Quantum Properties of the Orbital Angular Momentum of Light

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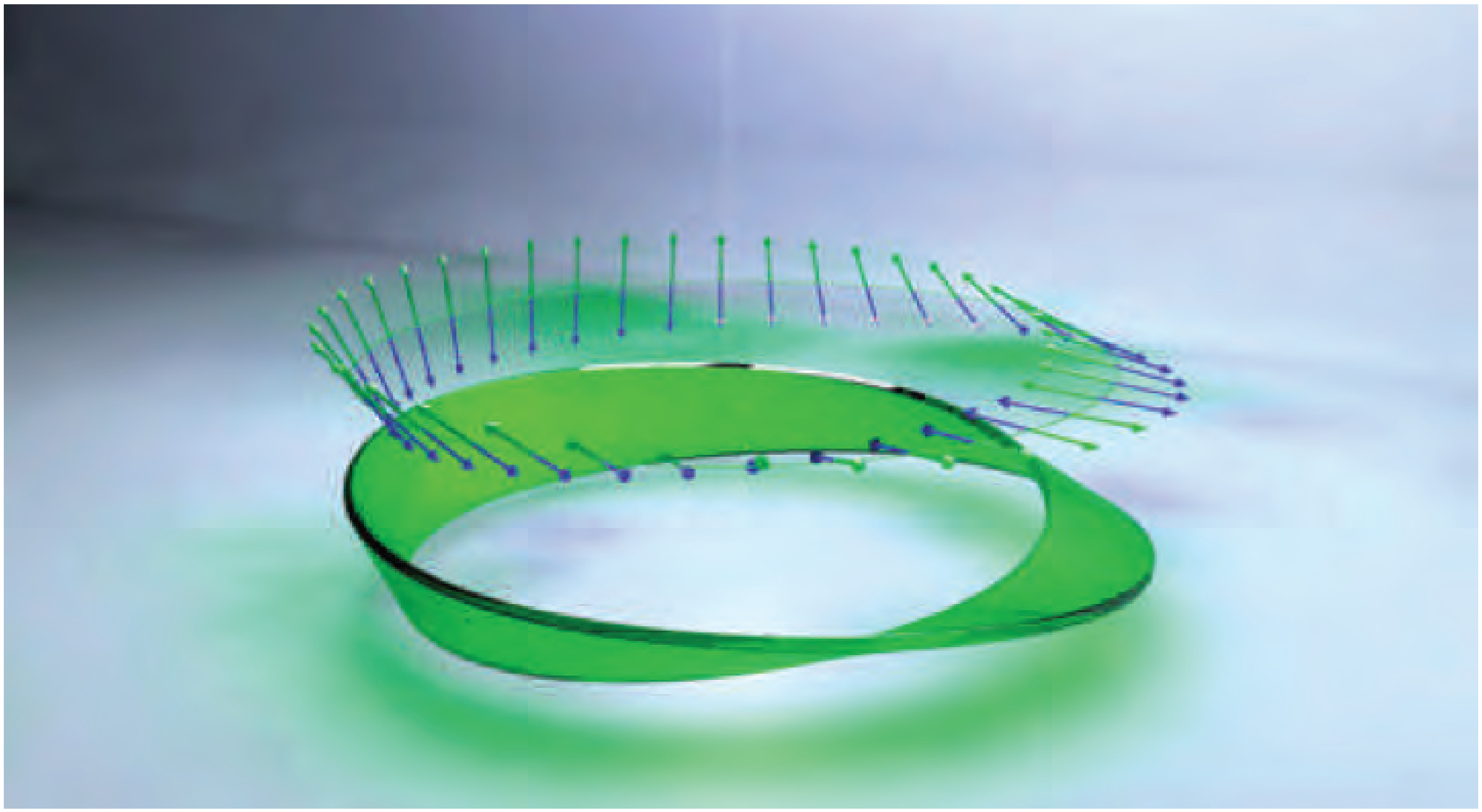
Prospectus

1. Möbius Strips of Polarization
2. “Slow Light” and Fresnel Drag
3. Foundations of Photonics
4. Quantum Applications of OAM

Observation of Optical Polarization Möbius Strips

Thomas Bauer, Peter Banzer, Ebrahim Karimi, Sergej Orlov, Andrea Rubano, Lorenzo Marrucci, Enrico Santamato, Robert W Boyd and Gerd Leuchs

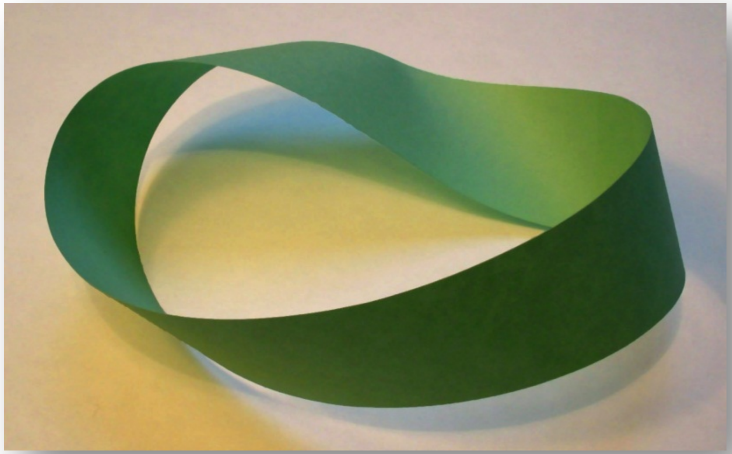
Science, 347, 964 (2015)



Möbius strips are familiar geometrical structures, but their occurrence in nature is extremely rare. We generate such structures in the nanoscale in tightly focused vector light beams and confirm experimentally their Möbius topology.

Optical Möbius Strips – When Light Turns One-Sided and Single-Edged

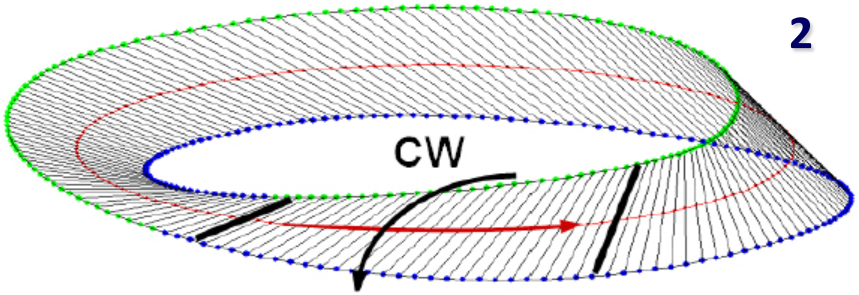
An 'ordinary' Möbius strip



1

A polarization Möbius strip

(introduced by Isaac Freund)



2

Isaac Freund discovered, described, and investiated these unusal objects

- Optical Möbius strips can be found in light fields
- One has to look at a very special field distribution in a very special way
- By doing so, one can observe optical Möbius strips in the field structure

[1] Wikipedia

[2] Isaac Freund, Bar-Ilan Univ., Talk: *Optical Moebius Strips and Twisted Ribbons*, Conf. on Singular Optics, ICTP Trieste, Part II, 30 May 2011

Isaac Freund, Opt. Commun. 242, 65-78 (2004)

Isaac Freund, Opt. Commun. 249, 7-22 (2005)

Isaac Freund, Opt. Commun. 256, 220-241 (2005)

Isaac Freund, Opt. Commun. 283, 1-15 (2010)

Isaac Freund, Opt. Commun. 283, 16-28 (2010)

Isaac Freund, Opt. Lett. 35, 148-150 (2010)

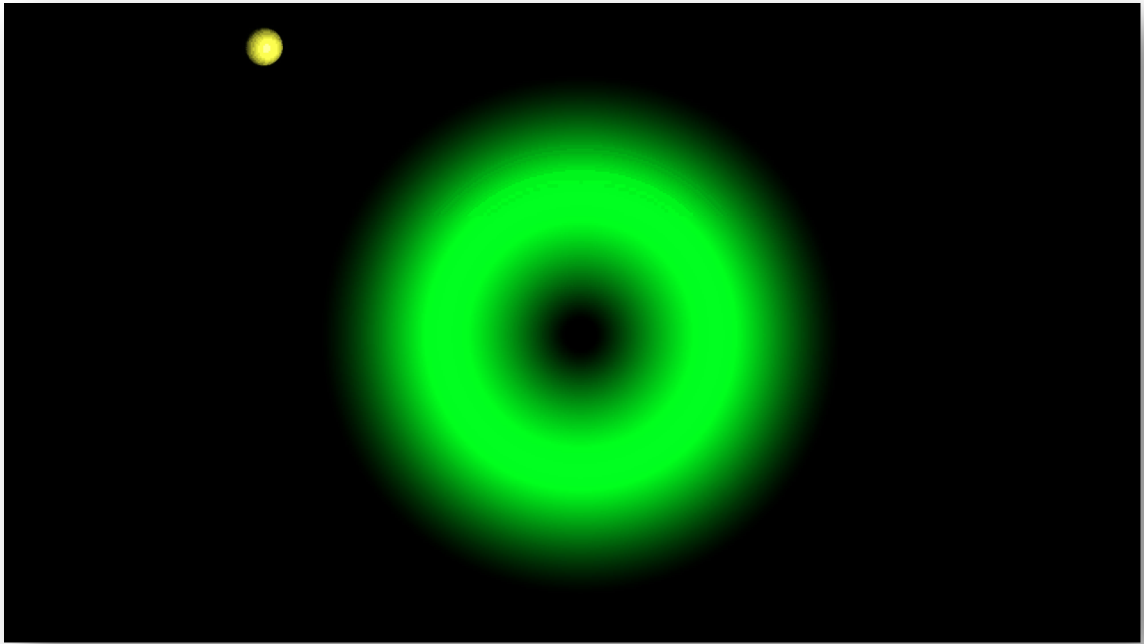
Isaac Freund, Opt. Commun. 284, 3816-3845 (2011)

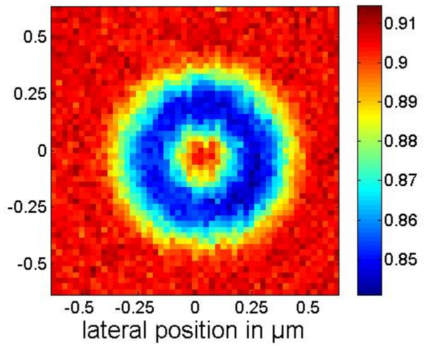
Full vectorial beam measurement on the nanoscale

Nanoparticle-based probing technique for vector beam reconstruction

1. A dipole-like spherical nanoparticle (90 nm diameter) is scanned through the beam
2. The forward- and backward-scattered light for each position of the nanoparticle relative to the beam in the focal plane is measured

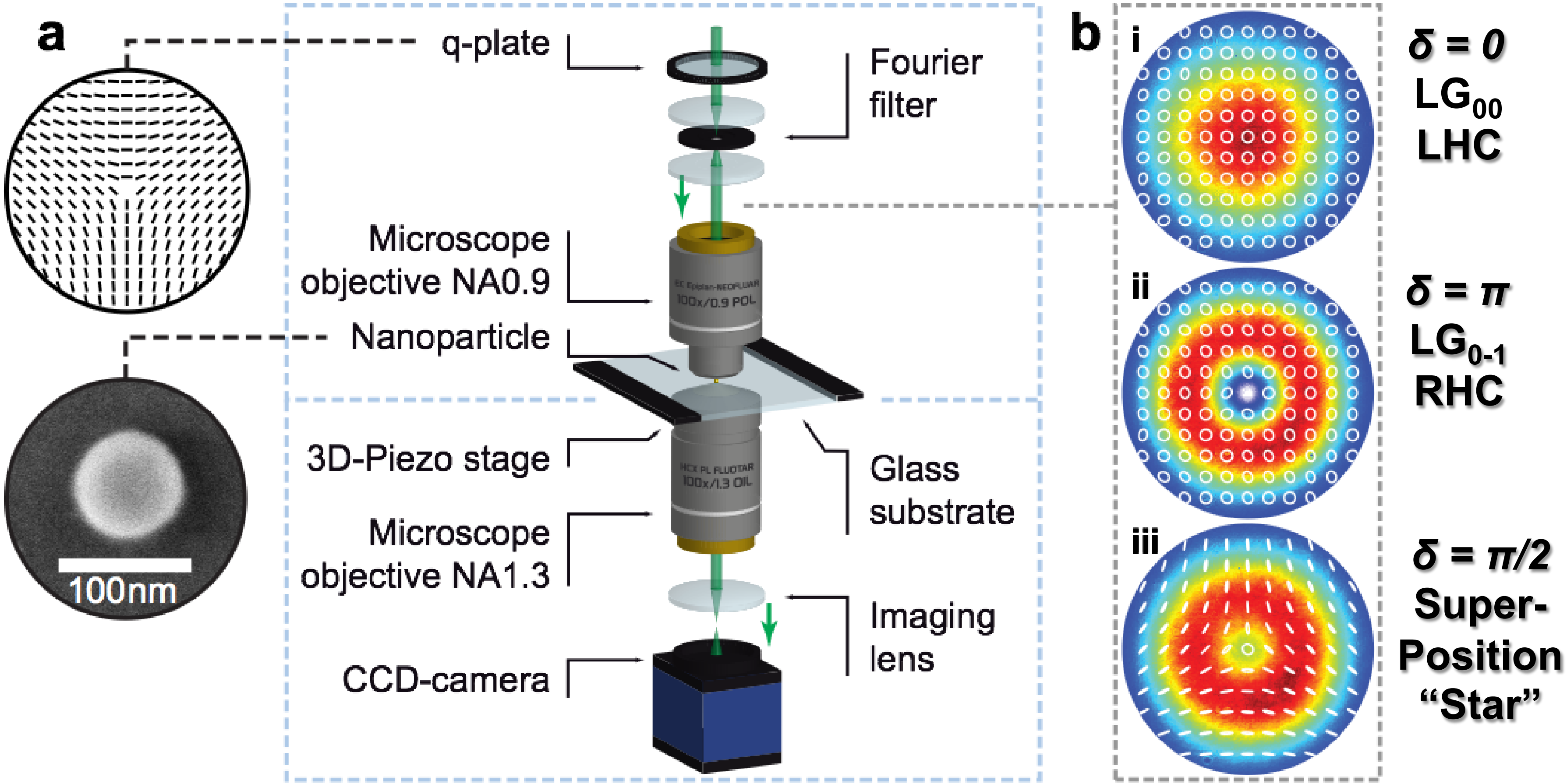
measured intensity
(can also measure
polarization and phase)





Full amplitude and phase reconstruction scheme:
T. Bauer, S. Orlov, U. Peschel, P. B. and G. Leuchs, "Nanointerferometric Amplitude and Phase Reconstruction of Tightly Focused Vector Beams", Nat. Photon 8, 23 - 27 (2014).

Observing a Polarization Möbius Strip

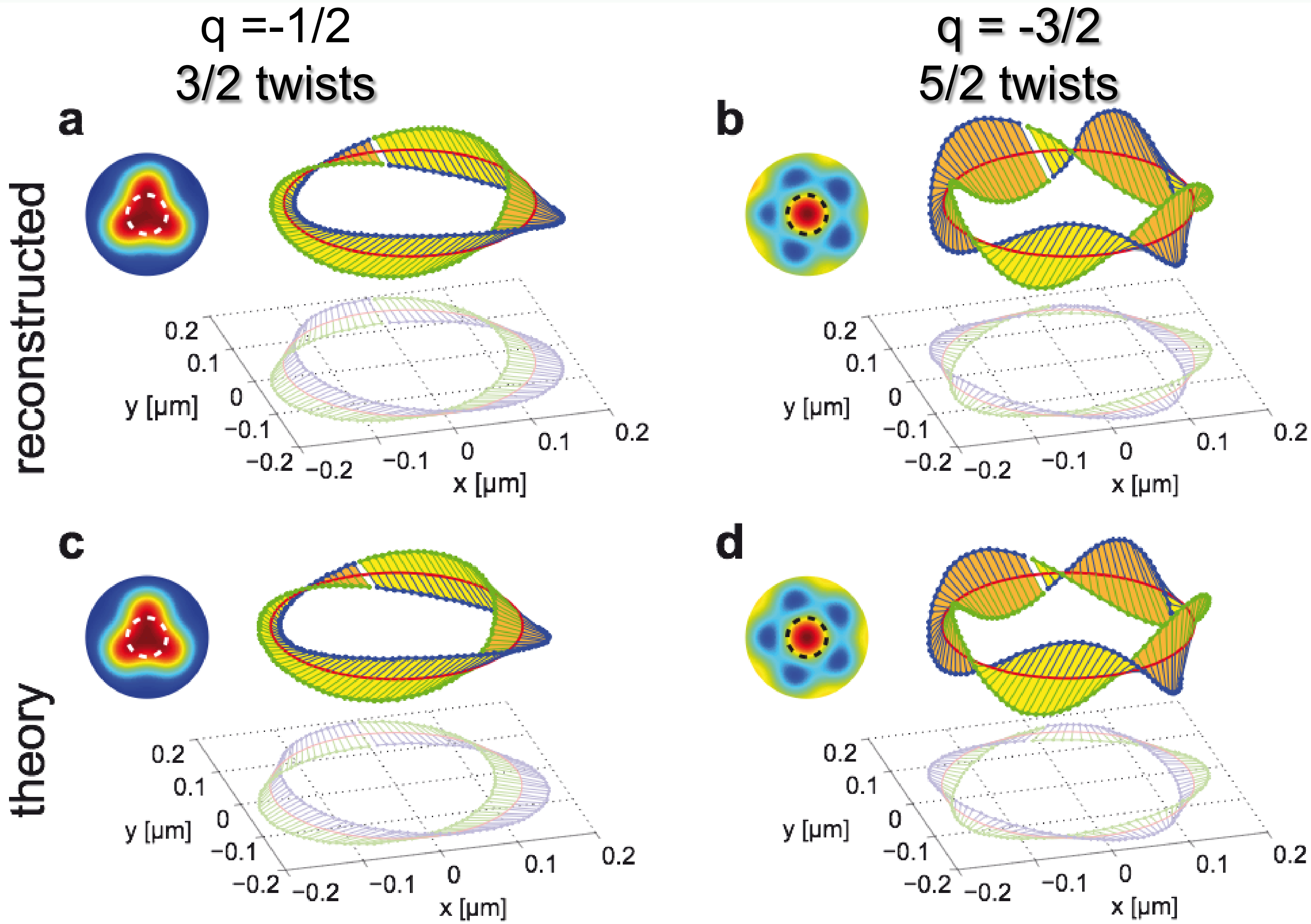


Crucial: tight focusing enhances the Möbius effect, which depends on the z component of the field

Bauer T, Banzer P, Karimi E, Orlovas S, Rubano A, Marrucci L, Santamato E, Boyd RW, and Leuchs G. Science, 2015.

Observation of Polarization Möbius Strips

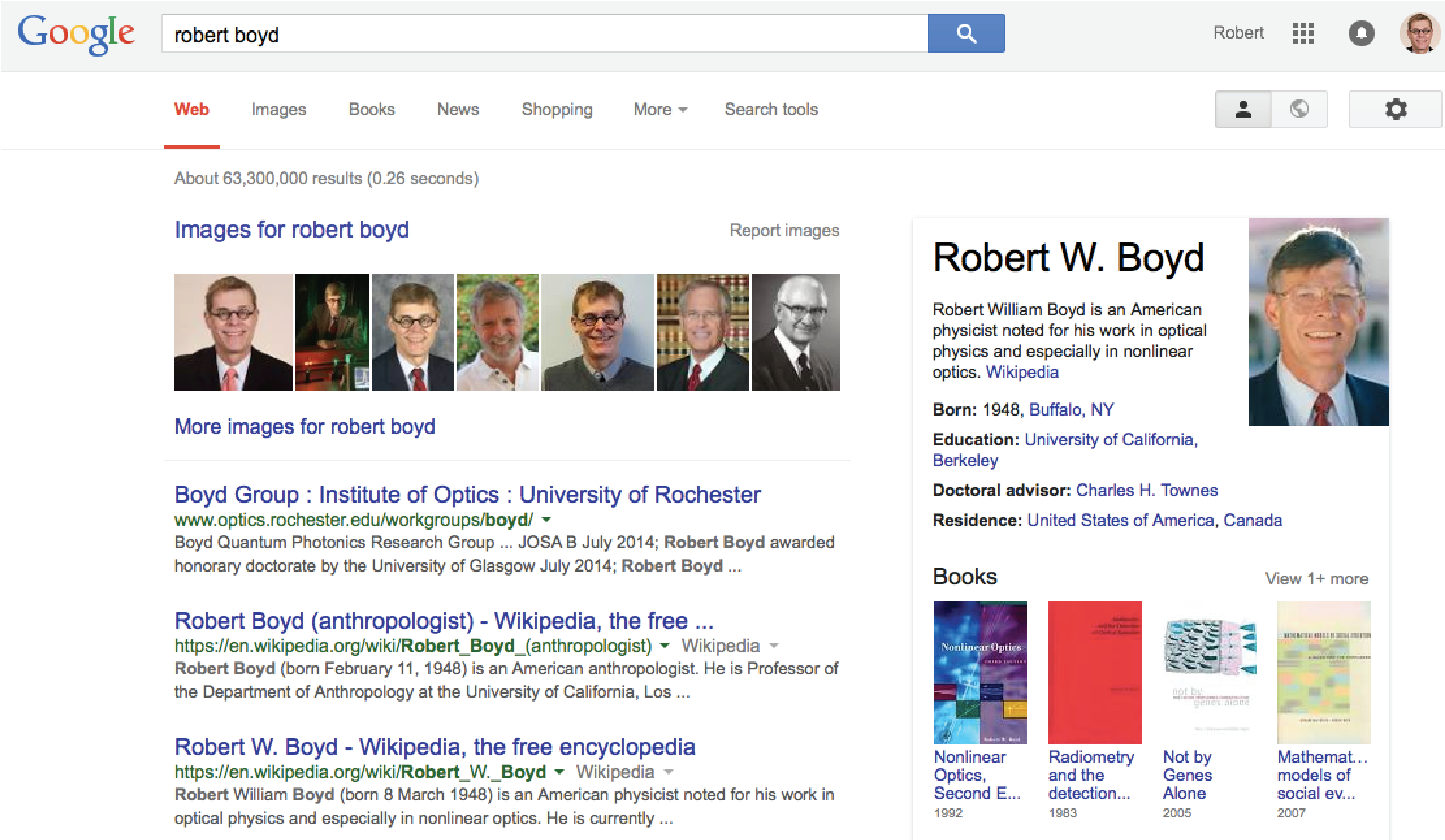
- Measured
- Theory



We find |q|+1 half twists

Bauer, Banzer, Karimi, Orlovas, Rubano, Marucci, Santamato, Boyd, Leuchs, Science (2015).

Why We Shouldn’t Always Trust Google



Our Work on “Slow Light” and Fresnel Drag

Controlling the Velocity of Light

"Slow," "Fast" and "Backwards" Light

– Light can be made to go:

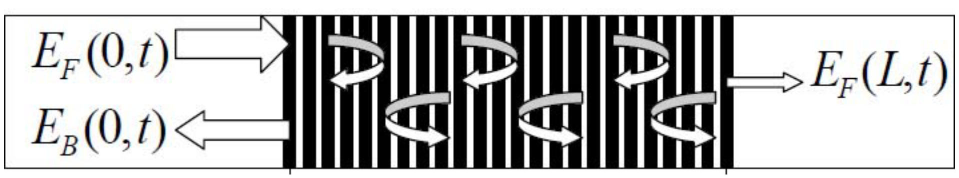
slow: $v_g << c$ (as much as 10^6 times slower!)

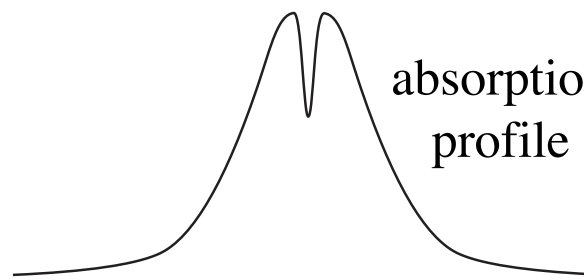
fast: $v_g > c$

backwards: v_g negative

Here v_g is the group velocity: $v_g = c/n_g \quad n_g = n + \omega\,(dn/d\omega)$

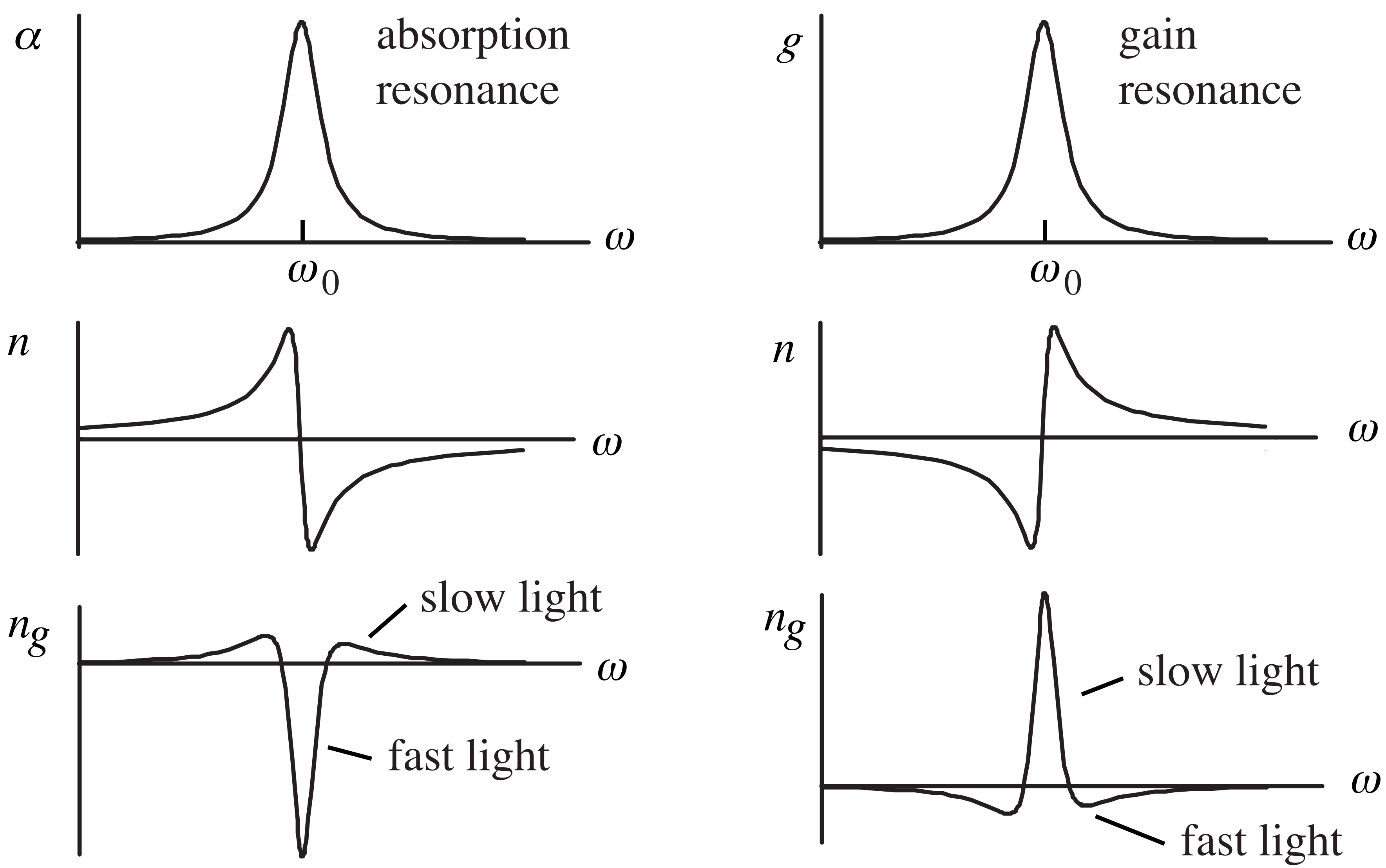
– Velocity controlled by structural or material resonances





Review article: Boyd and Gauthier, Science 326, 1074 (2009).

Slow and Fast Light Using Isolated Gain or Absorption Resonances

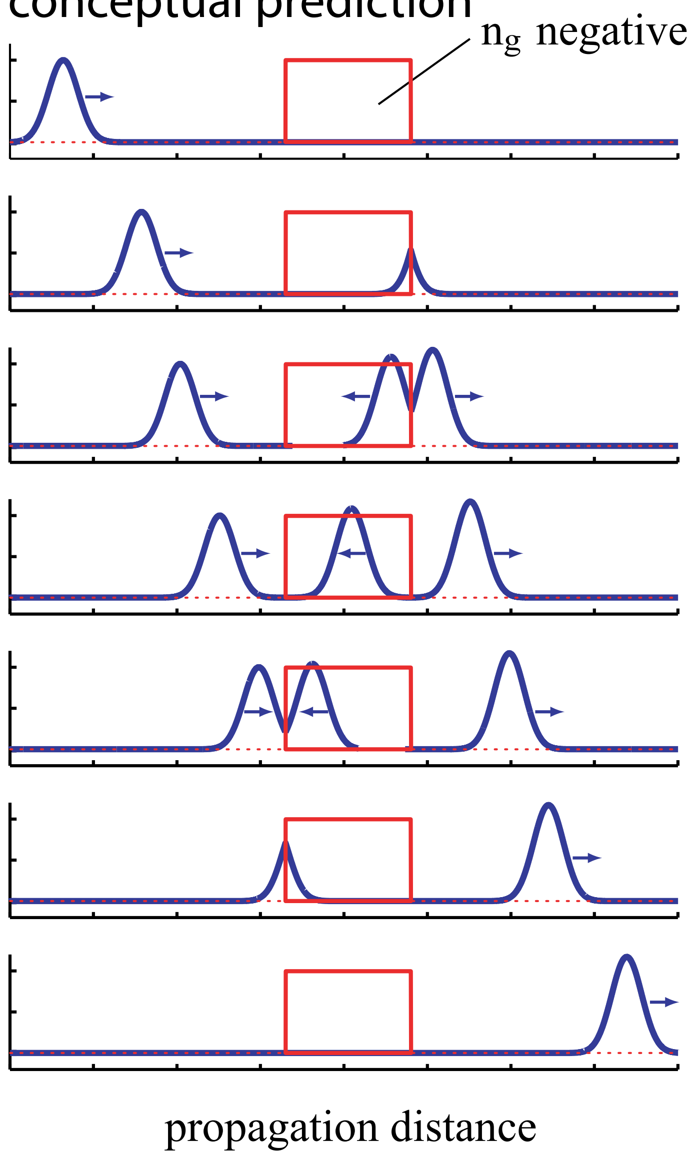


$$n_g = n + \omega\,(dn/d\omega)$$

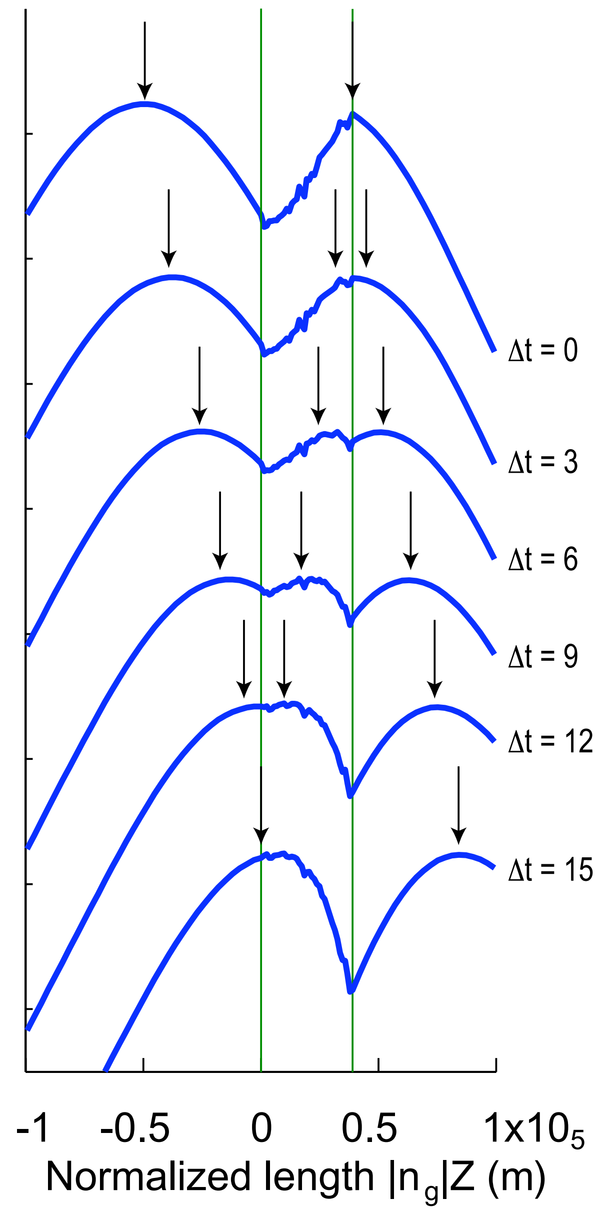
Observation of Superluminal and "Backwards" Pulse Propagation

- A strongly counterintuitive phenomenon
- But entirely consistent with established physics
- Predicted by Garrett and McCumber (1970) and Chiao (1993).
- Observed by Gehring, Schweinsberg, Barsi, Kostinski, and Boyd Science 312, 985 2006.

\- conceptual prediction



\- laboratory results

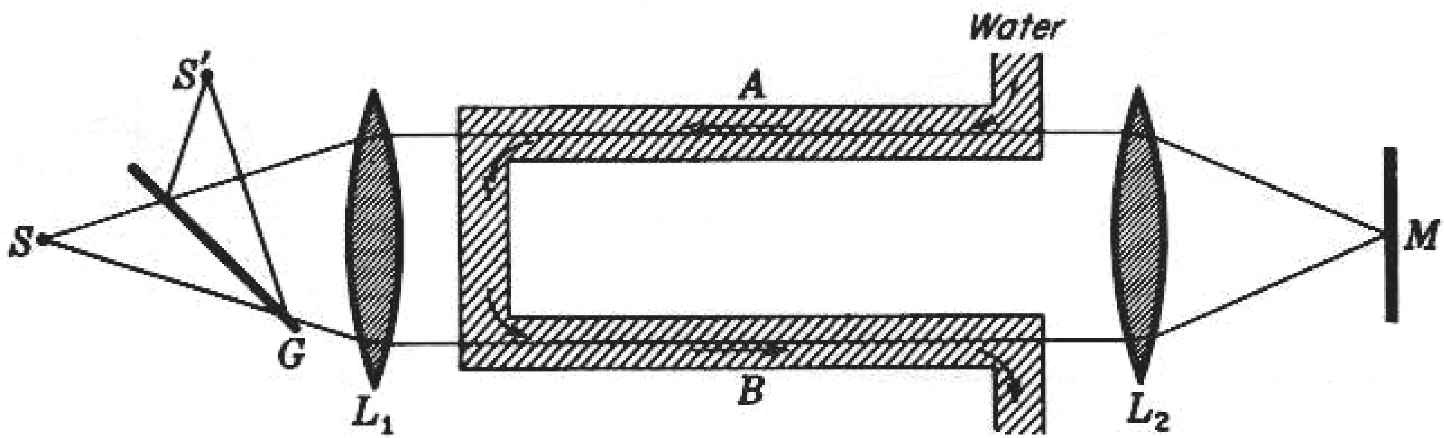


Velocity of (Slow) Light in Moving Matter: Photon Drag (or Ether Drag) Effects

Fizeau (1859): Longitudinal photon drag:

Velocity of light in flowing water.

V = 700 cm/sec; L = 150 cm; displacement of 0.5 fringe.



Modern theory: relativistic addition of velocities

$$v = \frac{c/n + V}{1 + (V/c)(1/n)} \approx \frac{c}{n} + V\left(1 - \frac{1}{n^2}\right)$$

Fresnel "drag" coefficient

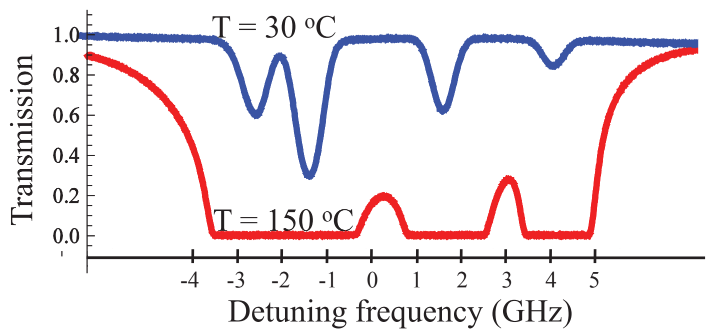
Dispersive Light Drag Experiment

Light Drag in a Slow Light Medium

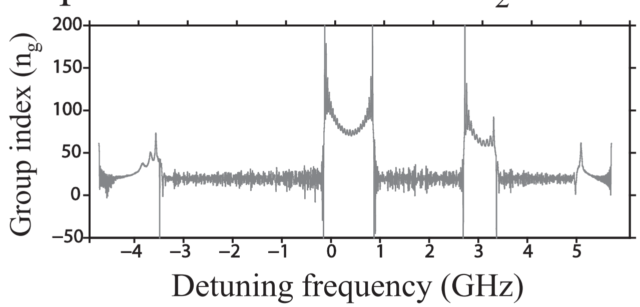
$$u \simeq \frac{c}{n} \pm v\left(1 - \frac{1}{n^2} + \frac{n_g - n}{n^2}\right)$$

We Use Rubidium as Our Slow Light Medium

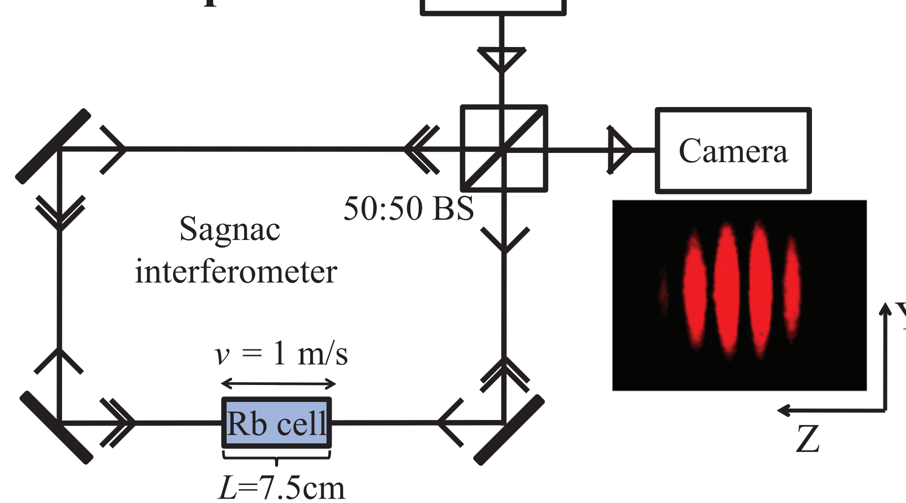
- Transmission spectrum of Rb around D_2 transition:

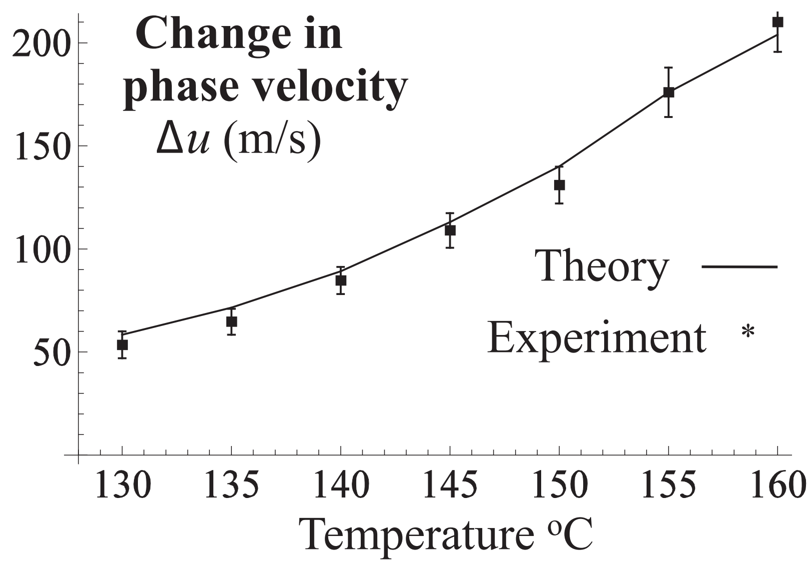


- Group index of Rb around D_2 line at T=130



Lab Setup





Safari, De Leon, Mirhosseini, Magana-Loaiza, and Boyd

Boyd Name Origin

(Road outside Glasgow)

Prospectus

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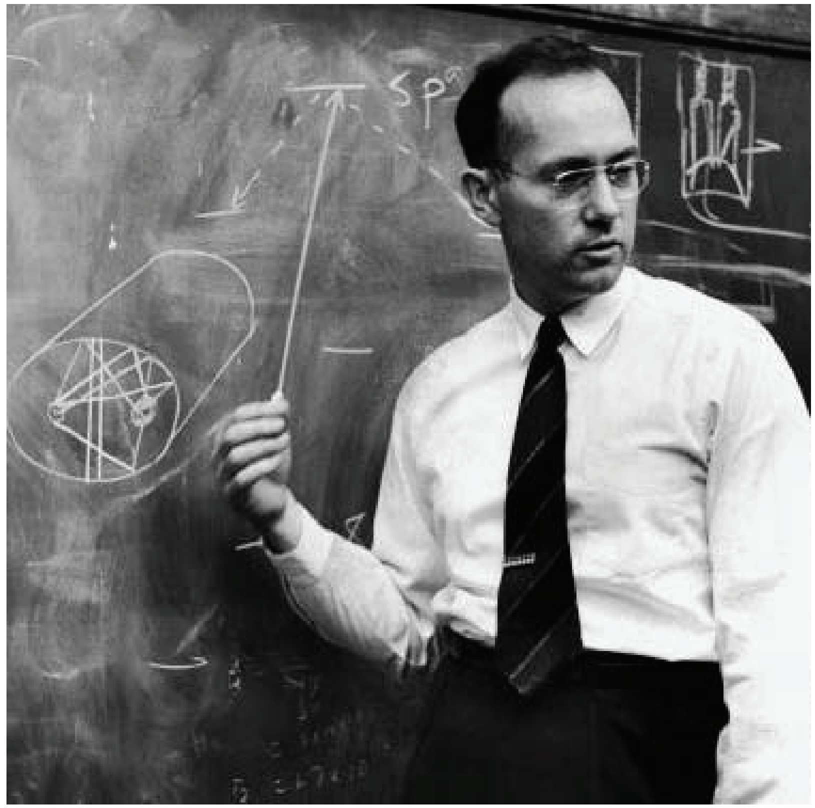
Charles H. Townes

July 28, 1915 to January 27, 2015



- Inventor of the maser and laser
- Nobel Prize, 1964
- Advisor to three US presidents
- Teacher, mentor, and friend

Charles Townes: Father of the Field of Photonics

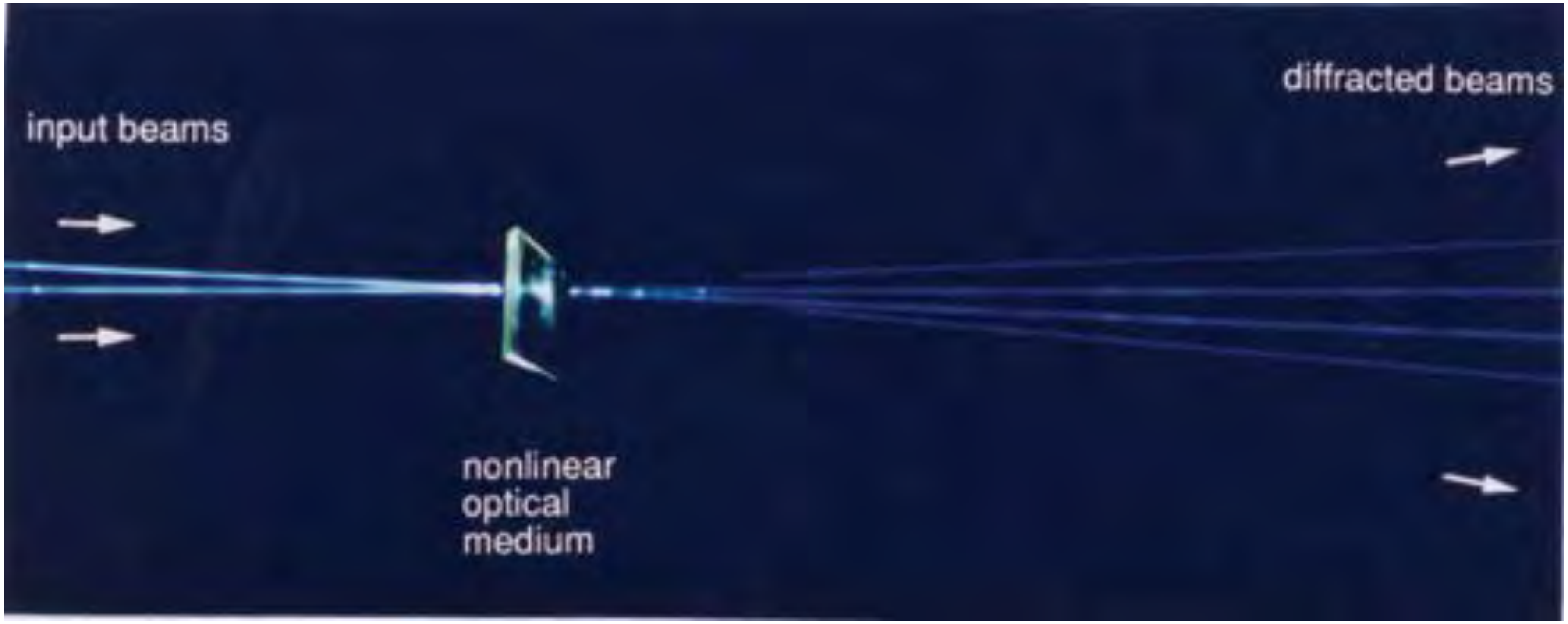
Part of the Charles Townes Legacy

Inventor of the Laser

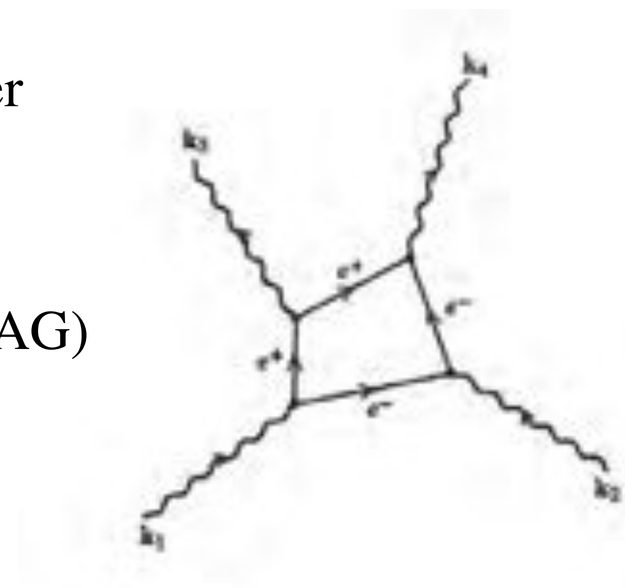
Discoverer of Fundamental Nonlinear Optical Processes
Autler-Townes Effect
Self Trapping of Light (Optical Solitons)
Stimulated Brillouin Scattering (SBS)

Research group discovered light-by-light scattering
(the essence of photonics)

Nonlinear Optics and Light-by-Light Scattering



The elementary process of light-by-light scattering has never been observed in vacuum, but is readily observed using the nonlinear response of material systems.



Nonlinear material is fluorescein-doped boric acid glass (FBAG)

n_2(FBAG) ≈ 10^{14} n_2(silica)   [But very slow response!]

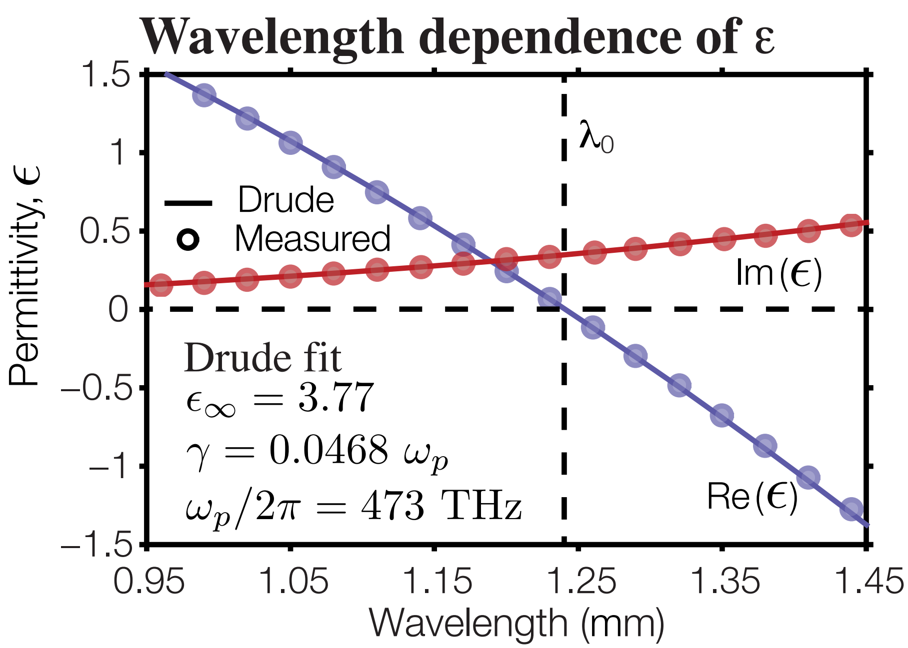
M. A. Kramer, W. R. Tompkin, and R. W. Boyd, Phys. Rev. A, 34, 2026, 1986.
W. R. Tompkin, M. S. Malcuit, and R. W. Boyd, Applied Optics 29, 3921, 1990.

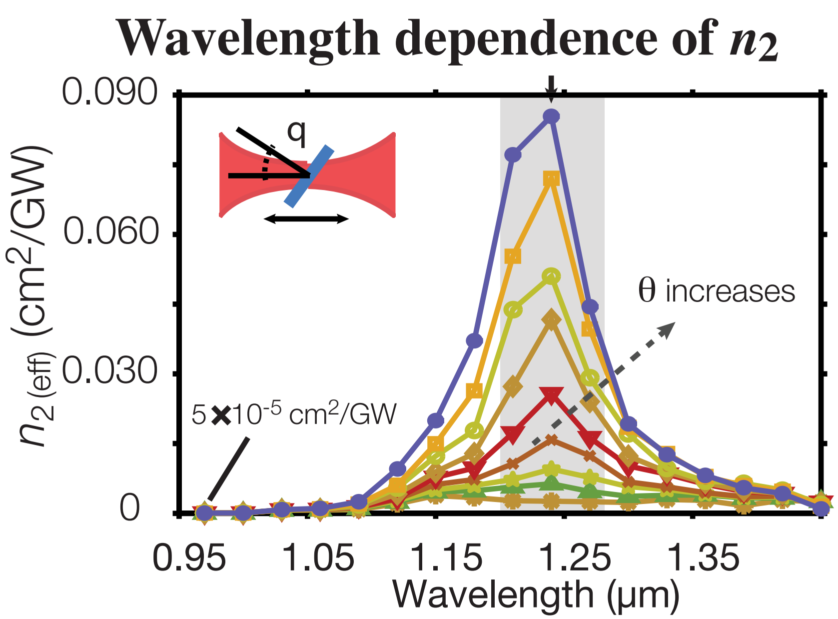
Huge Nonlinear Optical Response of ITO near its Epsilon-Near-Zero Wavelength

Indium Tin Oxide (ITO) displays enormously strong NLO properties:

- n_2 is 2.5 x 10^5 times that of fused silica
- nonlinear change in refractive index as large as 0.8
- response time of 270 fs

$$n_2 = \frac{3\chi^{(3)}}{4\epsilon_0 c\, n_0 \,\mathrm{Re}(n_0)}$$





Some possible new effects

- Waveguiding outside the "weakly-guiding" regime
- Efficient all-optical switching
- No need for phase-matching

Alam, De Leon, Boyd

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Quantum Information and Orbital Angular Momentum (OAM) of Light

- Utilize the transverse degree of freedom of light
  - Spatial division multiplexing

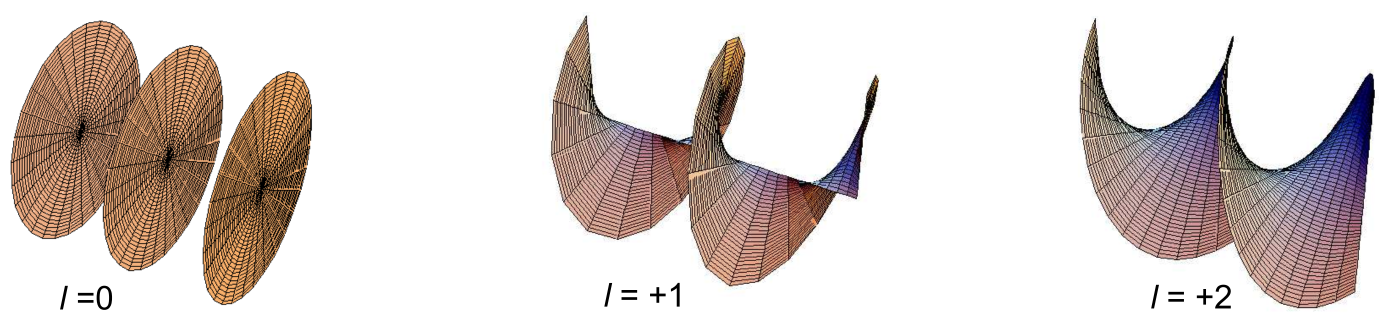
- In particular, encode in angular position and its conjugate vaiable, orbital angular momentum (OAM)

- Motivation: Encode more information and even more information per photon

What Are the OAM States of Light?

- Light can carry spin angular momentum (SAM) by means of its circular polarization.
- Light can also carry orbital angular momentum (OAM) by means of the phase winding of the optical wavefront.
- A well-known example are the Laguerre-Gauss modes. These modes contain a phase factor of $\exp(il\phi)$ and carry angular momentum of $l\hbar$ per photon. (Here ϕ is the azimuthal coordinate.)

Phase-front structure of some OAM states



See, for instance, A.M. Yao and M.J. Padgett, Advances in Photonics 3, 161 (2011).

Laguerre-Gauss Modes

The paraxial approximation to the Helmholtz equation $(\nabla^2 + k^2)E(\boldsymbol{k}) = 0$ gives the paraxial wave equation which is written in the cartesian coordinate system as

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + 2ik\frac{\partial}{\partial z}\right) E(x, y, z) = 0. \tag{1}$$

The paraxial wave equation is satisfied by the Laguerre-Gaussian modes, a family of orthogonal modes that have a well defined orbital angular momentum. The field amplitude $LG_p^l(\rho, \phi, z)$ of a normalized Laguerre-Gaussian modes is given by

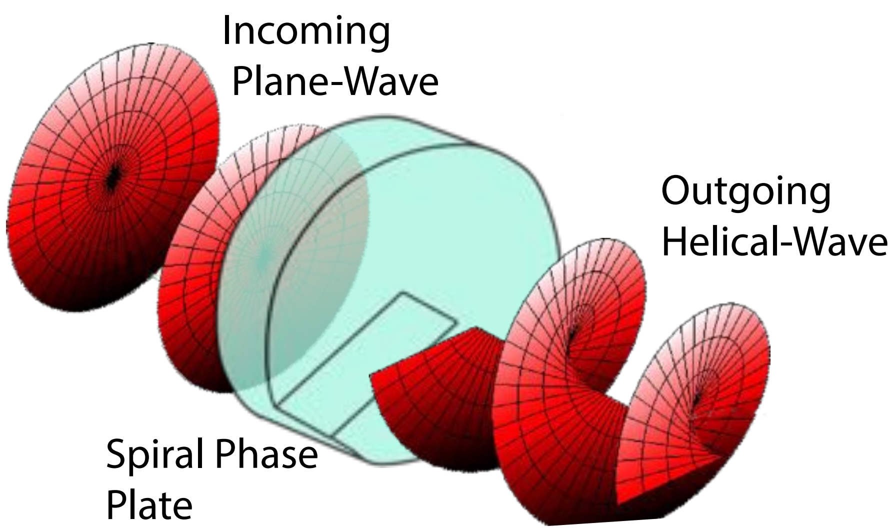
$$LG_p^l(\rho, \phi, z) = \sqrt{\frac{2p!}{\pi(|l| + p)!}} \frac{1}{w(z)} \left[\frac{\sqrt{2}\rho}{w(z)}\right]^{|l|} L_p^l \left[\frac{2\rho^2}{w^2(z)}\right] \\ \times \exp\left[-\frac{\rho^2}{w^2(z)}\right] \exp\left[-\frac{ik^2\rho^2 z}{2(z^2 + z_R^2)}\right] \exp\left[i(2p + |l| + 1)\tan^{-1}\left(\frac{z}{z_R}\right)\right] e^{-il\phi}, \tag{2}$$

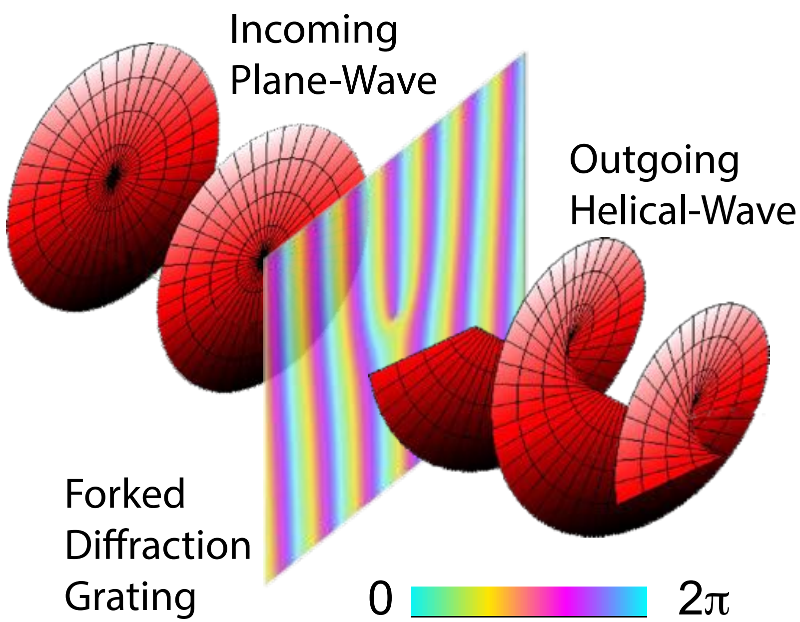
where k is the wave-vector magnitude of the field, z_R the Rayleigh range, $w(z)$ the radius of the beam at z, l is the azimuthal quantum number, and p is the radial quantum number. L_p^l is the associated Laguerre polynomial.

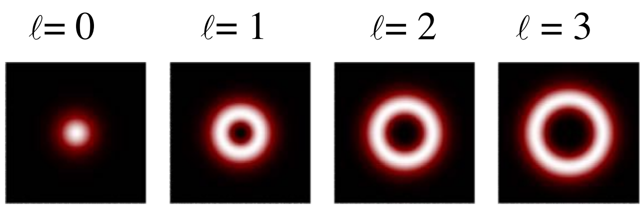
How to create a beam carrying orbital angular momentum?

- Pass beam through a spiral phase plate

- Use a spatial light modulator acting as a computer generated hologram (more versatile)







Exact solution to simultaneous intensity and phase masking with a single phase-only hologram, E. Bolduc, N. Bent, E. Santamato, E. Karimi, and R. W. Boyd, Optics Letters 38, 3546 (2013).

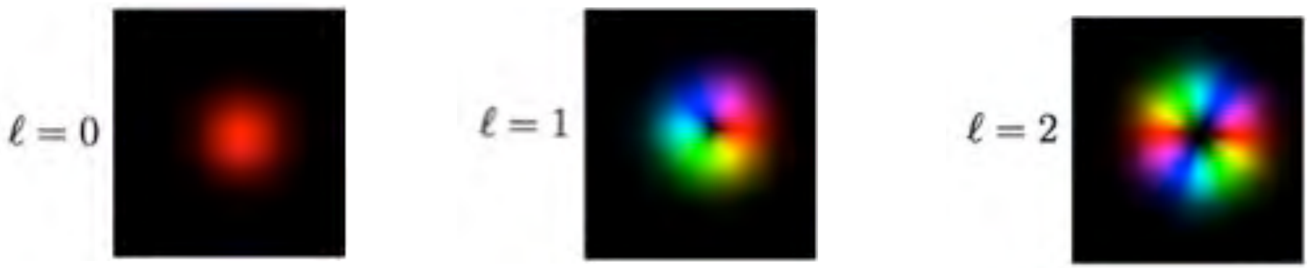
Background

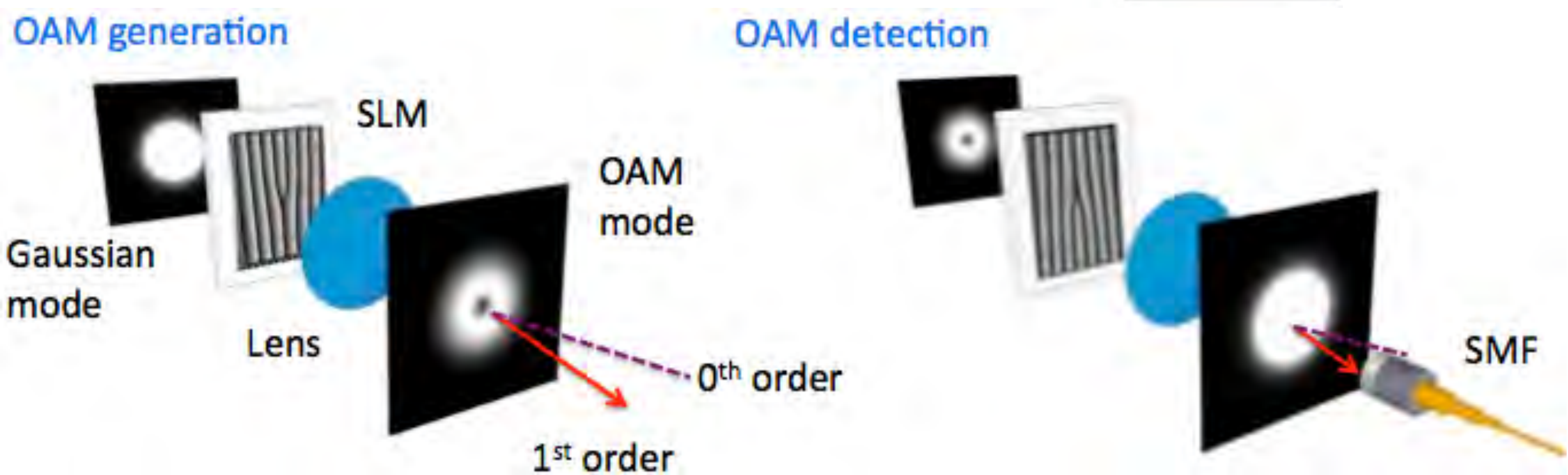
PHYSICAL REVIEW A VOLUME 45, NUMBER 11 1 JUNE 1992

Orbital angular momentum of light and the transformation of Laguerre-Gaussian laser modes

L. Allen, M. W. Beijersbergen, R. J. C. Spreeuw, and J. P. Woerdman
Huygens Laboratory, Leiden University, P.O. Box 9504, 2300 RA Leiden, The Netherlands
(Received 6 January 1992)

A beam of light with helicity of the phase front given by the azimuthal phase dependence of $e^{il\phi}$ carries orbital angular momentum (OAM).





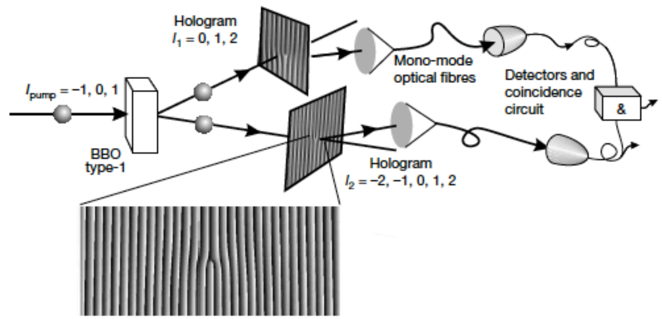
Background

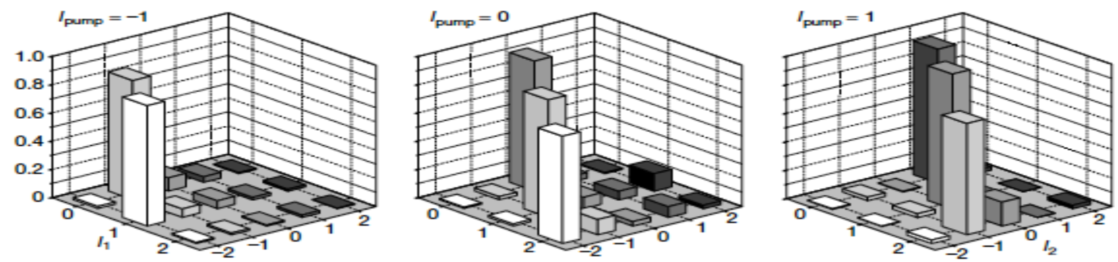
Entanglement of the orbital angular momentum states of photons

Alois Mair*, Alipasha Vaziri, Gregor Weihs & Anton Zeilinger

Institut für Experimentalphysik, Universität Wien, Boltzmanngasse 5, 1090 Wien, Austria

NATURE | VOL 412 | 19 JULY 2001 | www.nature.com



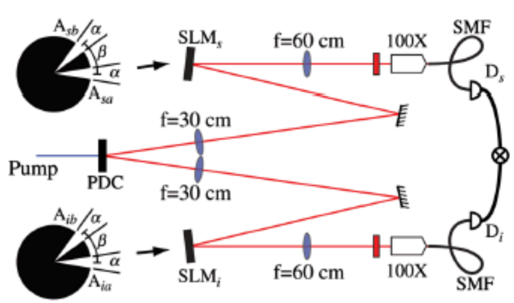


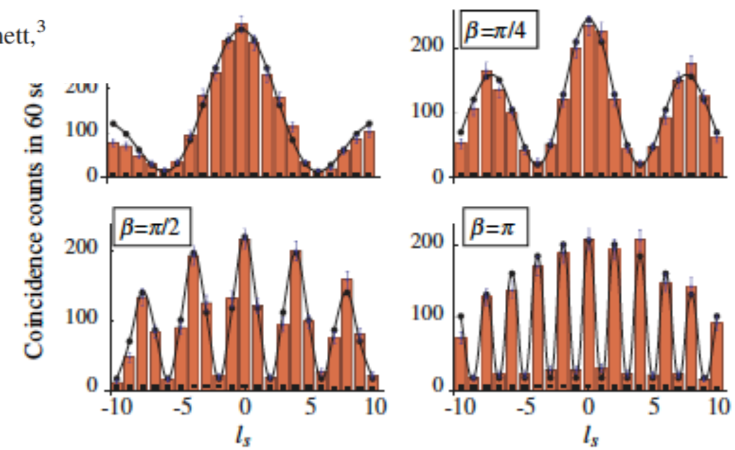
PRL **104,** 010501 (2010) PHYSICAL REVIEW LETTERS week ending 8 JANUARY 2010

Angular Two-Photon Interference and Angular Two-Qubit States

Anand Kumar Jha,[1] Jonathan Leach,[2] Barry Jack,[2] Sonja Franke-Arnold,[2] Stephen M. Barnett,[3] Robert W. Boyd,[1] and Miles J. Padgett[2]

(a)



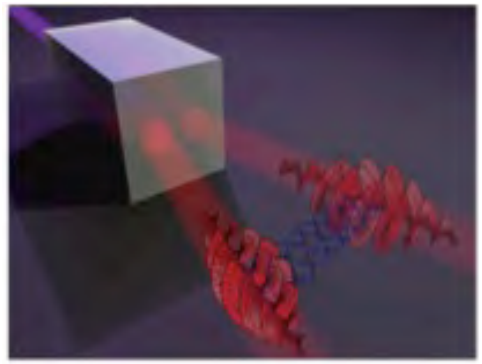


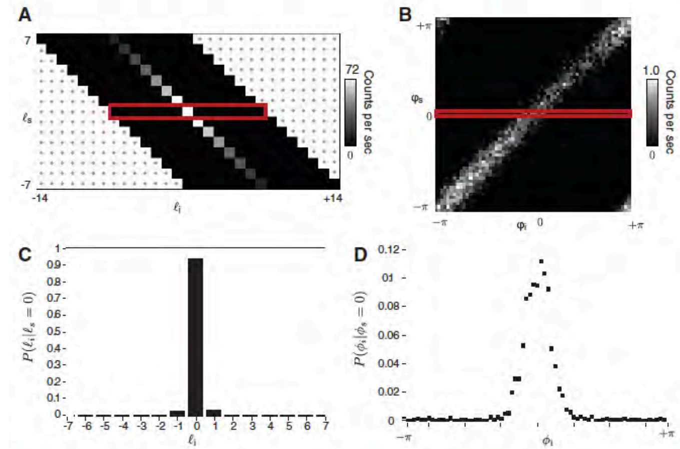
Background

Quantum Correlations in Optical Angle–Orbital Angular Momentum Variables

Jonathan Leach,[1] Barry Jack,[1] Jacqui Romero,[1] Anand K. Jha,[2] Alison M. Yao,[3] Sonja Franke-Arnold,[1] David G. Ireland,[1] Robert W. Boyd,[2] Stephen M. Barnett,[3] Miles J. Padgett[1*]

6 AUGUST 2010 VOL 329 SCIENCE www.sciencemag.org

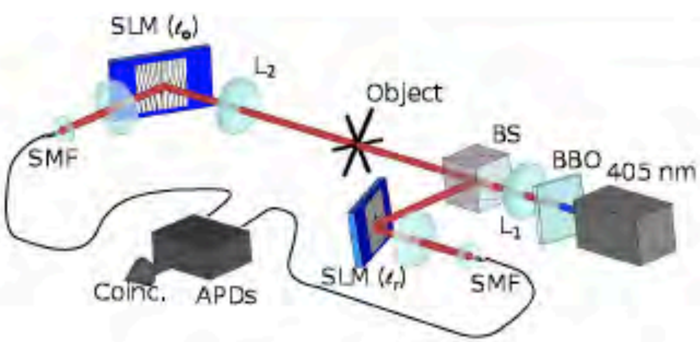


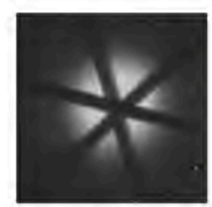


PRL 110, 043601 (2013) PHYSICAL REVIEW LETTERS week ending 25 JANUARY 2013

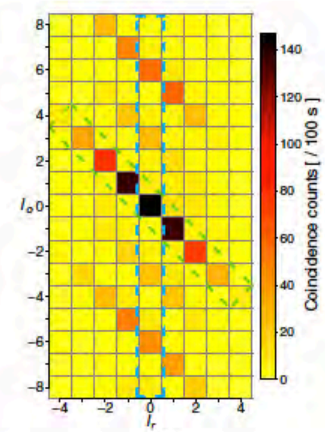
Object Identification Using Correlated Orbital Angular Momentum States

Néstor Uribe-Patarroyo,[1,*] Andrew Fraine,[1] David S. Simon,[1,2] Olga Minaeva,[3] and Alexander V. Sergienko[1,4]





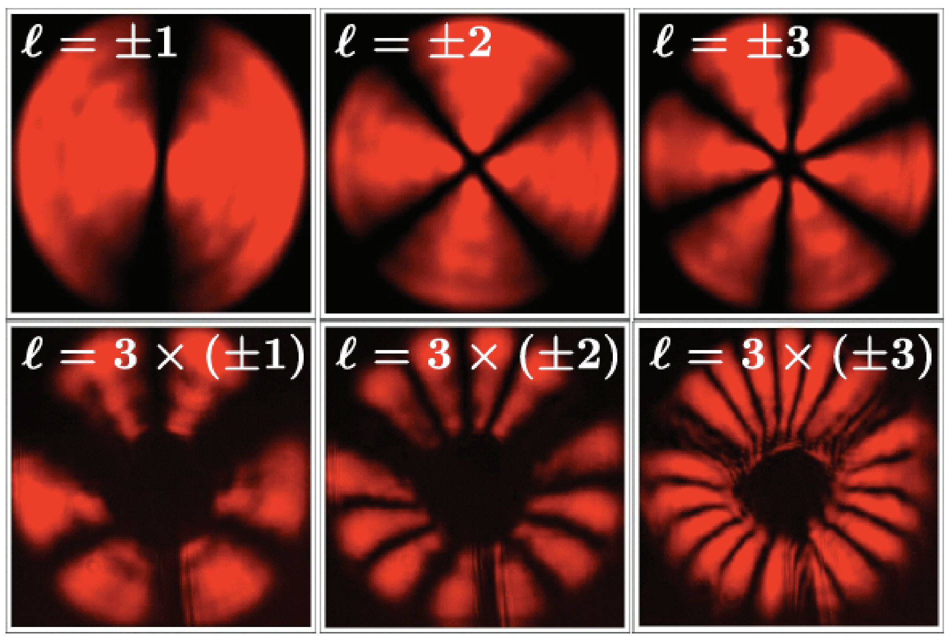
(a)



(b)

Quantum Hilbert Hotel

Václav Potoček,[1,2] Filippo M. Miatto,[3,4,*] Mohammad Mirhosseini,[5,†] Omar S. Magaña-Loaiza,[5]
Andreas C. Liapis,[5] Daniel K. L. Oi,[6] Robert W. Boyd,[4,5,1] and John Jeffers[6]
[1]*SUPA School of Physics and Astronomy, University of Glasgow, Glasgow G12 8QQ, United Kingdom*
[2]*Department of Physics, Czech Technical University in Prague, Faculty of Nuclear Sciences and Physical Engineering, Břehová 7, 115 19 Praha 1, Czech Republic*
[3]*Institute for Quantum Computing and Department of Physics and Astronomy, University of Waterloo, 200 University Avenue West, Waterloo, Ontario N2L 3G1, Canada*
[4]*Department of Physics, University of Ottawa, 150 Louis Pasteur, Ottawa, Ontario K1N 6N5, Canada*
[5]*The Institute of Optics, University of Rochester, 500 Joseph C. Wilson Boulevard, Rochester, New York 14627, USA*
[6]*SUPA Department of Physics, University of Strathclyde, Glasgow G4 0NG, United Kingdom*
(Received 15 June 2015; published 15 October 2015)

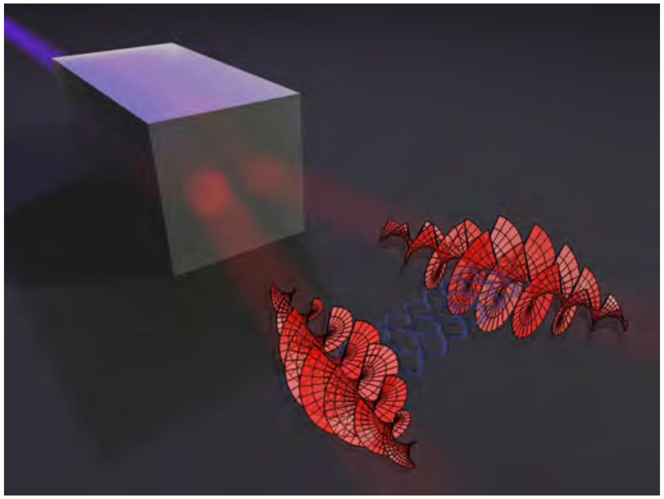


Use of Quantum States for Secure Optical Communication

- The celebrated BB84 protocol for quantum key distribution (QKD) transmits one bit of information per received photon

- We have built a QKD system that can carry more than one bit per photon.
  - Note that in traditional telecom, one uses many photons per bit!

- Our procedure is to encode using beams that carry orbital angular momentum (OAM), such as the Laguerre-Gauss states, which reside in an infinite dimensional Hilbert space.



QKD System Carrying Many Bits Per Photon

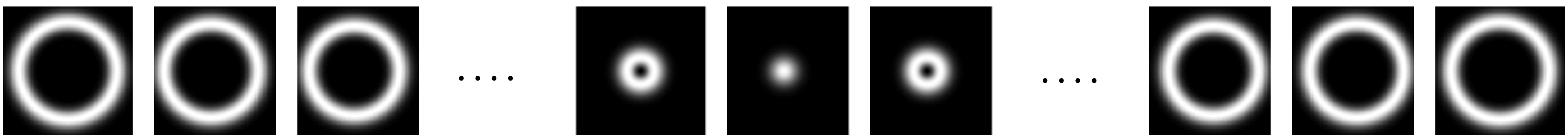
We are constructing a QKD system in which each photon carries many bits of information

We encode in states that carry OAM such as the Laguerre-Gauss states

As a diagnostic, we need to be able to measure the statevector of OAM states

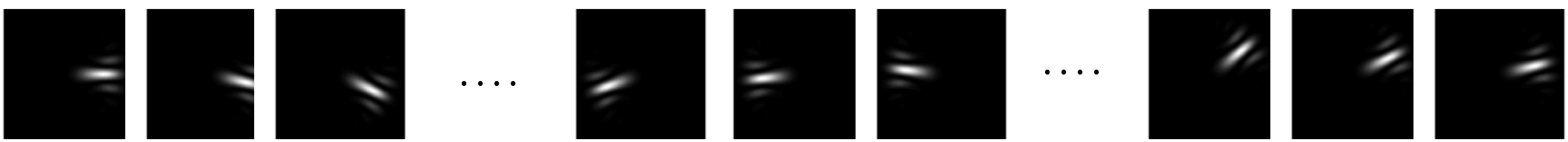
Single Photon States

Laguerre-Gaussian Basis $\ell = -13, \ldots, 13$



-13 -12 -11 .... -1 0 1 .... 11 12 13

"Angular" Basis (mutually unbiased with respect to LG)



0 1 2 .... 12 13 14 .... 25 26 27

$$\Psi_{\mathrm{AB}}^{N} = \frac{1}{\sqrt{27}} \sum_{l=-13}^{13} \mathrm{LG}_{l,0} \exp\left(i 2\pi N l / 27\right)$$

Alice (Alison) and Bob

Is there an eavesdropper?

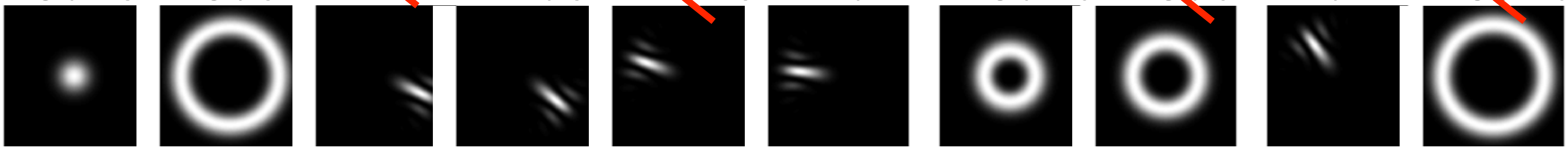
Alice (Alison) and Bob

(St)eavesdropper!

Is there an eavesdropper?

Protocol

Alice	LG:13	LG:3	~~AB:2~~	AB:3	~~AB:15~~	AB:14	LG:16	~~LG:8~~	AB:24	~~LG:26~~



Bob	LG	LG	~~LG~~	AB	~~LG~~	AB	LG	~~AB~~	AB	~~AB~~
	↓	↓		↓		↓	↓		↓	
Result	13	3	~~15~~	3	~~15~~	14	16	~~17~~	24	~~10~~

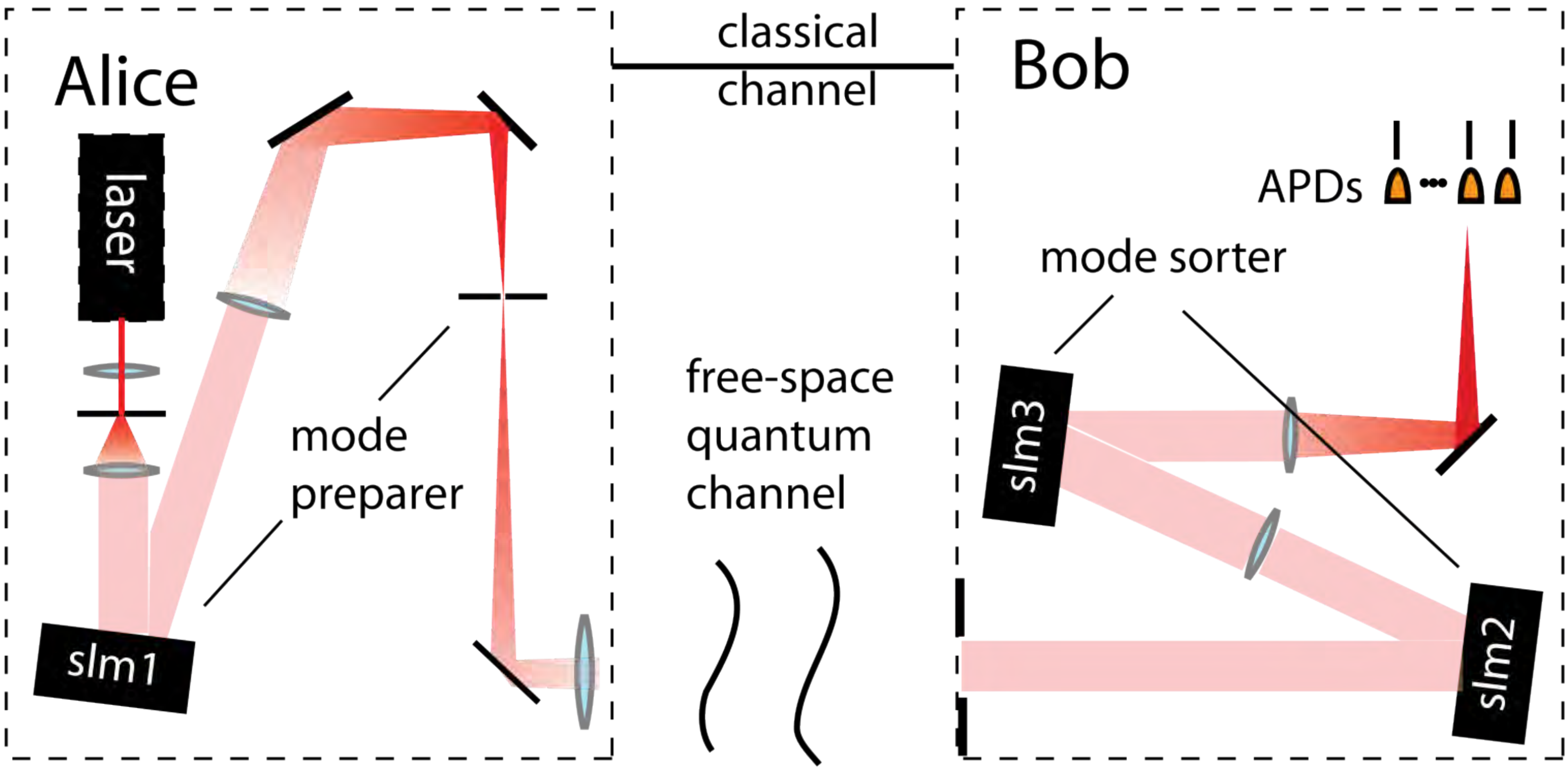
Sifted Key 13 3 3 14 16 24 … ← in principle contains no errors unless eavesdropper is present.

In any real system, Bob's key will have errors due to system imperfections.

1. Error Correction (Cascade Protocol)
2. Privacy Amplification

Under many conditions, these protocols can be successfully implemented if Alice/Bob share more bits of information than Alice and Eve.

Spatially Based QKD System



<u>Source</u>

Weak Coherent Light

Heralded Single Photon

<u>Protocol</u>

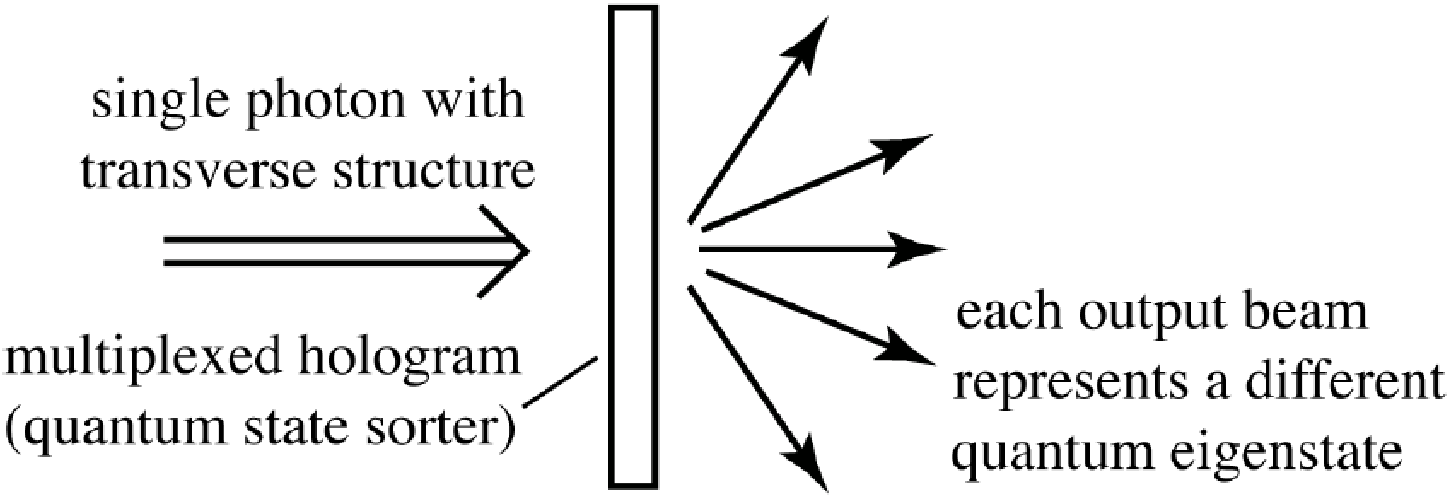
Modified BB84 as discussed

<u>Challenges</u>

1. State Preparation
2. State Detection
3. Turbulence

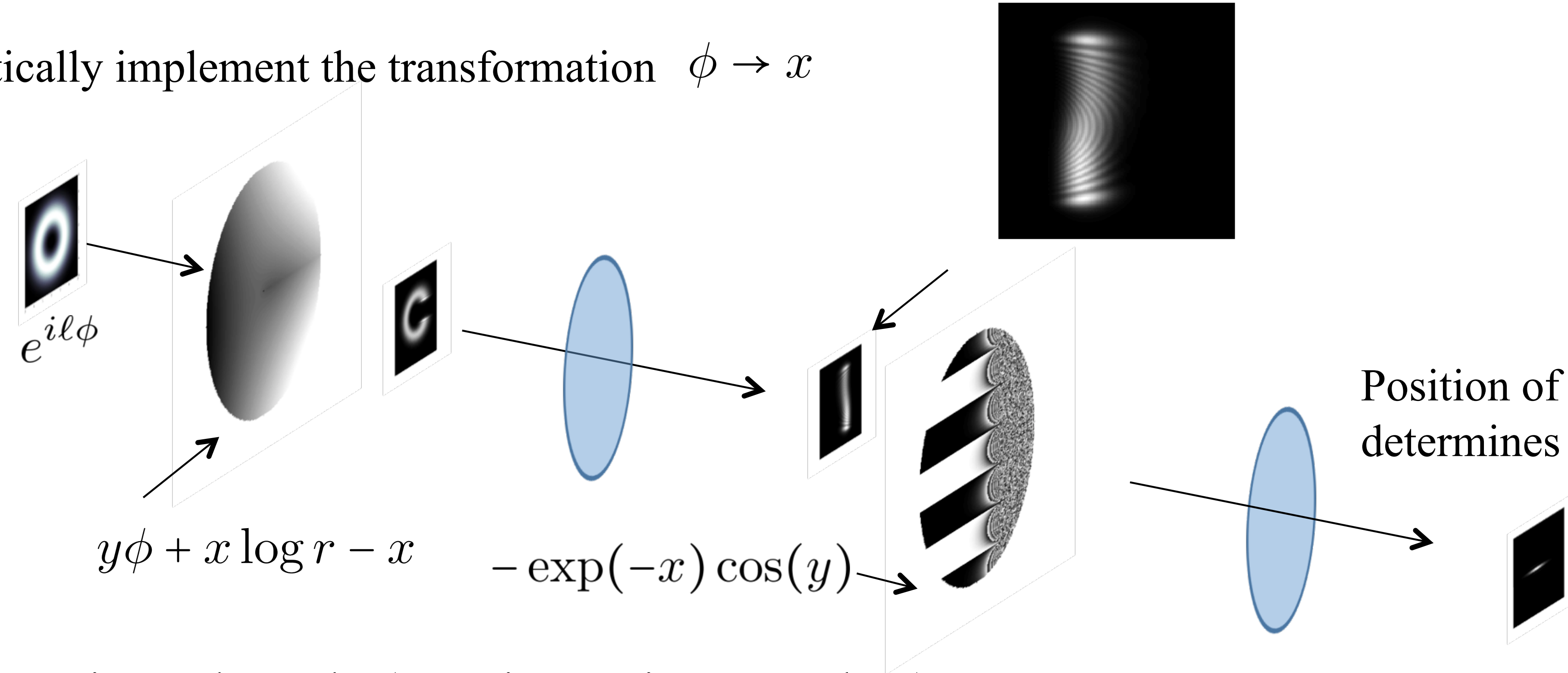
Mode Sorting

A mode sorter

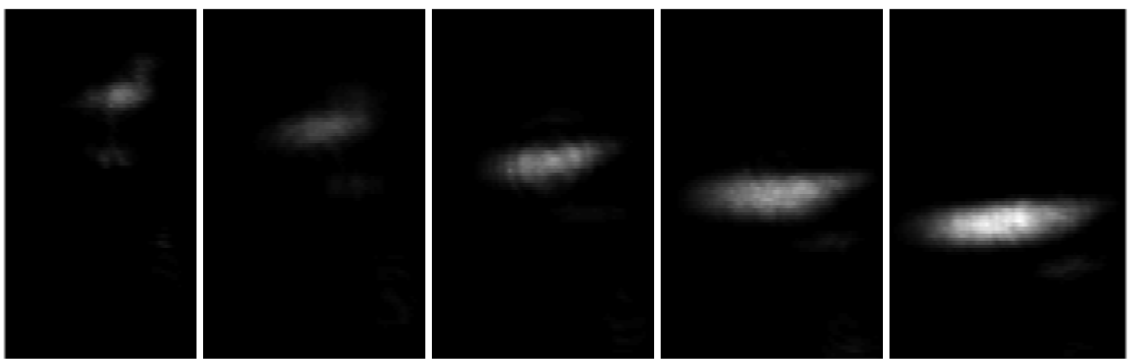


Sorting OAM using Phase Unwrapping

Optically implement the transformation $\phi \rightarrow x$



Experimental Results (CCD images in output plane)



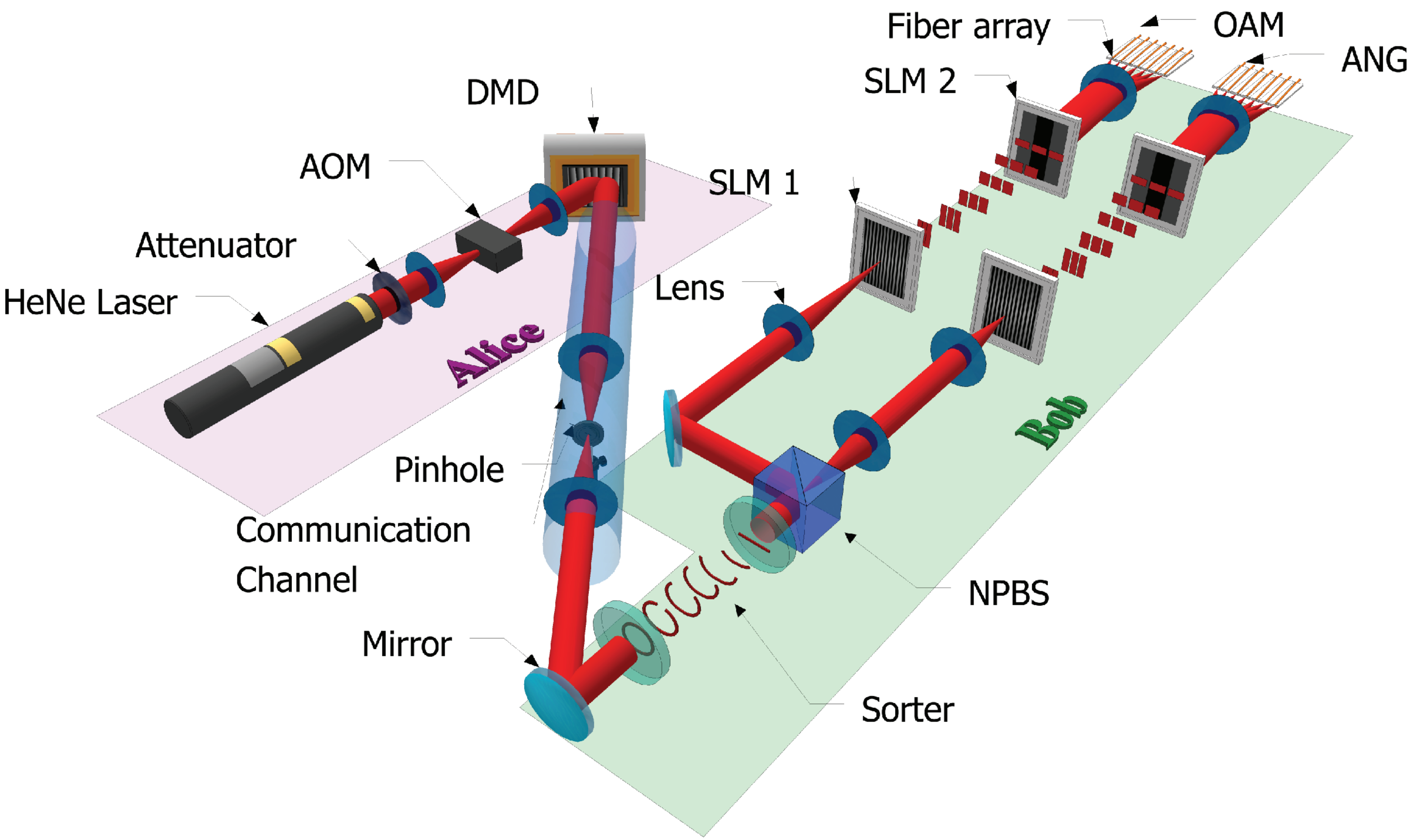
$\ell = 1$ $\ell = 2$ $\ell = 3$ $\ell = 4$ $\ell = 5$

-Can also sort angular position states.

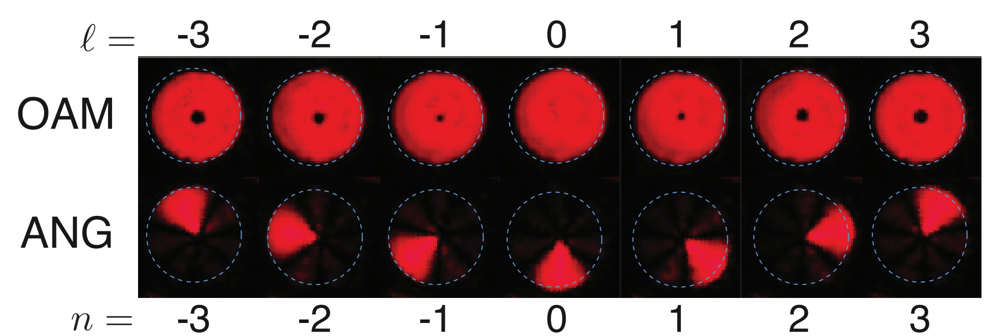
-Limited by the overlap of neighboring states.

*Berkhout *et al.* *PRL* **105,** 153601 (2010).
O. Bryngdahl, *J. Opt. Soc. Am.* **64**, 1092 (1974).

Our Laboratory Setup



We use a seven-dimensional state space



Laboratory Results - OAM-Based QKD

Alice

063133602132045444456141026645
545050363603025261643215524164
230146602513401613222451551026

Bob

063033601132021444456141026645
545022353603025261643215524164
230146602515403613422451551026

Error Correction

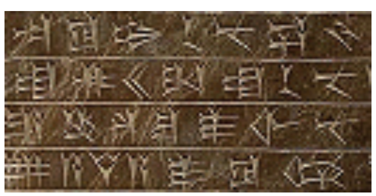
0000 0100 0111 0100 0101 0001 1100 0010 1100 1001 0010 0100 1101 0001 0000
0001 0011 0100 0000 0100 0010 0001 0001 0011 0000 0101 0010 0111 1000 0101
1101 0011 0001 1001 1011 0101 0000 0100 0111 0100 0011 0110 0101 0000 1101
0101 0010 1111 1010 1111 0010 1000

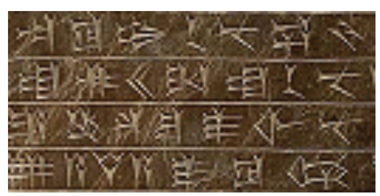
Privacy Amplification

0001 0100 1011 1111 1111 1010 1010 0111 1010 1011 1111 0001 0000 1101 1111
1110 1110 1101 1110 0001 0011 1000 0101 1101 1100 0011 0111 1101 0001 1001
0100 0110 1011 0000 0110 1010 1

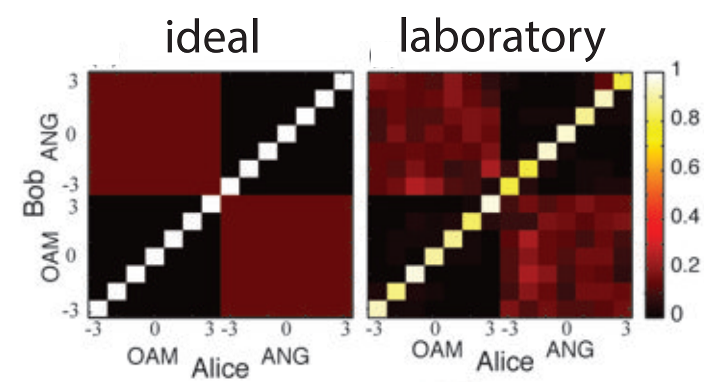
Alice Encrypts

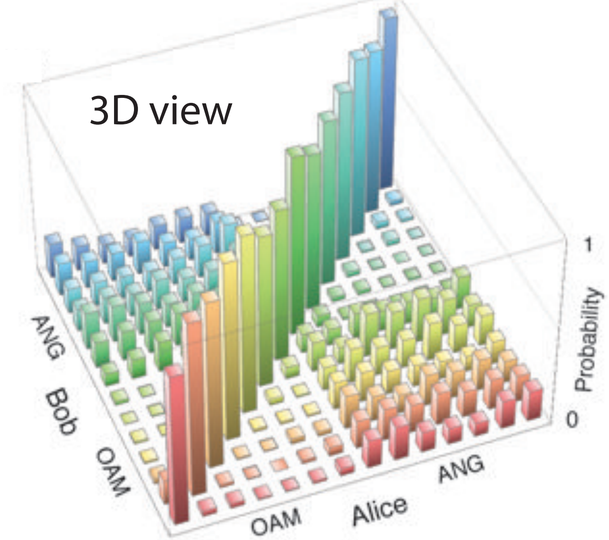
Bob Decrypts



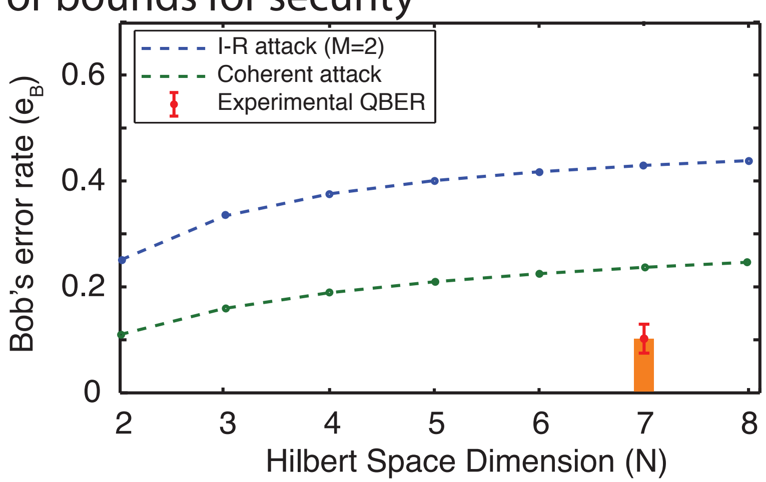


cross-talk matrices





- error bounds for security



We use a 7-letter alphabet, and achieve a channel capacity of 2.1 bits per sifted photon.

We do not reach the full 2.8 bits per photon for a variety of reasons, including dark counts in our detectors and cross-talk among channels resulting from imperfections in our sorter.

Nonetheless, our error rate is adequately low to provide full security,

ARTICLES
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nature photonics

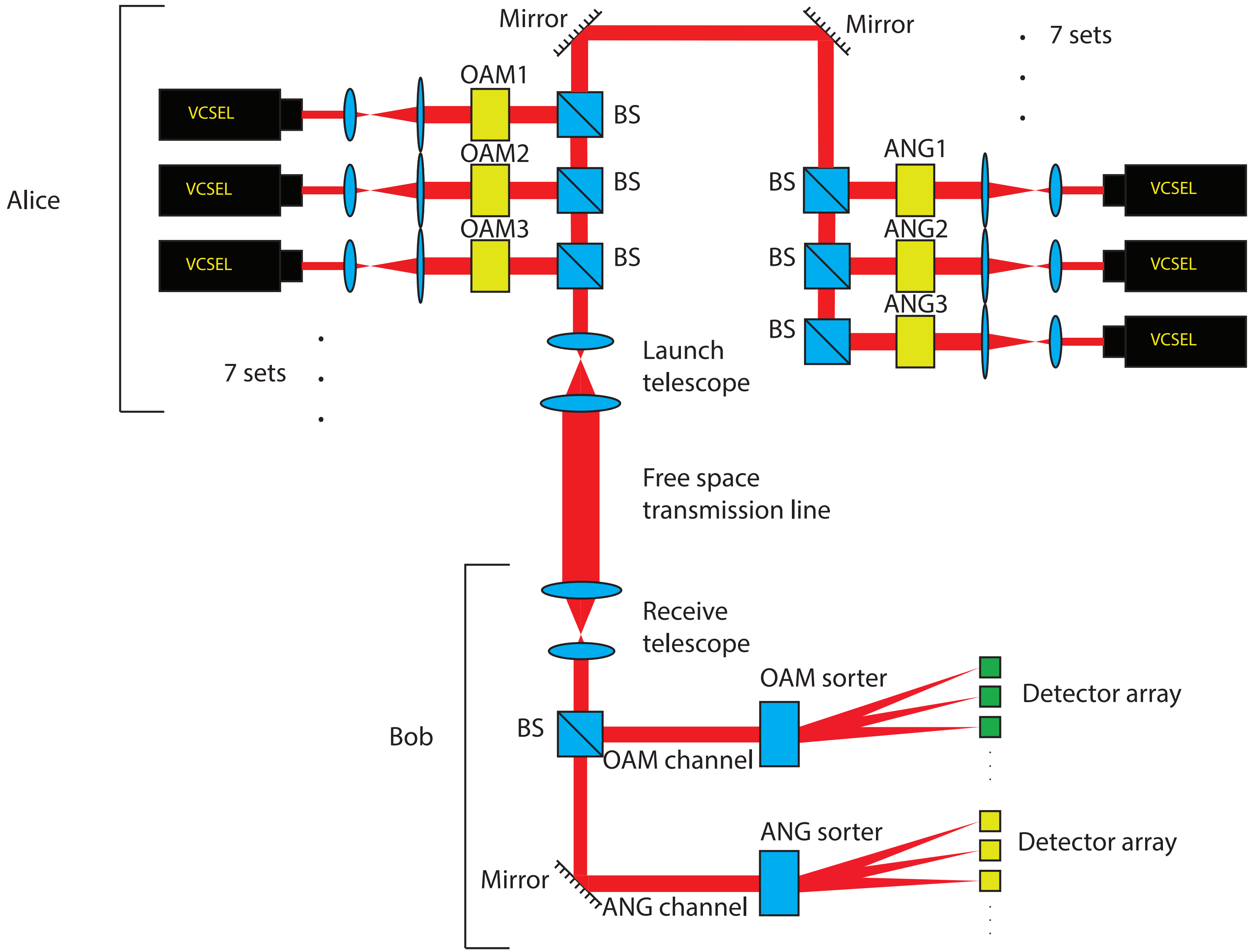
Terabit free-space data transmission employing orbital angular momentum multiplexing

Jian Wang[1,2]*, Jeng-Yuan Yang[1], Irfan M. Fazal[1], Nisar Ahmed[1], Yan Yan[1], Hao Huang[1], Yongxiong Ren[1], Yang Yue[1], Samuel Dolinar[3], Moshe Tur[4] and Alan E. Willner[1]*

The recognition in the 1990s that light beams with a helical phase front have orbital angular momentum has benefited applications ranging from optical manipulation to quantum information processing. Recently, attention has been directed towards the opportunities for harnessing such beams in communications. Here, we demonstrate that four light beams with different values of orbital angular momentum and encoded with 42.8×4 Gbit s^{-1} quadrature amplitude modulation (16-QAM) signals can be multiplexed and demultiplexed, allowing a 1.37 Tbit s^{-1} aggregated rate and 25.6 bit s^{-1} Hz^{-1} spectral efficiency when combined with polarization multiplexing. Moreover, we show scalability in the spatial domain using two groups of concentric rings of eight polarization-multiplexed 20×4 Gbit s^{-1} 16-QAM-carrying orbital angular momentum beams, achieving a capacity of 2.56 Tbit s^{-1} and spectral efficiency of 95.7 bit s^{-1} Hz^{-1}. We also report data exchange between orbital angular momentum beams encoded with 100 Gbit s^{-1} differential quadrature phase-shift keying signals. These demonstrations suggest that orbital angular momentum could be a useful degree of freedom for increasing the capacity of free-space communications.

Next Step: gigabit-per-second OAM-based QKD system

- Use direct modulation of laser diode to encode at gigabits per sec.



Our Program in “Weak Values”

How the Result of a Measurement of a Component of the Spin of a Spin-$\frac{1}{2}$ Particle Can Turn Out to be 100

Yakir Aharonov, David Z. Albert, and Lev Vaidman
Physics Department, University of South Carolina, Columbia, South Carolina 29208, and School of Physics and Astronomy, Tel-Aviv University, Ramat Aviv 69978, Israel
(Received 30 June 1987)

We have found that the usual measuring procedure for preselected and postselected ensembles of quantum systems gives unusual results. Under some natural conditions of weakness of the measurement, its result consistently defines a new kind of value for a quantum variable, which we call the weak value. A description of the measurement of the weak value of a component of a spin for an ensemble of preselected and postselected spin-$\frac{1}{2}$ particles is presented.

PACS numbers: 03.65.Bz

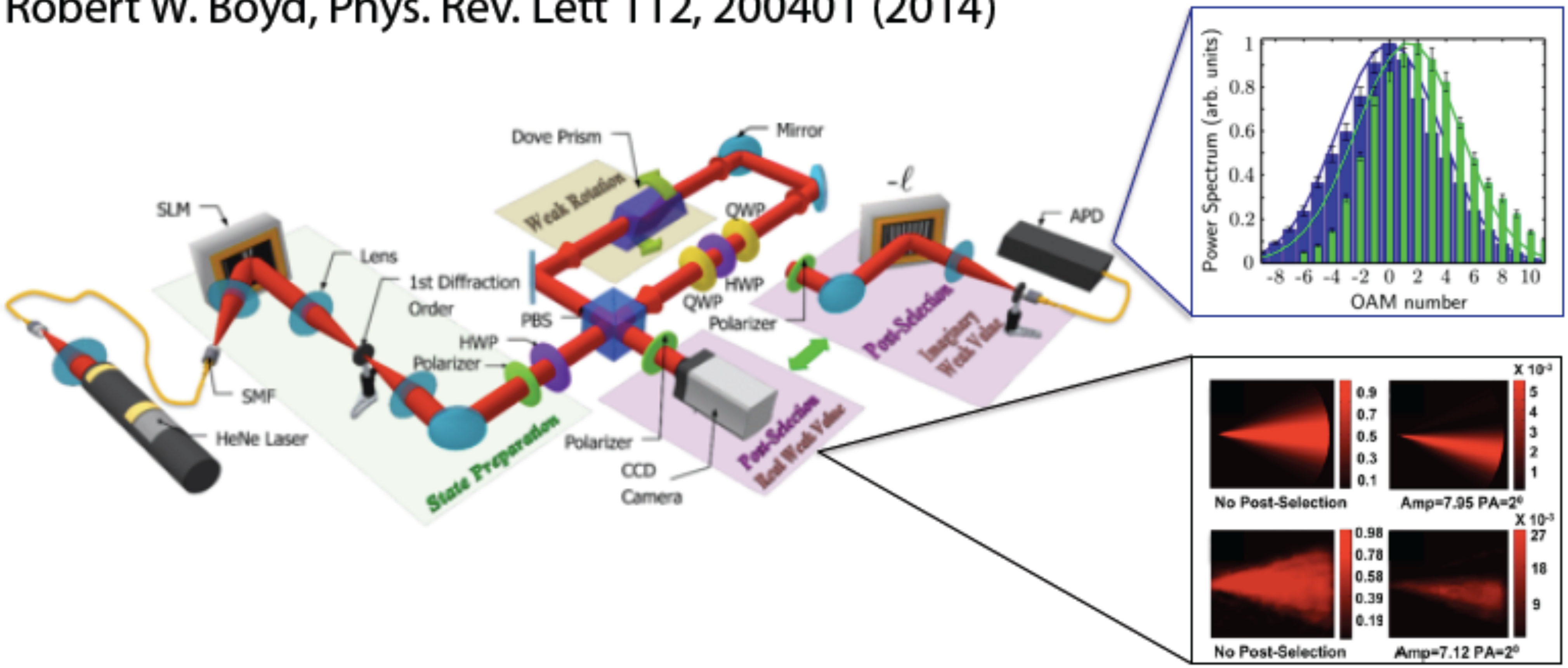
standard expectation value: $< A >=< \Psi|\hat{A}|\Psi >$

weak value: $A_w \equiv \langle \psi_f \mid A \mid \psi_{\text{in}} \rangle / \langle \psi_f \mid \psi_{\text{in}} \rangle .$

Why are weak values important?
can lead to amplification of small signals
can lead to direct measurement of the quantum wavefunction

Amplification of Angular Rotations using Weak Measurements

Omar S. Magaña-Loaiza, Mohammad Mirhosseini, Brandon Rodenburg, and Robert W. Boyd, Phys. Rev. Lett 112, 200401 (2014)



First demonstration of weak--value amplification in the azimuthal variables of angular position and orbital angular momentum.

LETTER

doi:10.1038/nature10120

Direct measurement of the quantum wavefunction

Jeff S. Lundeen[1], Brandon Sutherland[1], Aabid Patel[1], Corey Stewart[1] & Charles Bamber[1]

$$\langle A \rangle_{W} = \frac{\langle c|A|\Psi \rangle}{\langle c|\Psi \rangle}$$

Returning to our example of a single particle, consider the weak measurement of position ($A = \pi_x \equiv |x\rangle\langle x|$) followed by a strong measurement of momentum giving $P = p$. In this case, the weak value is:

$$\langle \pi_x \rangle_{W} = \frac{\langle p|x\rangle \langle x|\Psi \rangle}{\langle p|\Psi \rangle} \tag{2}$$

$$= \frac{e^{ipx/\hbar}\,\Psi(x)}{\Phi(p)} \tag{3}$$

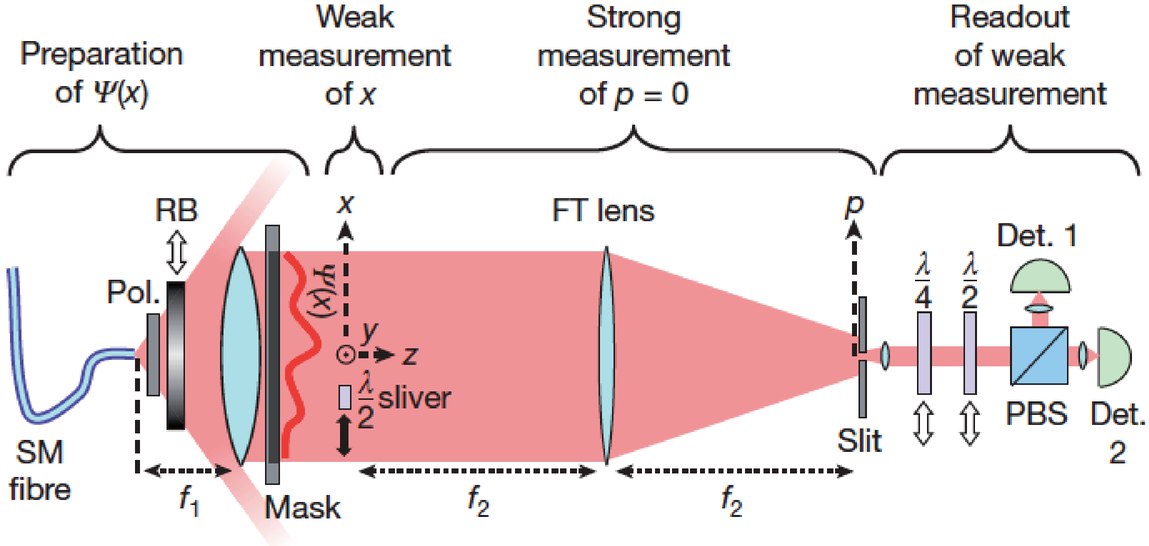
In the case $p = 0$, this simplifies to

$$\langle \pi_x \rangle_{W} = k\Psi(x) \tag{4}$$

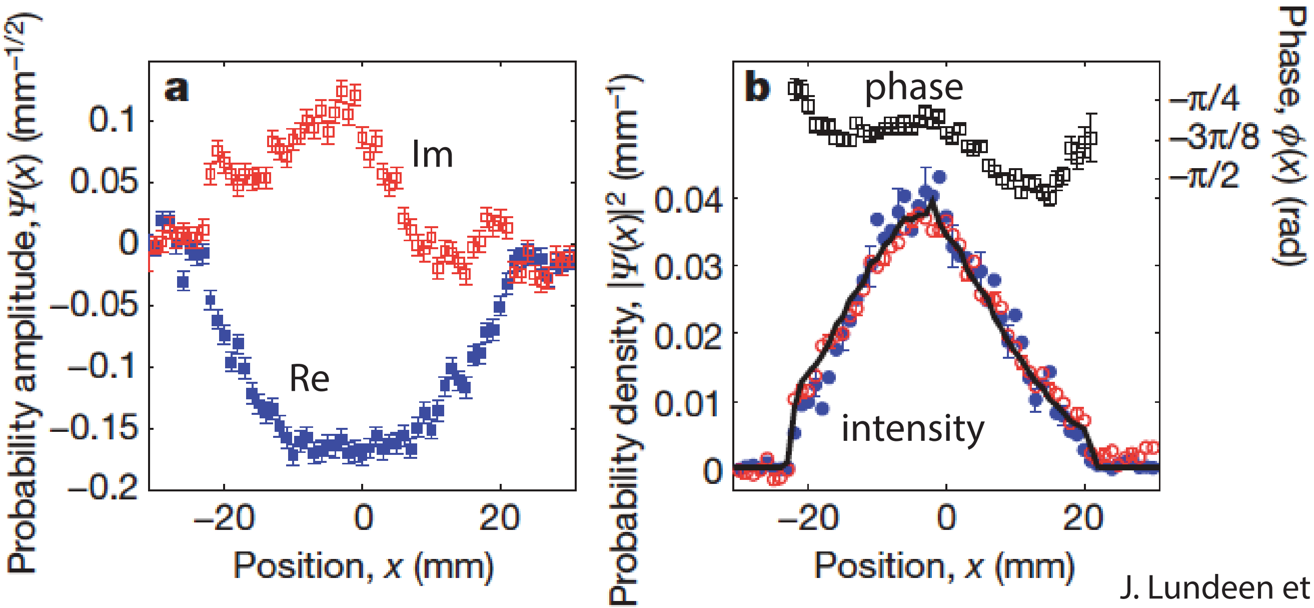
where $k = 1/\Phi(0)$ is a constant (which can be eliminated later by normalizing the wavefunction). The average result of the weak mea-

Direct Measurement of the Photon "Wavefunction"

Measurement setup



Typical results



J. Lundeen et al., Nature 474, 188 (2011)

Thank you for your attention!

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PAPER

Orbital angular momentum modes do not increase the channel capacity in communication links

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E-mail: gbjork@kth.se

Keywords: orbital angular momentum, mode density, spectral efficiency, free-space communication, broadcasting, signal-to-noise ratio, crosstalk