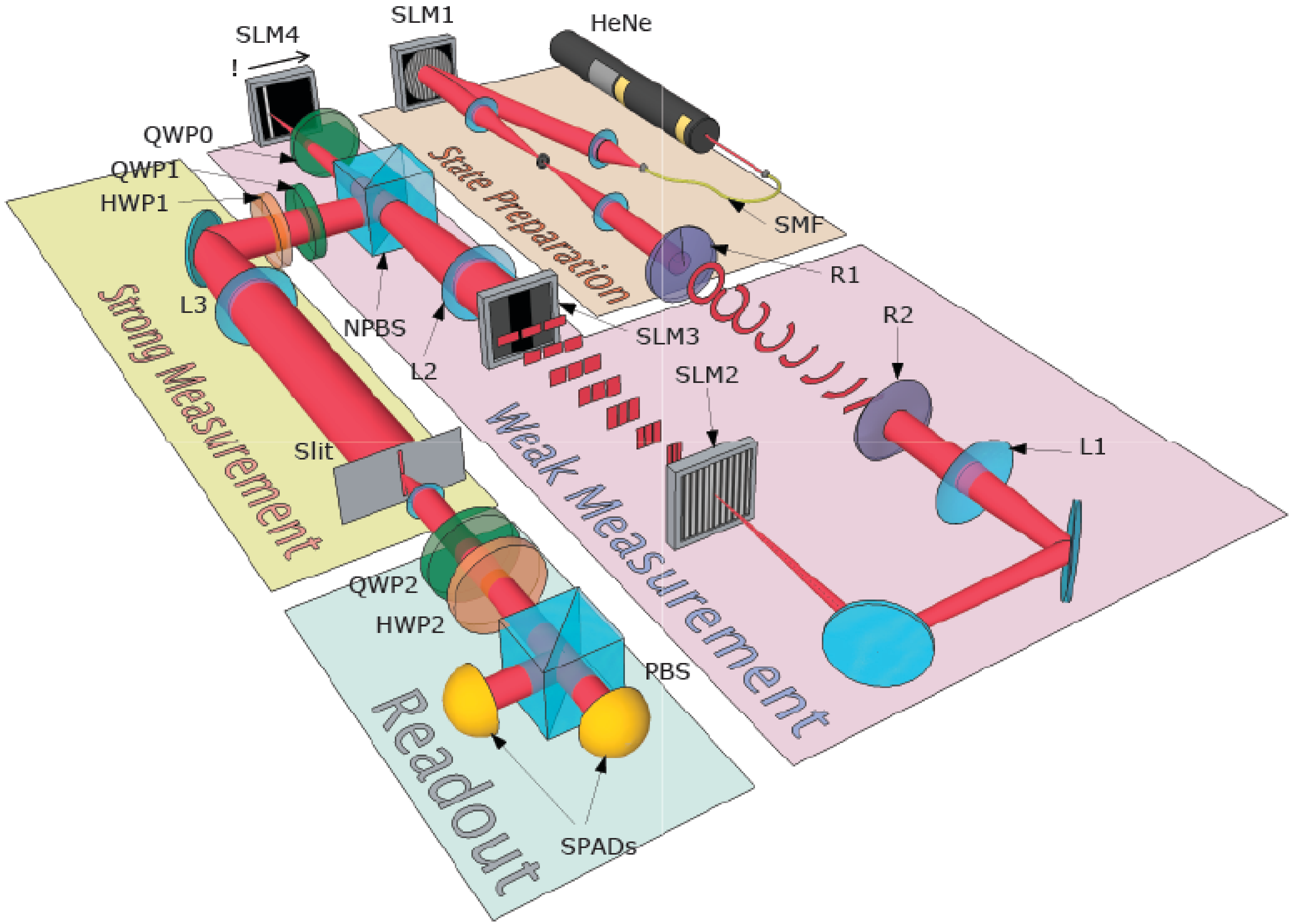


- Weak measurement of OAM—> *rotate* the polarization of an OAM component by a *small* angle

- Strong measurement of angle —> *post-select* on an angular wedge

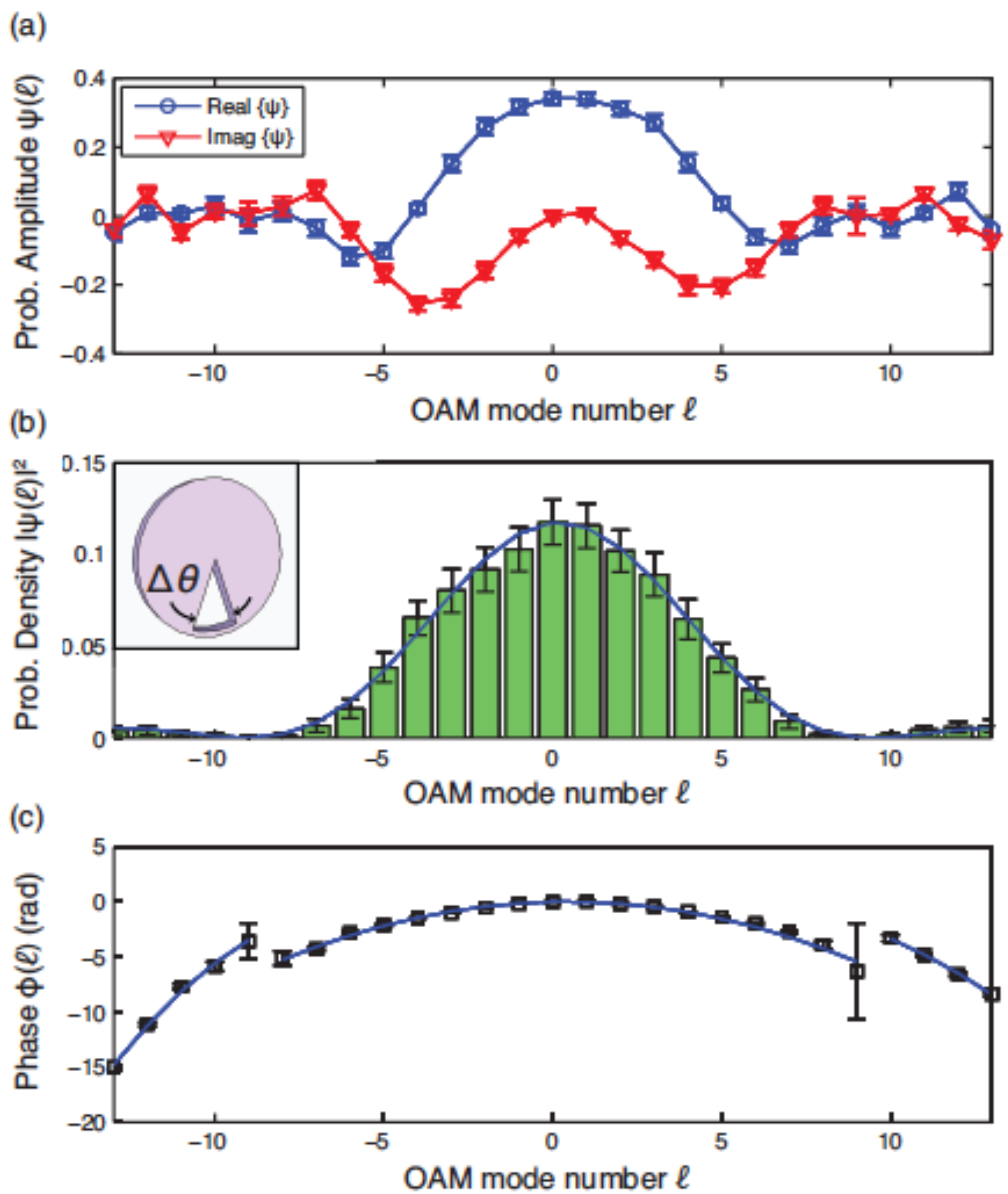
M. Malik, **M. Mirhosseini,** et al. *Nature Communications*, *5*, 3115, 2014.

Experimental setup

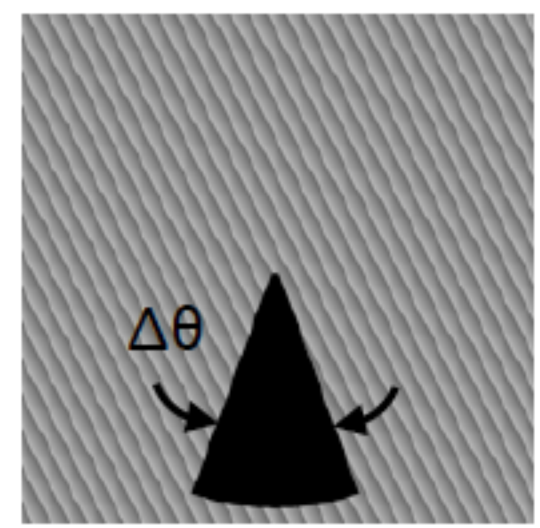


M. Malik, **M. Mirhosseini,** et al. *Nature Communications*, *5*, 3115, 2014.

Direct measurement of OAM state vector



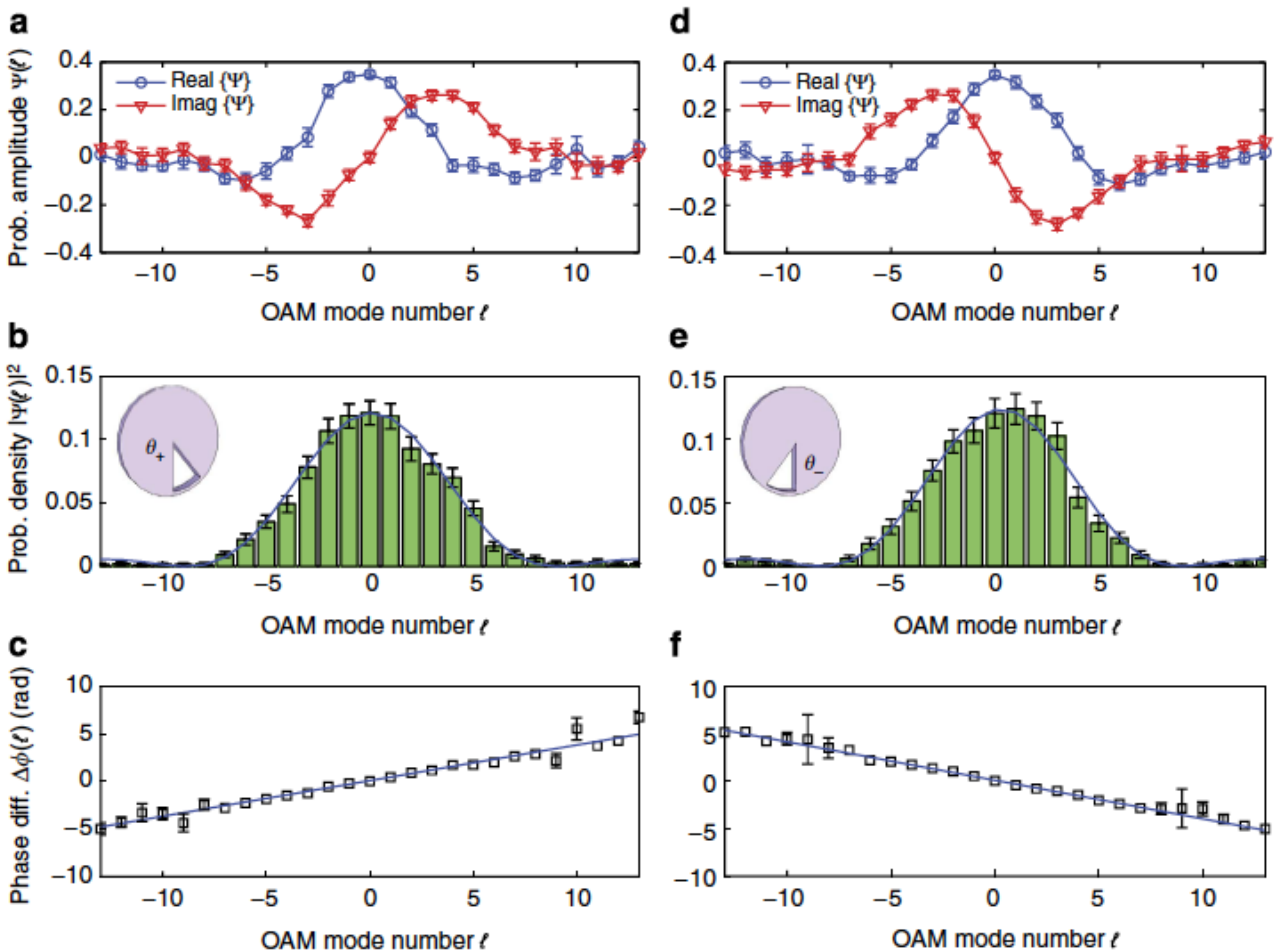
- We prepare a test wave function shaped as an angular wedged with



$$|\Psi\rangle = \sum_{\ell} a_\ell |\ell\rangle$$

$$a_\ell = \frac{\sqrt{2\pi}}{d}\operatorname{sinc}\left(\frac{\ell\pi}{d}\right)$$

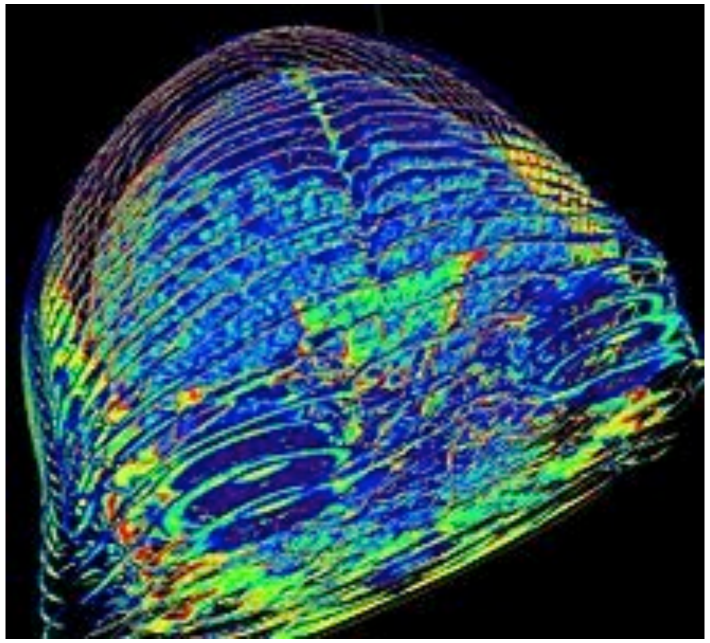
Visualizing rotation by measuring the OAM state vector

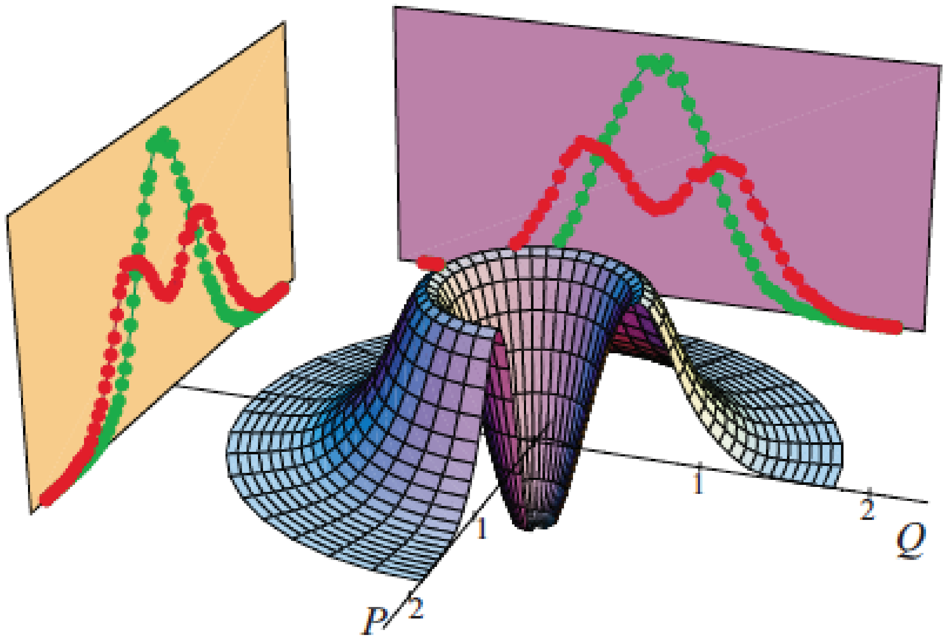


M. Malik, **M. Mirhosseini,** et al. *Nature Communications*, *5*, 3115, 2014.

Characterization of partially coherent states

- The knowledge of amplitude and phase is not adequate for partially coherent states.
- The quantum state is described using more complicated mathematical functions such as density matrix or Wigner distribution function.
- Quantum state tomography is the standard way of finding the Wigner function in $\hat{a}$ and $\hat{a}^\dagger$ basis.





Lvovsky and Raymer. *Reviews of Modern Physics, 2009*

Wigner distribution of twisted photons

- Wigner distribution simultaneously store position and momentum representations

$$P(x,p) = \frac{1}{\pi\hbar}\int_{-\infty}^{\infty} \langle x+y|\hat{\rho}|x-y\rangle e^{-2ipy/\hbar}\,dy,$$

Wigner, E. (1932). Physical Review, 40(5), 749–759.

Alonso, M. A. (2011). Advances in Optics and Photonics, 3(4), 272–365.

- The conjugate of OAM is azimuthal angle. Angle is a tricky coordinate.

$$[\hat{\theta}, \hat{\ell}] = ?$$

Barnett, S. M., & Pegg, D. T. (1990). Physical Review A, 41(7), 3427–3435.

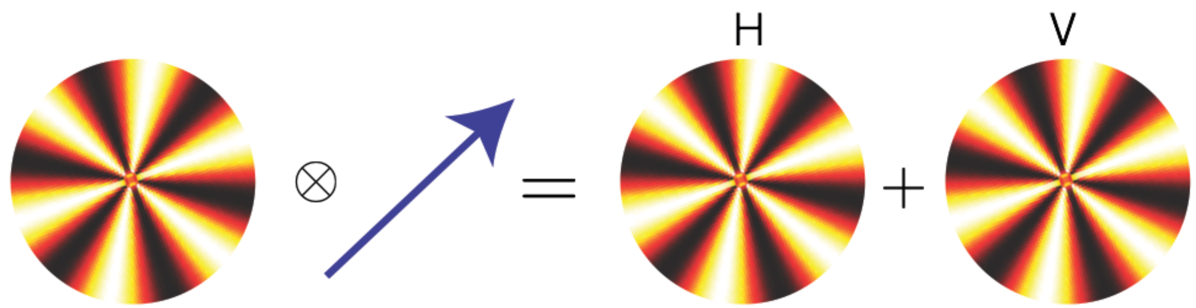
$$W(\theta,\ell) = \frac{1}{d}\sum_{\tau=-N}^{N} \exp\left(-\frac{4\pi i}{d}\ell\tau\right)\langle\theta-\tau|\hat{\rho}|\theta+\tau\rangle.$$

Leonhardt, U. (1995). Physical Review Letters, 74(21), 4101–4105.

Wigner distribution of twisted photons

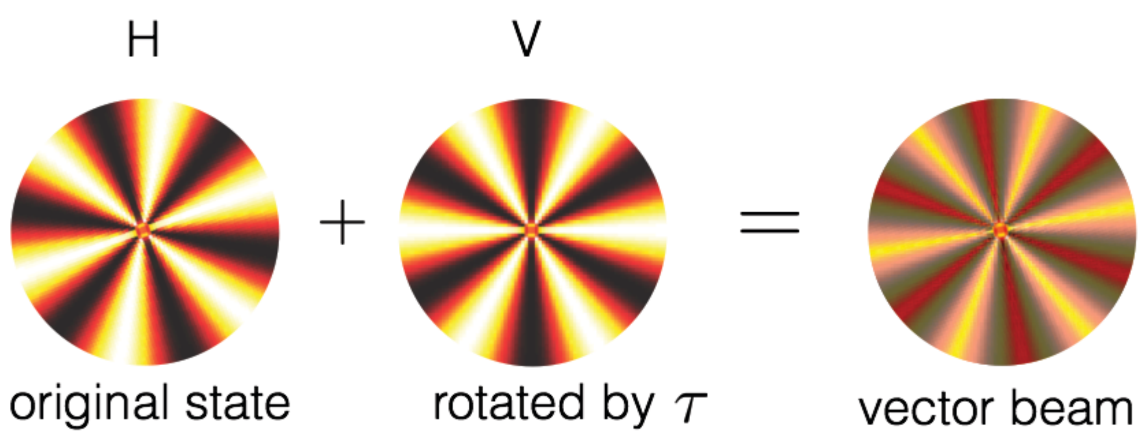
$$W(\theta,\ell)=\frac{1}{d}\sum_{\tau=-N}^{N}\exp\left(-\frac{4\pi i}{d}\ell\tau\right)\langle\theta-\tau|\hat{\rho}|\theta+\tau\rangle.$$

- We perform a polarization-sensitive rotation.



$$\hat{\rho}\times|D\rangle\langle D|$$

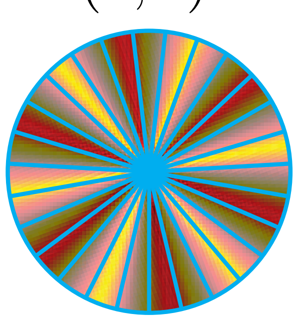
- We perform a polarization-sensitive rotation.



$$\hat{U}(\tau)=\exp\left(-\frac{2\pi i}{d}\tau\hat{L}\otimes\hat{\sigma}_z\right)$$

- We post-select on an angle eigenstate.

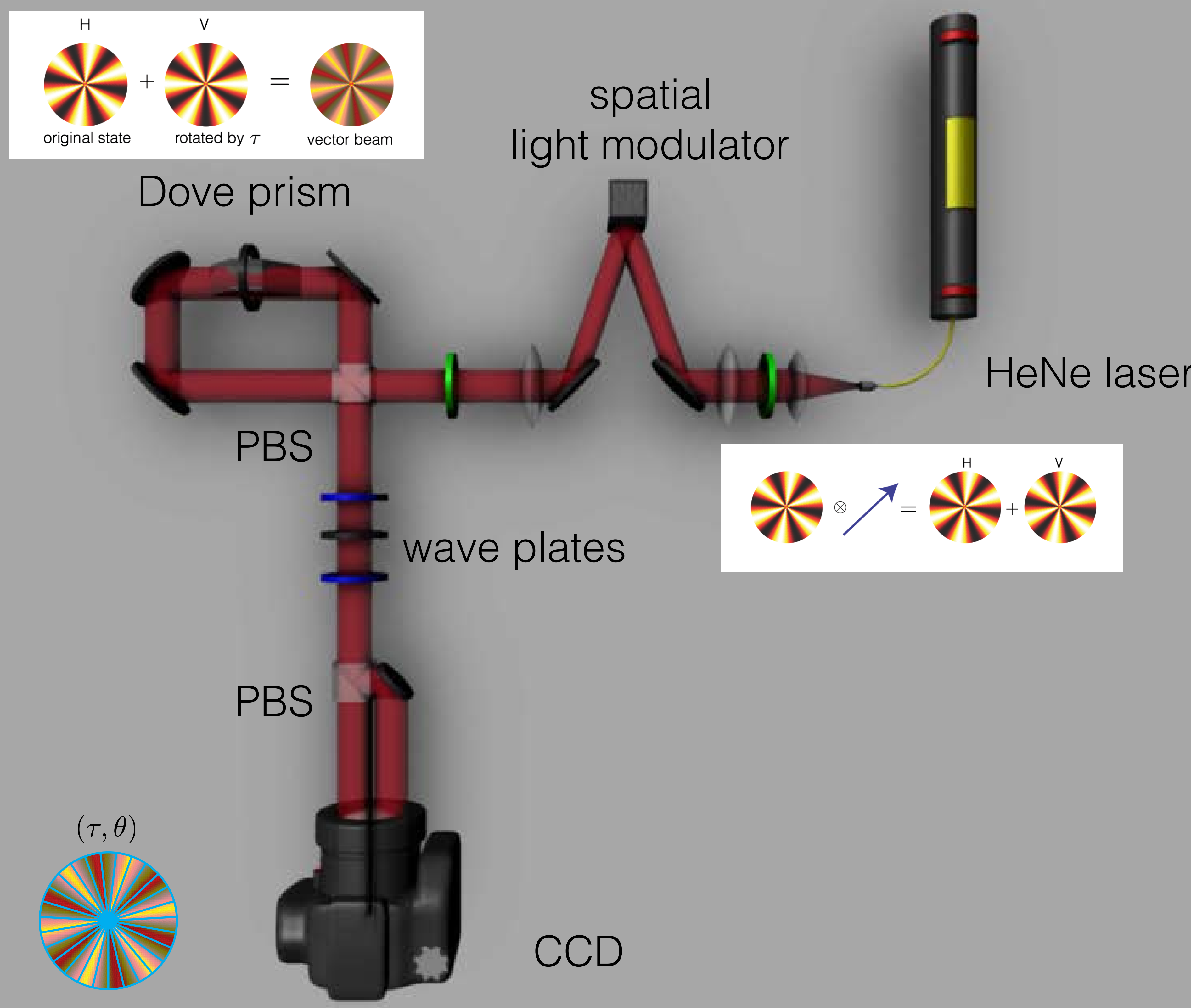
(τ,θ)



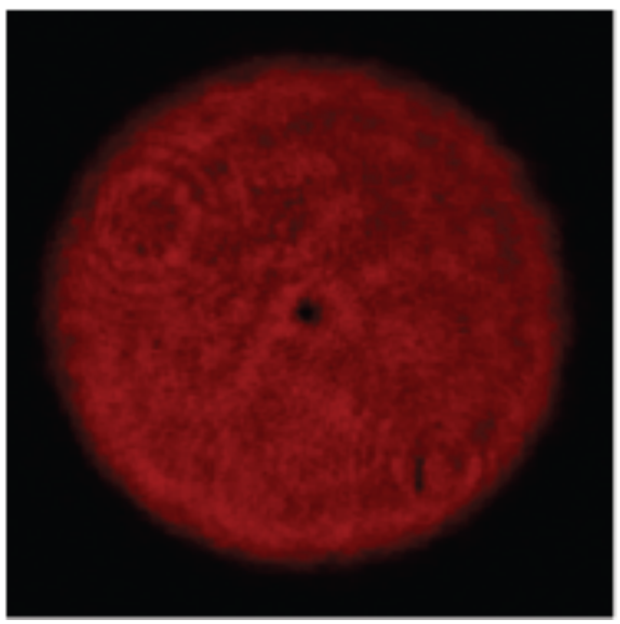
$$\langle\hat{\sigma}_x(\theta,\tau)\rangle=\frac{2}{N(\theta,\tau)}\mathrm{Re}\left[\langle\theta-\tau|\hat{\rho}|\theta+\tau\rangle\right]$$

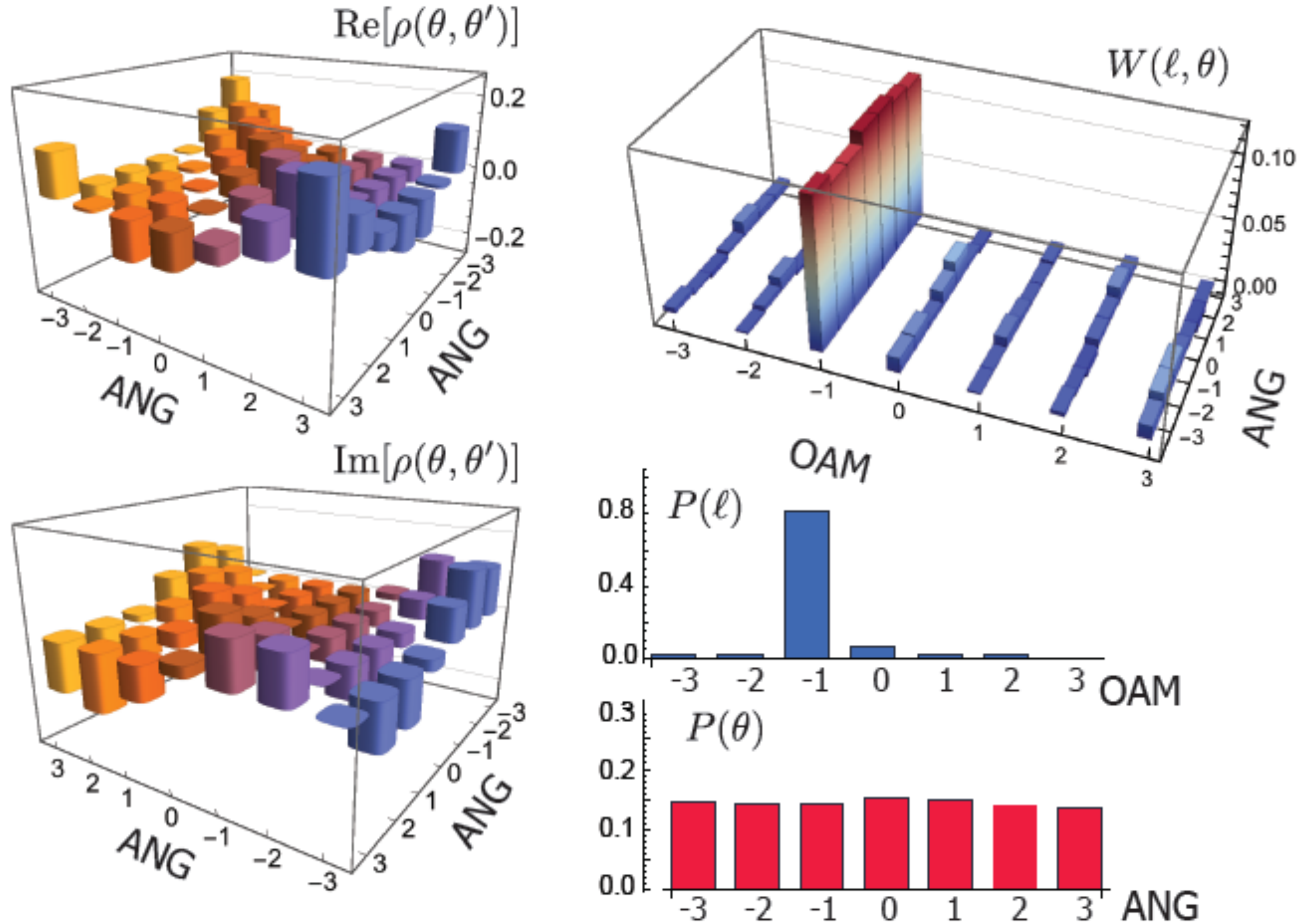
$$\langle\hat{\sigma}_y(\theta,\tau)\rangle=\frac{2}{N(\theta,\tau)}\mathrm{Im}\left[\langle\theta-\tau|\hat{\rho}|\theta+\tau\rangle\right]$$

Experimental setup



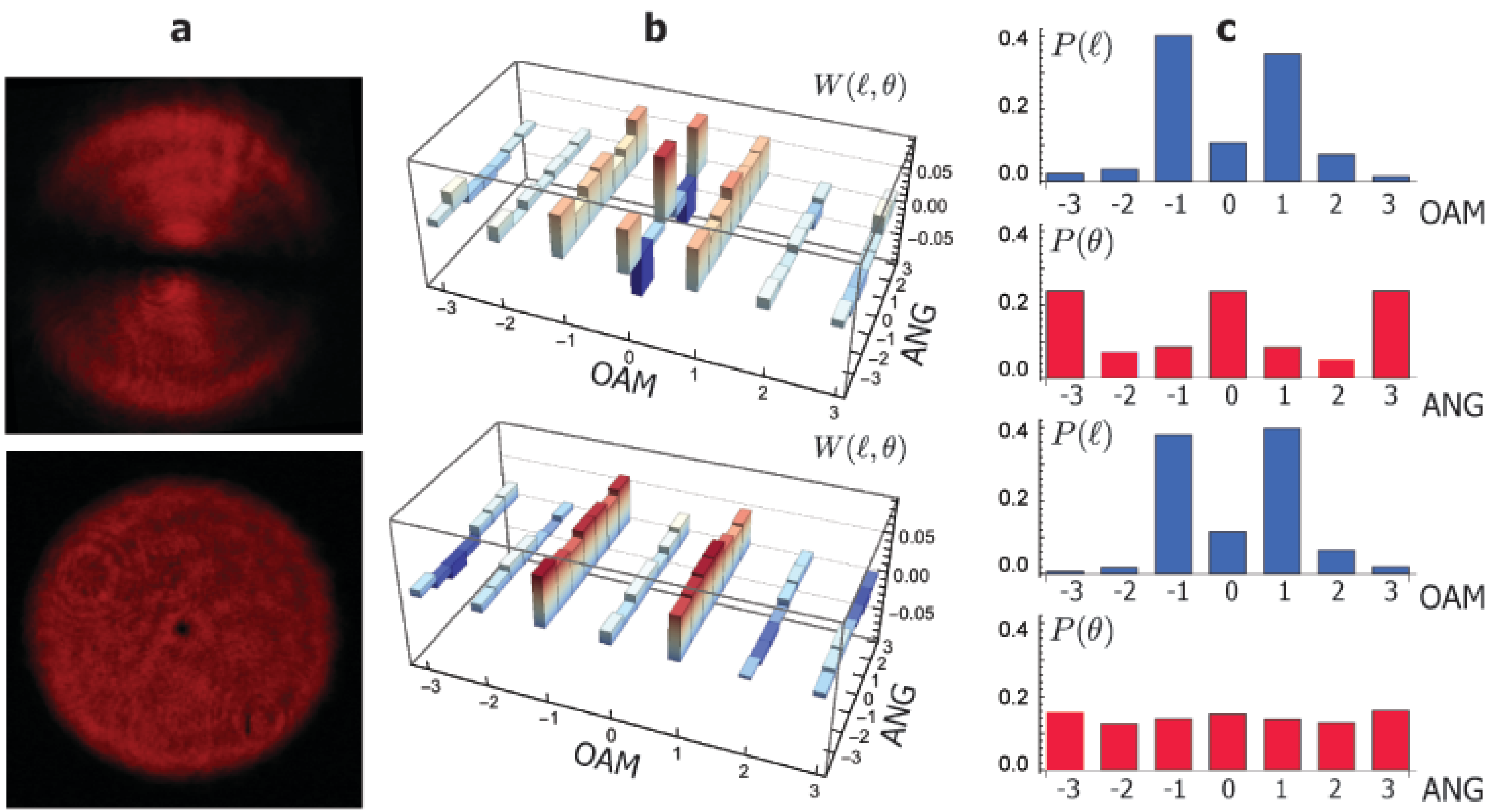
Results with classical beams—angle to OAM





Coherent superposition vs incoherent mixture

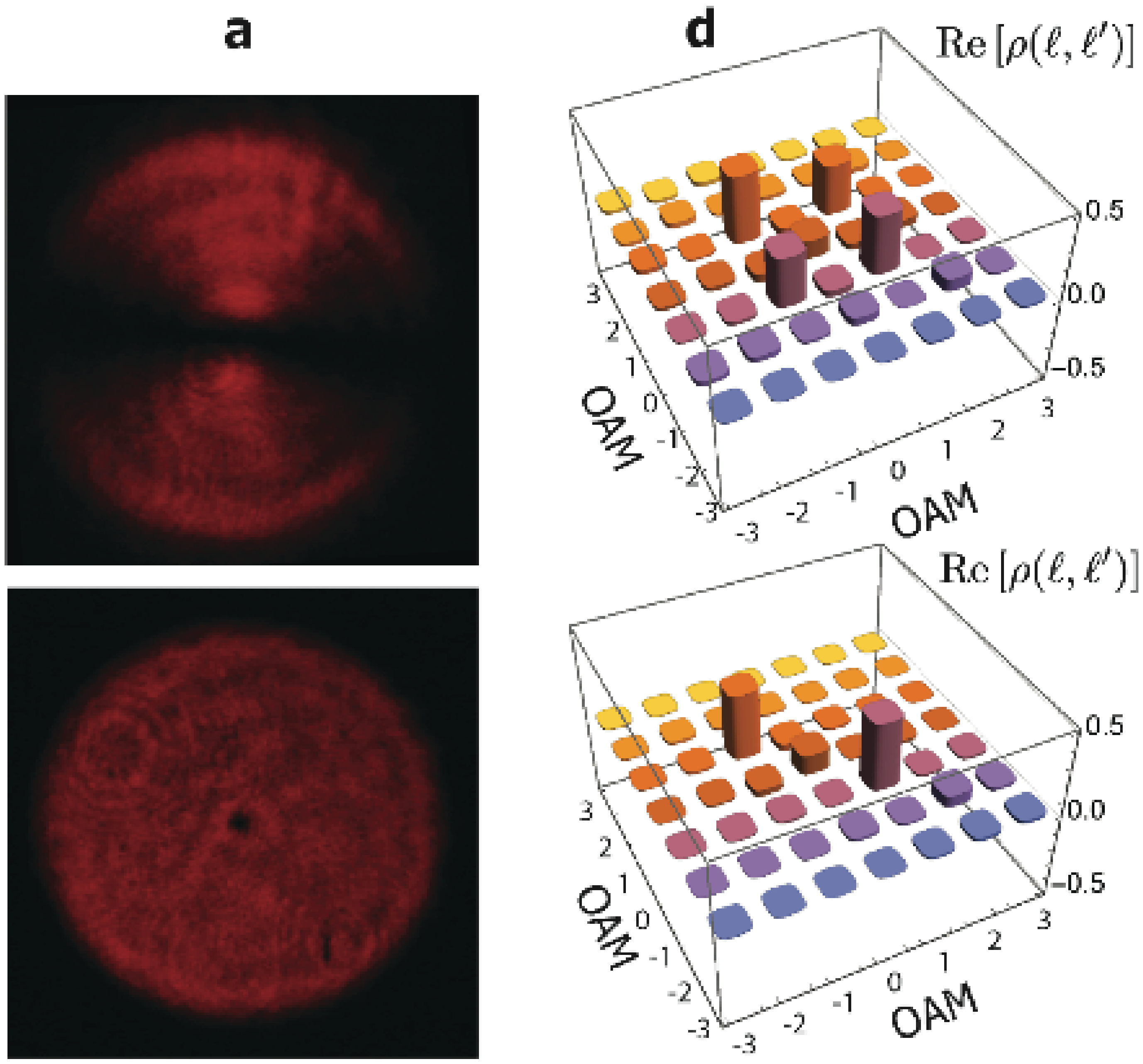
- Full characterization requires understanding of the state's behavior in both OAM and ANG bases.



Mirhosseini et al. Physical Review Letters, (2016).

Coherent superposition vs incoherent mixture

- Negativity of Wigner's function is an indicator for the degree of coherence.

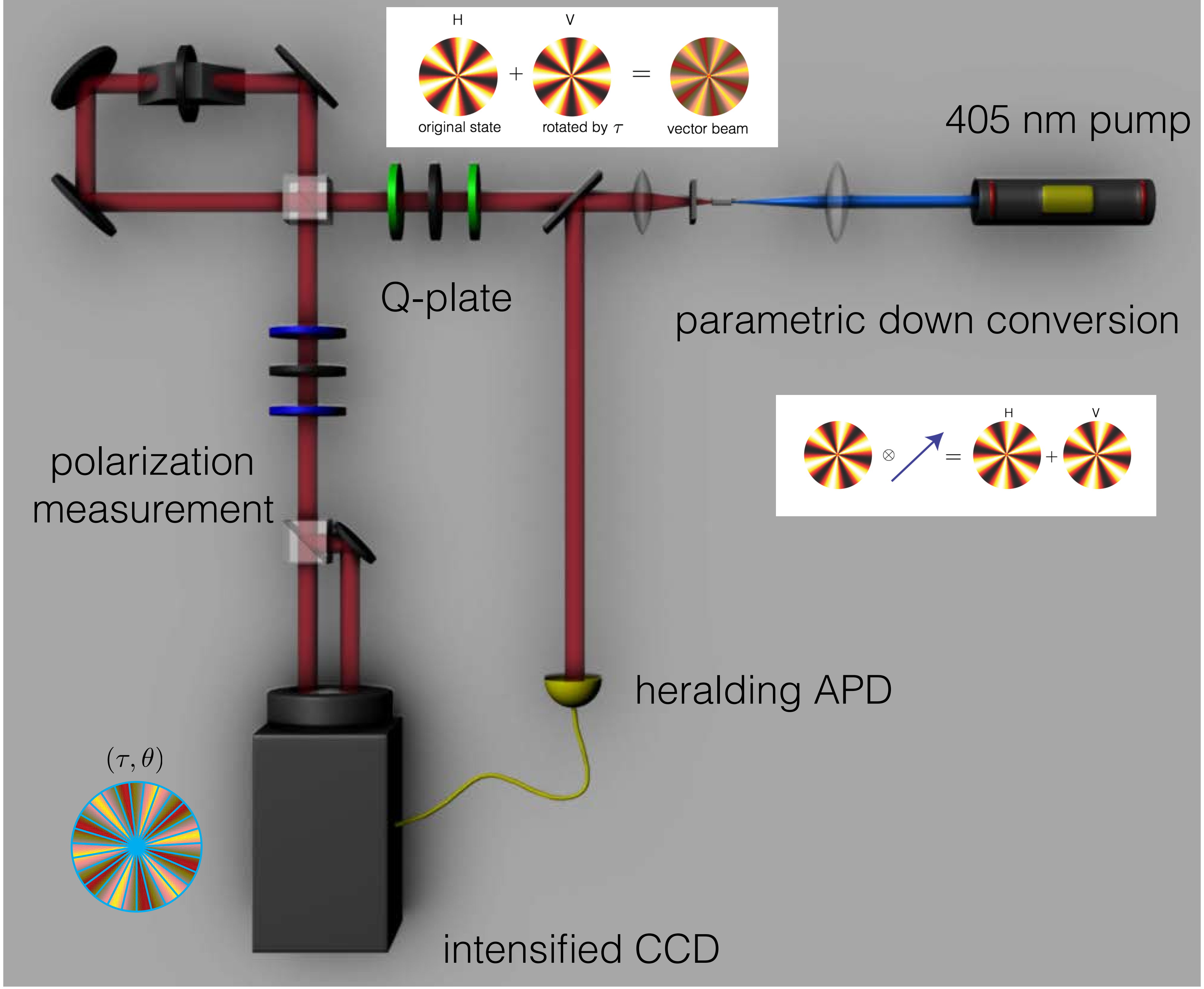


$$\gamma = \frac{|\rho(-1,1)|}{\sqrt{|\rho(1,1)||\rho(-1,-1)|}} = 0.8$$

$$\gamma = \frac{|\rho(-1,1)|}{\sqrt{|\rho(1,1)||\rho(-1,-1)|}} = 0.06$$

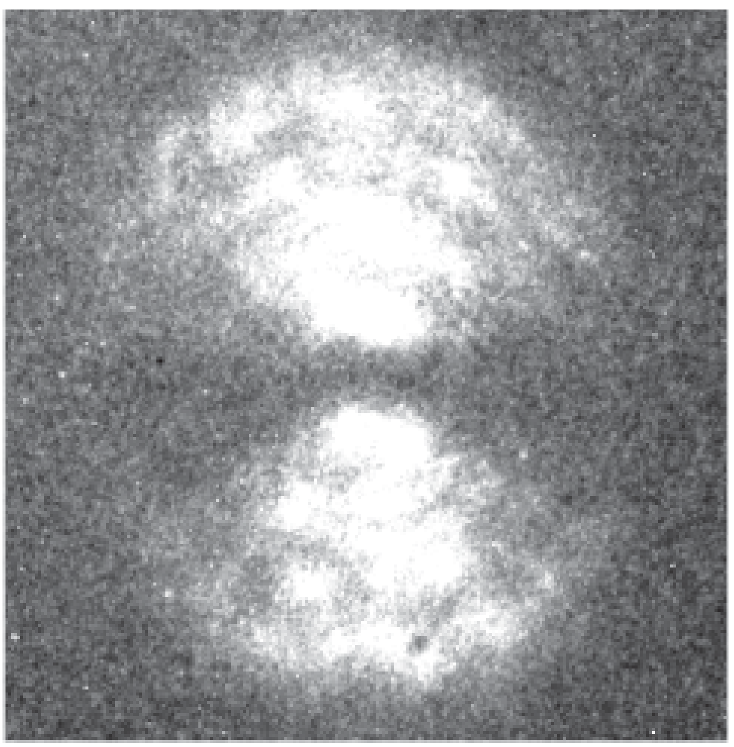
Mirhosseini et al. Physical Review Letters, (2016).

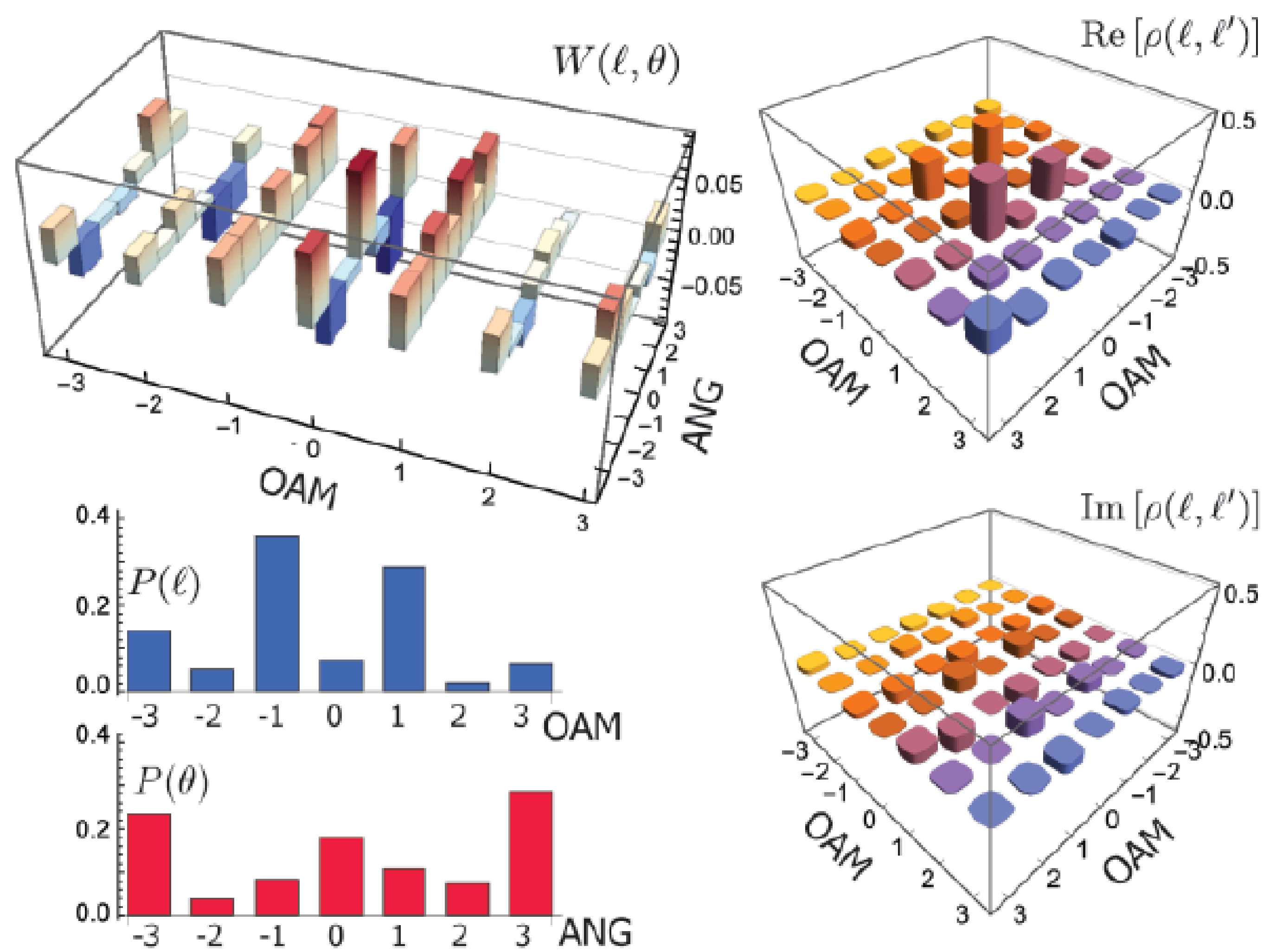
Experimental setup



Coherent OAM- ANG interference of a "single" photon

- Analogue of Young's experiment in the ANG-OAM basis.





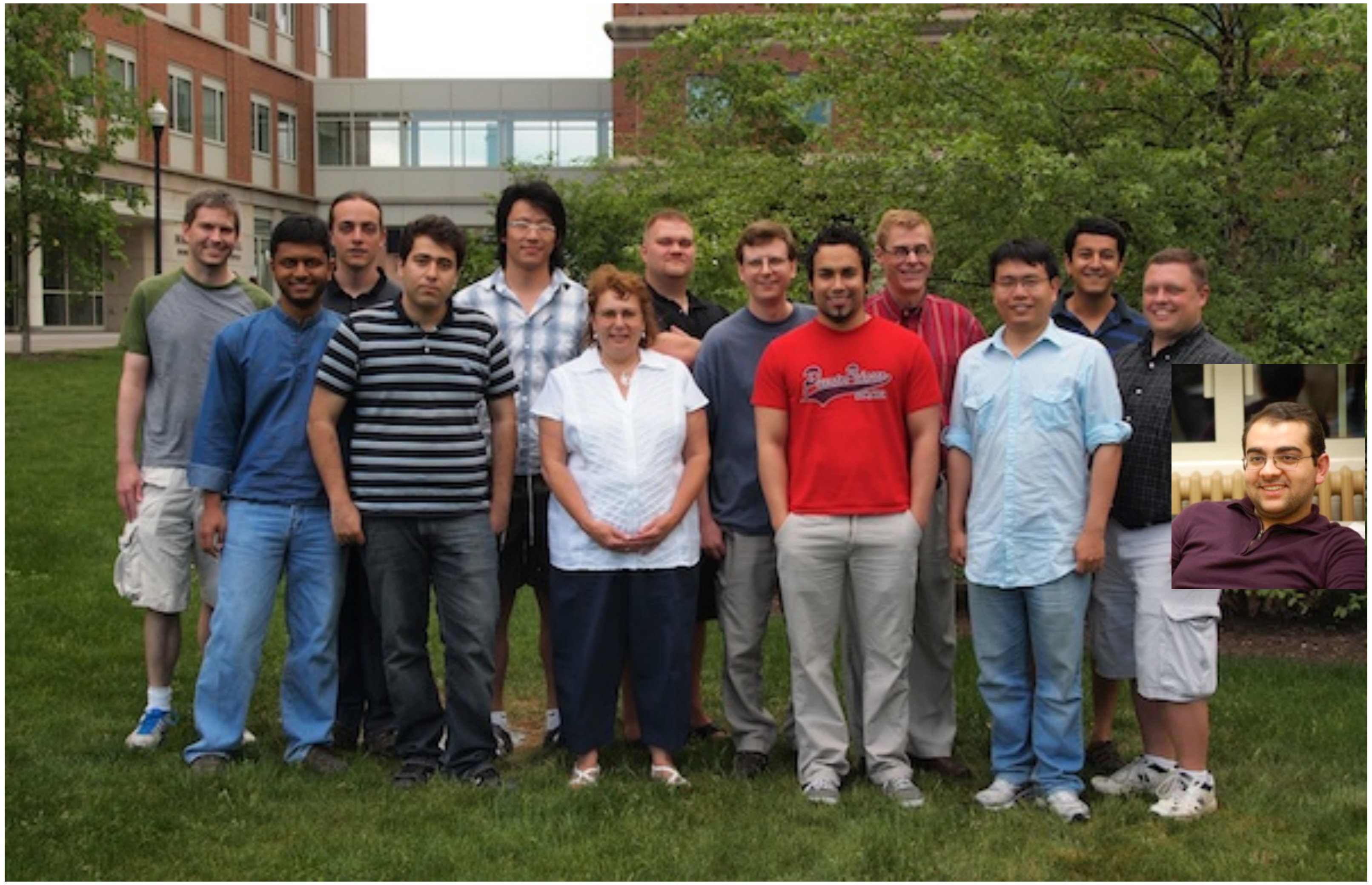
Mirhosseini et al. Physical Review Letters, (2016).

Conclusions

- High-dimensional spatial modes provide a great resource for quantum information processing.
- We have doubled the capacity of QKD with OAM encoding. The next step would be to go to GHz symbol transmission rates.
- Characterization of high-dimensional states of OAM.
- Further research is required to achieve scalable and efficient methods for characterizing multi-photon, multi-level states.

Acknowledgement

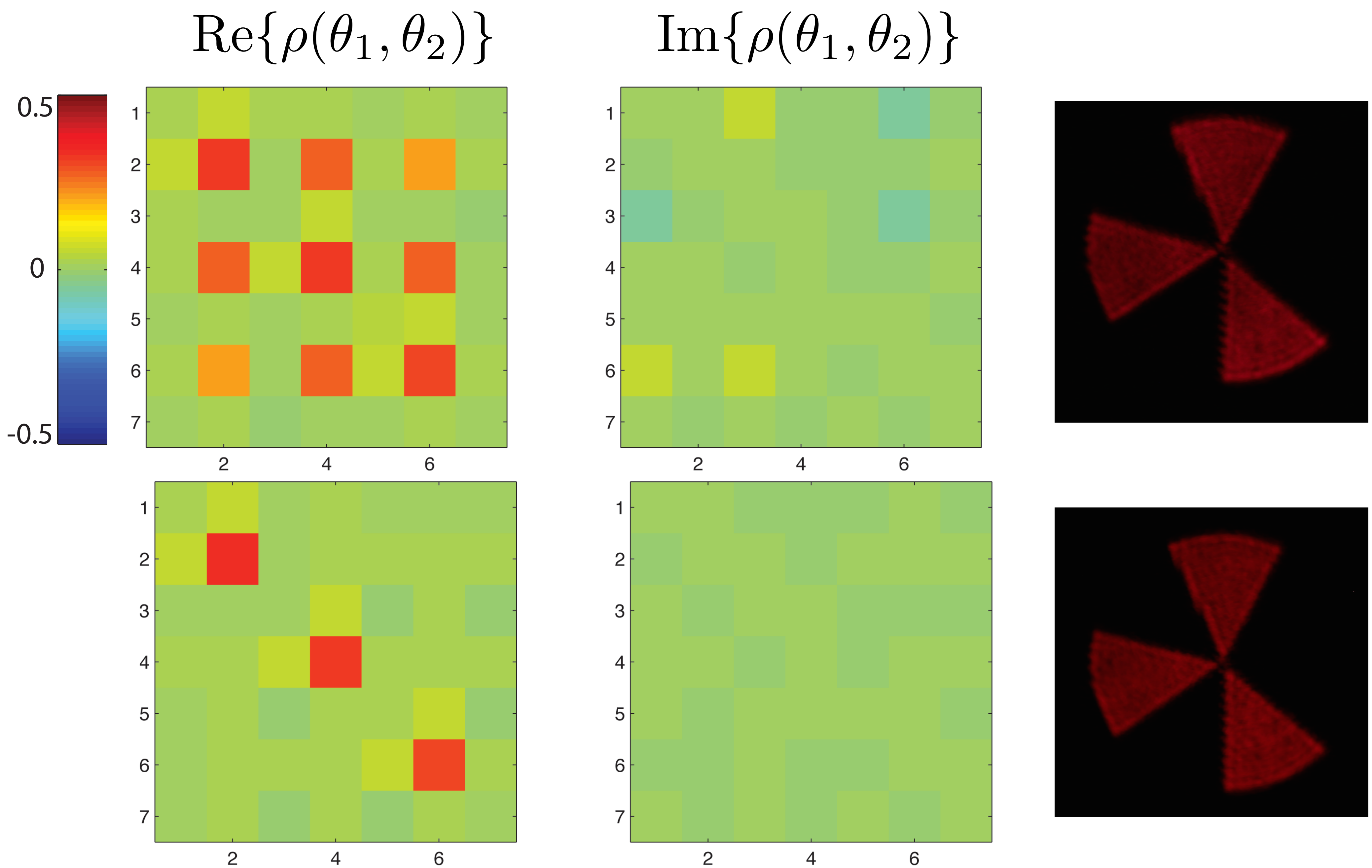
- Robert Boyd Quantum Photonics Group
- DARPA InPho project
- Canadian Research Excellence Center



Selected Publications

1. Mirhosseini et al. *Nature Communications*, 4, 2781 (2013).
2. Mirhosseini et a. *New Journal of Physics*, 17, 033033 (2015).
3. Mirhosseini, et al. *Physical Review Letters*, 113, 090402 (2014).
4. Mirhosseini, et al. *Physical Review Letters*, 116, 130402 (2016).
5. Mirhosseini, et al. *Physical Review A*, 93, 053836 (2016)
6. Mirhosseini et al. *Optics Express*, Vol. 21, Issue 25, 30196-30203 (2013).
7. Mirhosseini et al. *Journal of Modern Optics*, Vol. 61, Issue 1, 43-48 (2013).
8. Malik, Mirhosseini et al. *Nature Communications*, 5, 3115 (2014).
9. Magaña-Loaiza, Mirhosseini, et al., Science Advances 2 (no. 4), e1501143 (2016).
10. Magaña-Loaiza, Mirhosseini et al. *Physical Review Letters*,, 112, 200401 (2014).
11. Potocek et al. *Physical Review Letters*,, 115,160505 (2015).
12. Safari et al. *Physical Review Letters*,, accepted (2015).
13. Shi, Mirhosseini et al. *Optica*, Vol. 2, Issue 4, pp. 388-392 (2015).
14. Rodenburg, Mirhosseini et al. *New Journal of Physics*, 16, 033020 (2014).
15. Bouchard, et al. *New Journal of Physics*, 16, 123006 (2014).

Angular projections of density matrices

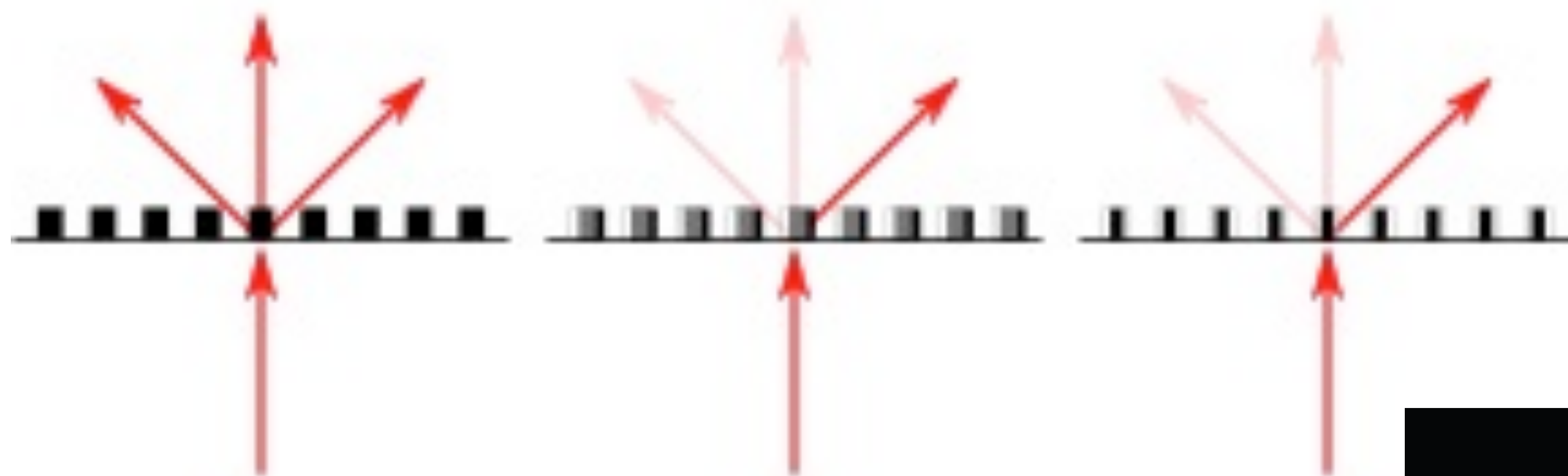


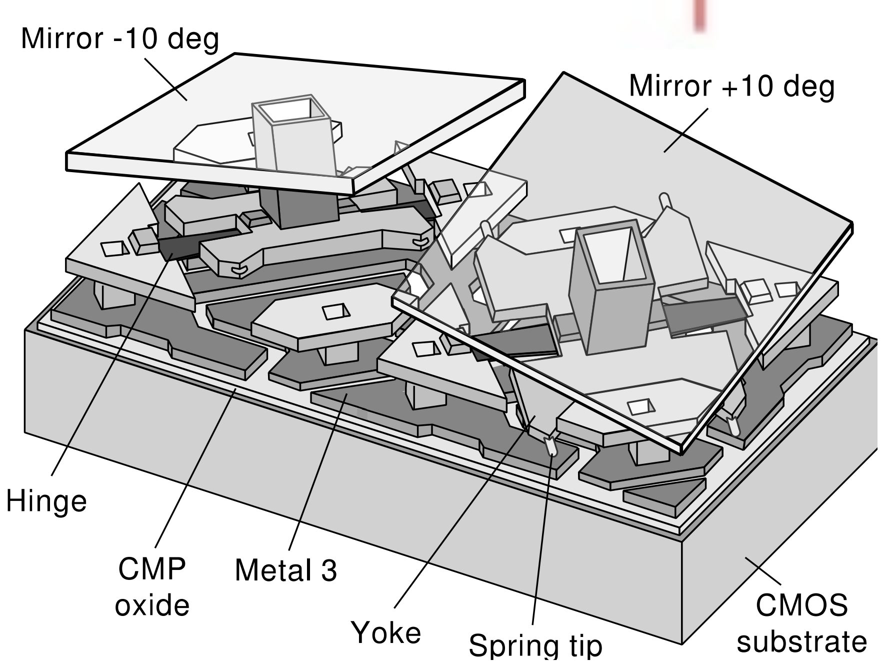
- We can reliably measure projections of density matrix in the angular basis

Generation of spatial modes

- Adjusting the position and the width (duty cycle) of a grating modulates amplitude and phase of the diffracted beams.

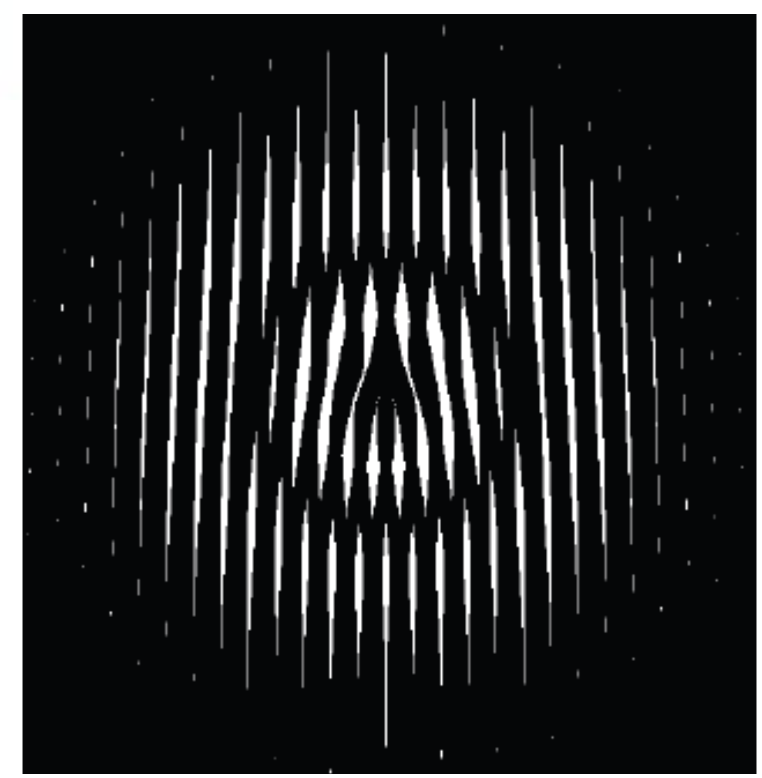
$\delta \rightarrow$ position of grating $\quad qT \rightarrow$ width (duty cycle) of grating





$$f(\mathbf{r}) = \sum_m \frac{\sin \pi m q}{\pi m} e^{im(\mathbf{G}.\mathbf{r}+2\pi\delta)}$$

$$\mathbf{G} = \frac{2\pi}{T}\left(\cos(\theta)\hat{x} + \sin(\theta)\hat{y}\right)$$



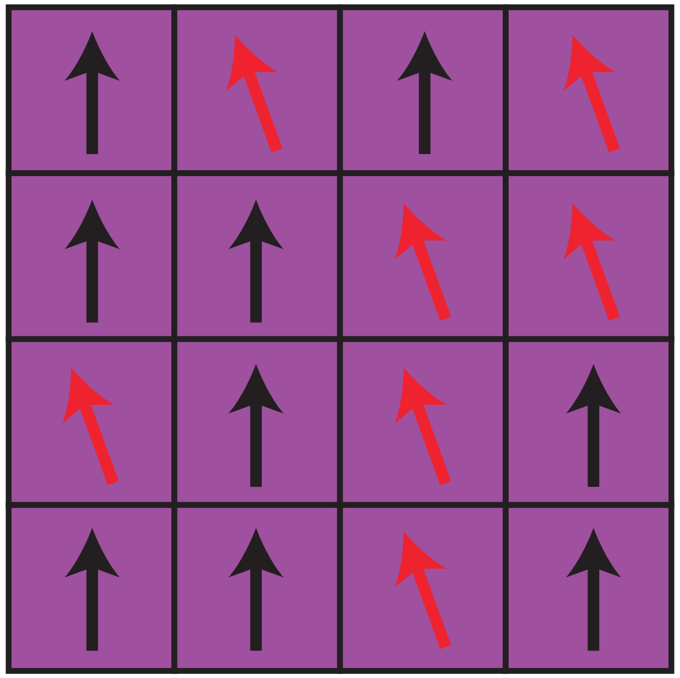
Mirhosseini et al, *Optics Express*, 21(2013)

Compressive direct measurement

- We use a liquid crystal SLM to make random measurements.

$$\phi_m = \frac{1}{\kappa}[\bar{\sigma}_{x,m} - i\bar{\sigma}_{y,m}]$$

$$\begin{pmatrix} \phi_1 \\ \phi_2 \\ \vdots \\ \phi_M \end{pmatrix} = \begin{pmatrix} Q_{1,1} & Q_{1,2} & \cdots & Q_{1,N} \\ Q_{2,1} & Q_{2,2} & \cdots & Q_{2,N} \\ \vdots & \vdots & \ddots & \vdots \\ Q_{M,1} & Q_{M,2} & \cdots & Q_{M,N} \end{pmatrix} \begin{pmatrix} \psi_1 \\ \psi_2 \\ \vdots \\ \psi_N \end{pmatrix}$$



$$\boldsymbol{\psi} = \mathbf{Q}^\dagger(\mathbf{Q}\mathbf{Q}^\dagger)^{-1}\boldsymbol{\phi}$$

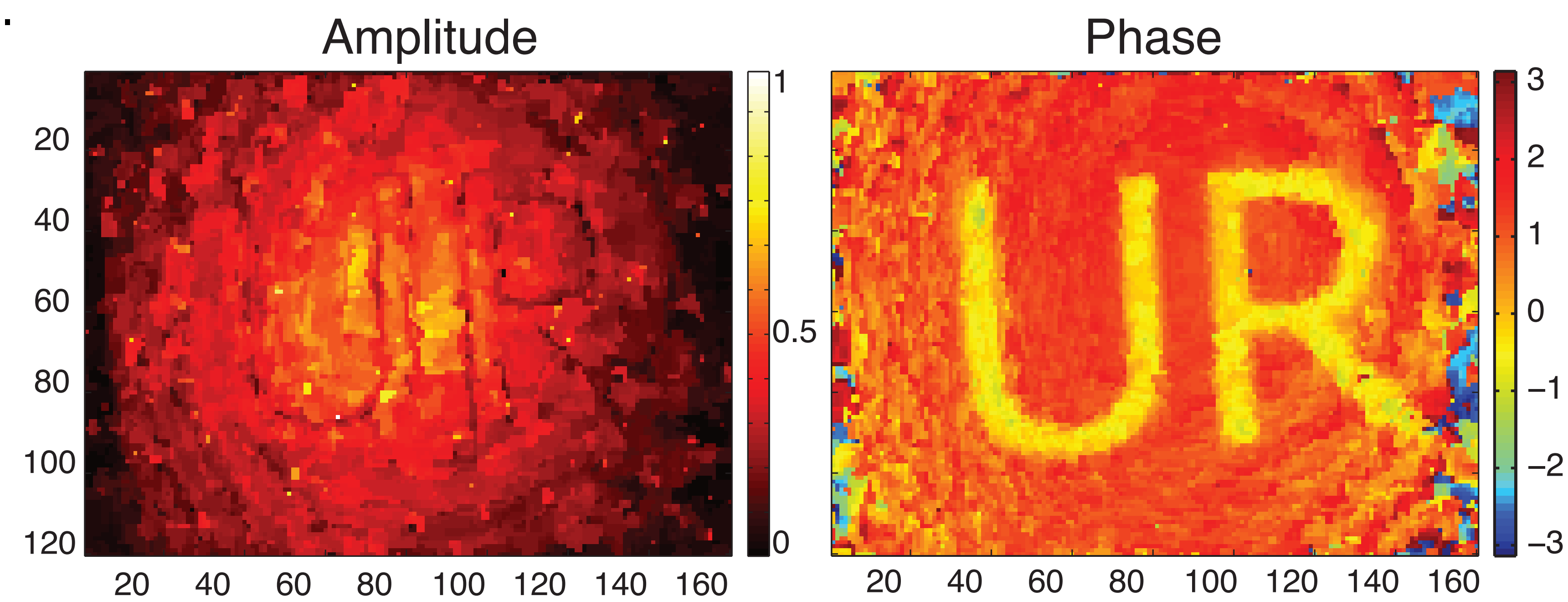
$$\min_{\boldsymbol{\psi}'} \sum_j ||\nabla \boldsymbol{\psi}'_j||_{\ell_1} + \frac{\mu}{2}||\mathbf{Q}\boldsymbol{\psi}' - \boldsymbol{\phi}||^2_{\ell_2}.$$

Candès, Romberg, Tao, Communications on Pure and Applied Mathematics 59 (8), (2006)

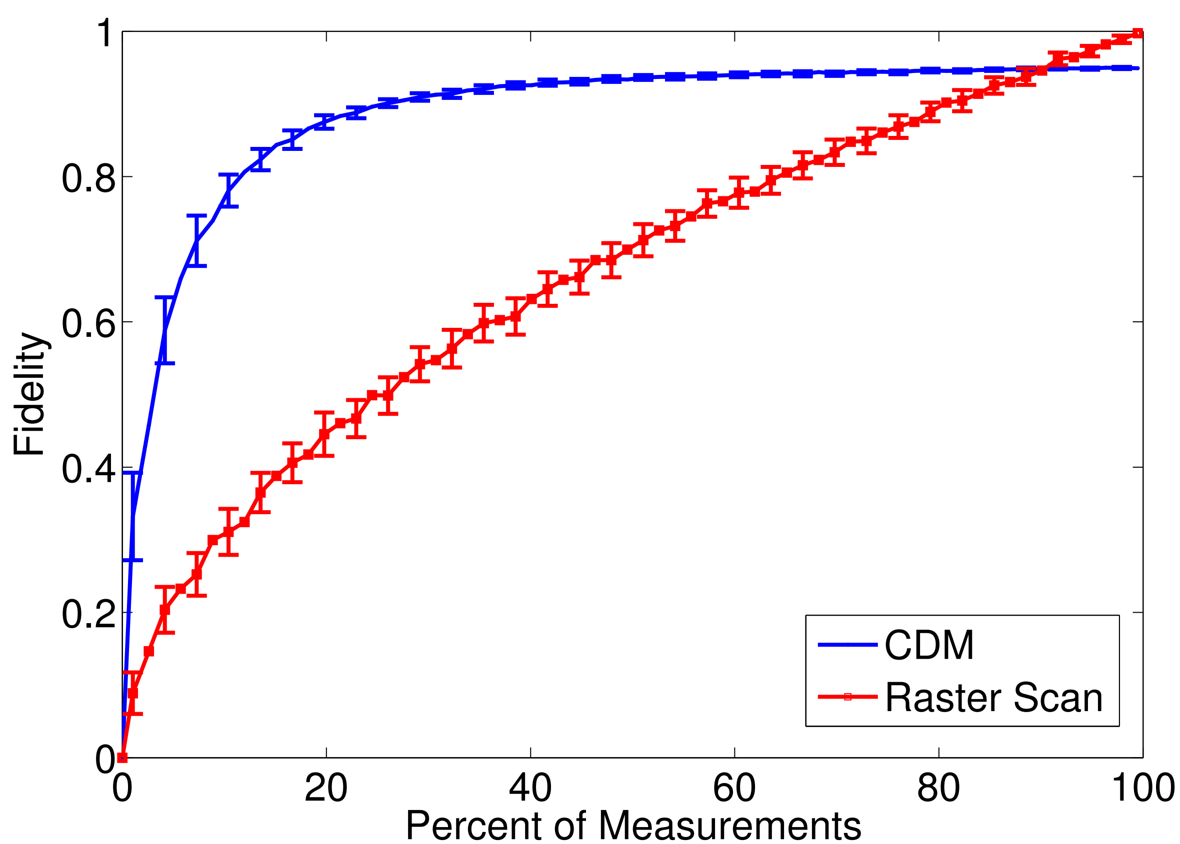
Romberg , *IEEE Signal Process. Mag.* (2008).

Experimental results

- 350 times faster than the standard direct measurement for a 19200 dimensional states.



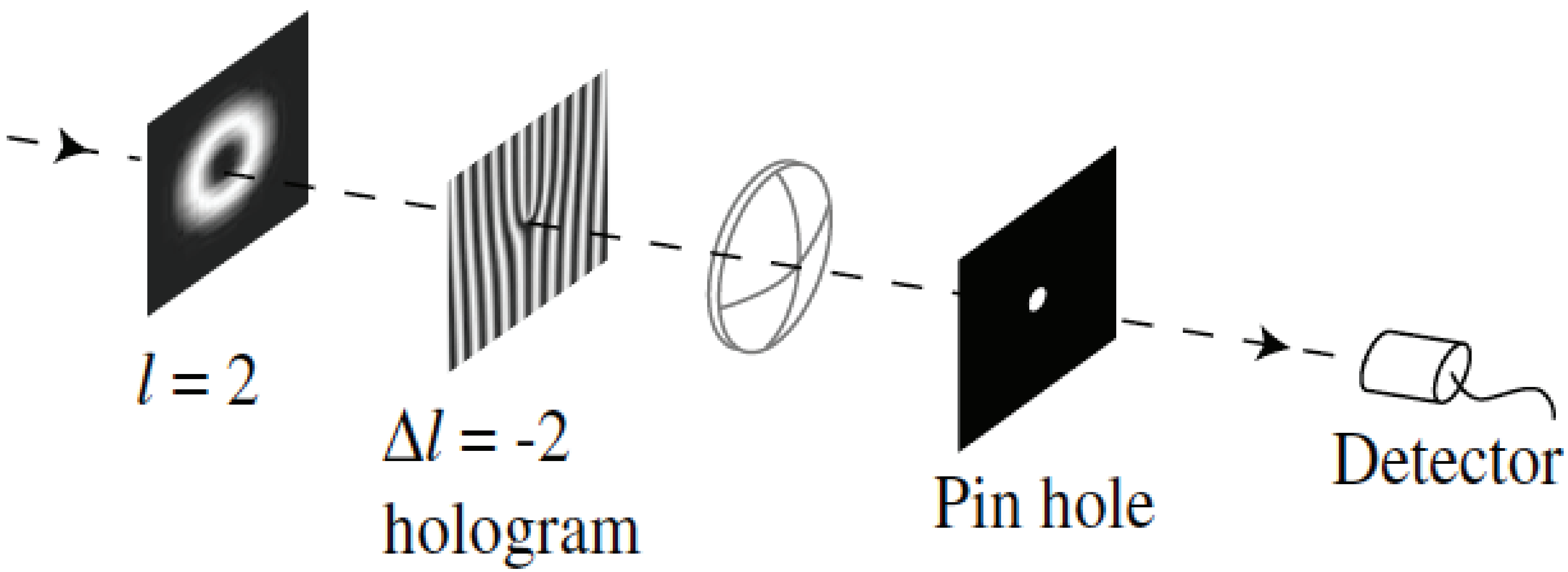
$$\frac{M}{N} = 20\%, N = 19200$$



$$F(|\psi'\rangle, |\psi\rangle) = |\langle\psi'|\psi\rangle|$$

M. Mirhosseini, et al. *Physical Review Letters*, *113*(9), 2014.

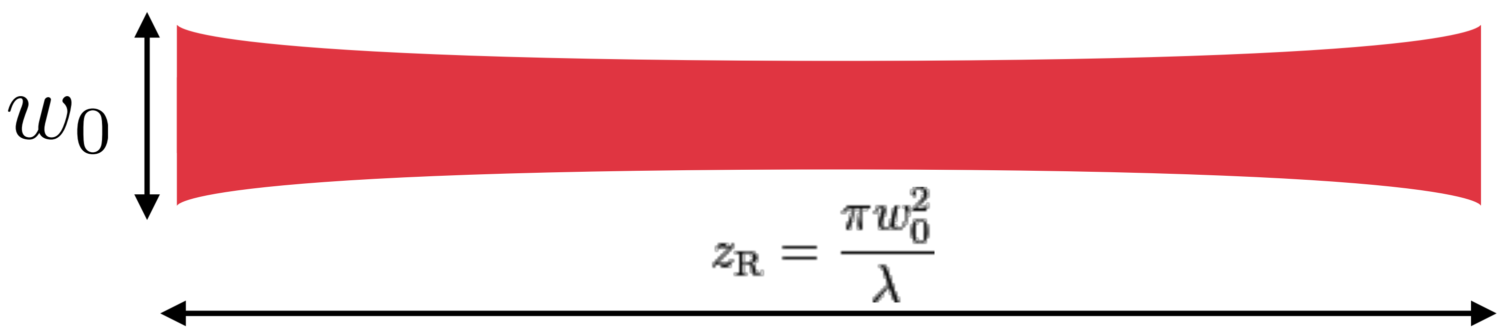
Projective measurements

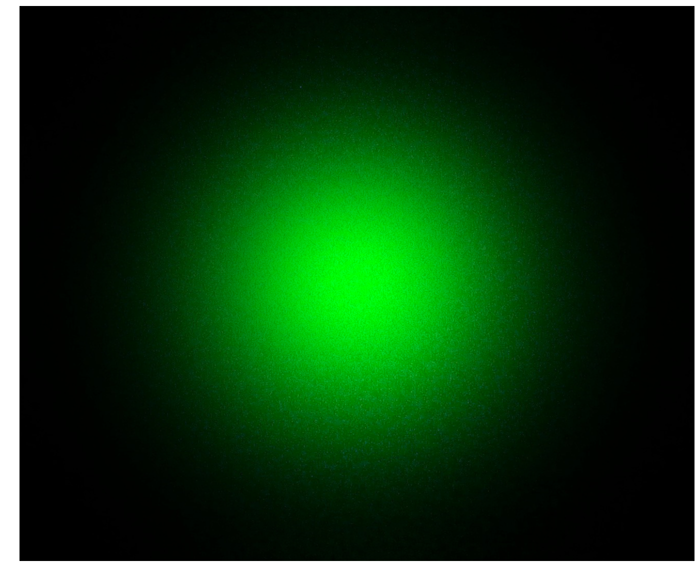


What is structured light?

- Gaussian laser beam

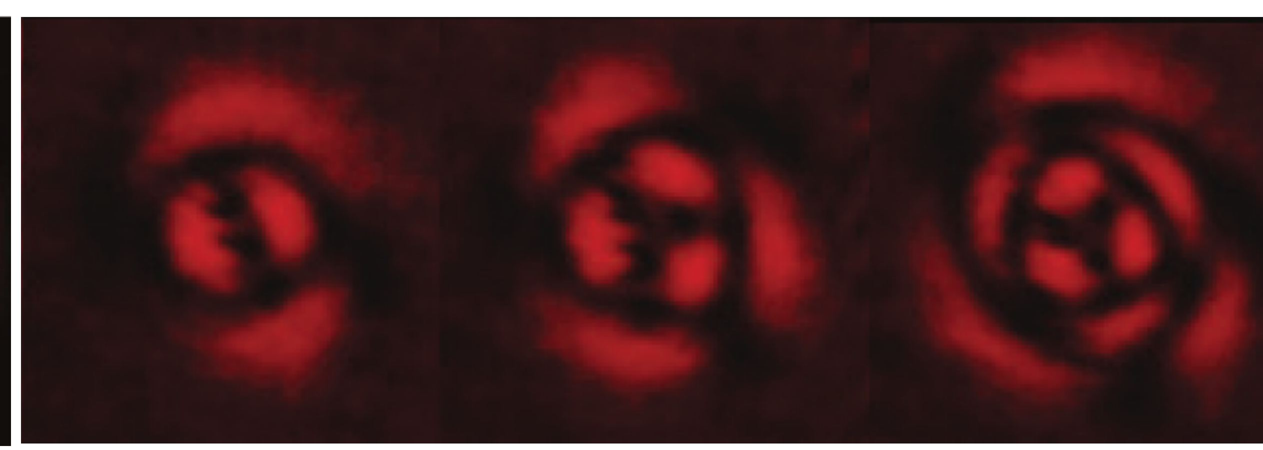
$$\mathbf{E}(r,z) = E_0\,\hat{x}\,\frac{w_0}{w(z)}\exp\left(\frac{-r^2}{w(z)^2}\right)\exp\left(-i\left(kz + k\frac{r^2}{2R(z)} - \psi(z)\right)\right),$$



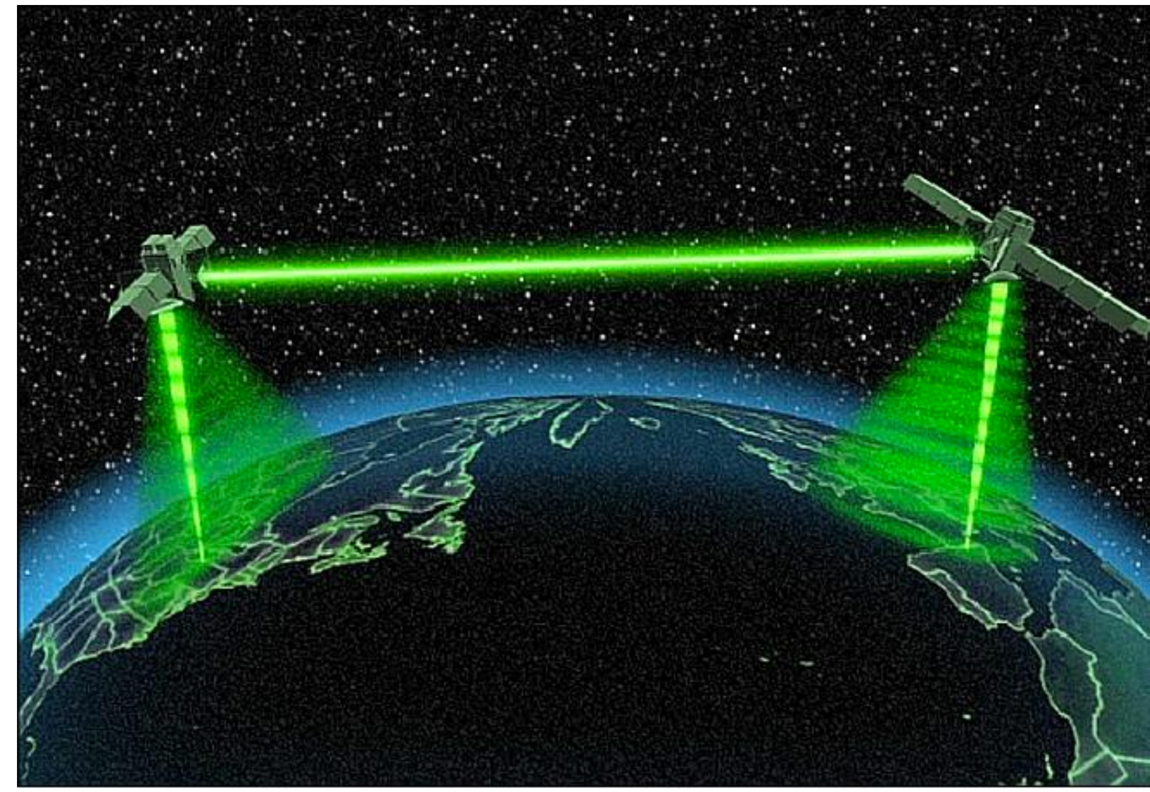


- Laguerre-Gaussian beams are a set of solutions to paraxial wave equation

LG_{21} $\quad LG_{31}$ $\quad LG_{32}$



How is orbital angular momentum related to communication?



- The number of spatial modes that can be effectively communicated between two apertures is finite.
- This number can be estimated using diffraction considerations:

$$\delta \simeq \lambda L / D.$$

$$N_F \simeq D/\delta = \frac{D^2}{\lambda L}$$