Optical Forces and Fresnel Drag in Atomic Vapor “Slow-Light” Media

Akbar Safari,1 Israel De Leon,1 Mohammad Mirhosseini,2 Omar S. Magana-Loaiza,2 and Robert W. Boyd1,2,3

1. Department of Physics, University of Ottawa
2. Institute of Optics, University of Rochester
3. School of Physics and Astronomy, University of Glasgow

Why Care about Optical Forces?

- Optical levitation
- Optical tweezers
- Optomechanical systems
- But can we control optical forces.

  - Yes! Photon momentum and optical forces depend on both refractive index and group index of optical materials.

  - The “slow light” community knows how to manipulate the group velocity of light.
Controlling the Velocity of Light

“Slow,” “Fast” and “Backwards” Light

– Light can be made to go:
  slow: \( v_g \ll c \) (as much as \( 10^6 \) times slower!)
  fast: \( v_g > c \)
  backwards: \( v_g \) negative

Here \( v_g \) is the **group velocity**: \( v_g = c/n_g \) \( n_g = n + \omega (dn/d\omega) \)

– Velocity controlled by structural or material resonances

Group Velocity

Pulse (wave packet) \[ \rightarrow u_g \]

Group velocity given by \[ u_g = \frac{d\omega}{dk} \]

For \[ k = \frac{n\omega}{c} \] \[ \Rightarrow \frac{dk}{d\omega} = \frac{1}{c} \left(n + \omega \frac{dn}{d\omega}\right) \]

Thus \[ u_g = \frac{c}{n + \omega \frac{dn}{d\omega}} \equiv \frac{c}{n_g} \]

Thus \[ n_g \neq n \] in a dispersive medium!
Slow Light Fundamentals: How to Create Slow and Fast Light

Use Isolated Gain or Absorption Resonance

\[ n_g = n + \omega \left( \frac{dn}{d\omega} \right) \]
Poynting’s Theorem when derived for a dispersive medium leads to the conclusion that

\[ S = \frac{1}{2} n \varepsilon_0 c E^2 \]  
(intensity)

\[ u = \frac{1}{2} n n_g \varepsilon_0 E^2 \]  
(energy density)

where

\[ v_g = c/n_g \]  
(group velocity).

It thus follows that

\[ S = u v_g. \]

Note:
Large enhancement of stored energy
But no enhancement of E!

See, e.g., Haus, Landau and Lifshitz, Milonni, or Harris and Hau
What is the Momentum of a Photon?

In vacuum: \( p = (\hbar \omega / c) \)

Abraham form (for matter)
\[ P = \mathbf{E} \times \mathbf{H} / c^2 \quad (\text{EM momentum density}) \]
\[ p = (\hbar \omega / c)(1/n_g) \quad (\text{photon momentum}) \]

Minkowski form (for matter)
\[ P = \mathbf{D} \times \mathbf{B} \quad (\text{EM momentum density}) \]
\[ p = (\hbar \omega / c)(n^2/n_g) \quad \text{or} \quad p = (\hbar \omega / c) n \quad (\text{photon momentum}) \]

One way or other, photon momentum very small in slow-light medium

Einstein-Balazs Argument Supports the Abraham Form

- Argue that center of mass-energy must move with a constant velocity.
- When photon wavepack enters block, it slows down. Block thus receives a kick into the forward direction.
- When photon leaves block, block receives backward kick and returns to rest.
- Block undergoes longitudinal displacement of
  \[ \Delta z = (n_g - 1) L \, \hbar \omega / (M c^2) \]
- Simple kinematic argument shows that momentum of photon in block is
  \[ p = \hbar \omega / (n_g c) \] Abraham form!
Fermi’s Argument Supports the Minkowski Form

- Fermi describes Doppler effect in terms of atomic recoil (RMP, 1932)
- Atom can absorb only if $\omega \approx \omega_0(1 - n\nu/c)$
- Conservation of energy and momentum
  
  Initial energy $= \hbar \omega + \frac{1}{2}mv^2$ \quad Final energy $= \hbar \omega_0 + \frac{1}{2}mv'^2$
  
  Initial momentum $= p + mv$ \quad Final momentum $= mv'$

- Solve: find photon momentum $p$ in medium given by

  $$p = n \frac{\hbar \omega}{c} \quad \text{Minkowski form!}$$
Which is Correct, Abraham or Minkowski?

Resolution of the Abraham-Minkowski Dilemma

Stephen M. Barnett
Department of Physics, SUPA, University of Strathclyde, Glasgow G4 0NG, United Kingdom
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The dilemma of identifying the correct form for the momentum of light in a medium has run for a century and has been informed by many distinguished contributions, both theoretical and experimental. We show that both the Abraham and Minkowski forms of the momentum density are correct, with the former being the kinetic momentum and the latter the canonical momentum. This identification allows us to explain why the experiments supporting each of the rival momenta gave the results that they did. The inclusion of dispersion and absorption provides an interesting subtlety, but does not change our conclusion.

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\[ p_{\text{med}} + p_{\text{Abr}} = p_{\text{can}} + p_{\text{Min}}. \]

Total momentum (field plus material) the same in either treatment!
What is the Momentum of a Photon?

In vacuum: \( p = (\hbar \omega / c) \)

Abraham form (for matter)
\[ \mathbf{P} = \mathbf{E} \times \mathbf{H} / c^2 \quad \text{(EM momentum density)} \]
\[ p = (\hbar \omega / c)(1/n_g) \quad \text{(photon momentum)} \]
It is the kinetic (as in mv) momentum
It is the momentum of the field (alone)
It is what comes out of Balazs’s moving block analysis

Minkowski form (for matter)
\[ \mathbf{P} = \mathbf{D} \times \mathbf{B} \quad \text{(EM momentum density)} \]
\[ p = (\hbar \omega / c)(n^2/n_g) \quad \text{or} \quad p = (\hbar \omega / c) n \quad \text{photon momentum} \]
It is the canonical momentum (as in \( h/\lambda_{\text{deBroglie}} \))
It is the momentum of field and (at least part of that of the) matter
It is what comes out of a Doppler shift analysis

One way or other, photon momentum very small in slow-light medium
We would like to use the dependence of the photon momentum on the group index as a means to control optical forces.

As a first step down this pathway, we are studying how to control photon drag effects using slow light.
Velocity of (Slow) Light in Moving Matter: Photon Drag (or Ether Drag) Effects

Fizeau (1859): Longitudinal photon drag:

Velocity of light in flowing water.

\[ V = 700 \text{ cm/sec}; \quad L = 150 \text{ cm}; \quad \text{displacement of 0.5 fringe.} \]

Modern theory: relativistic addition of velocities

\[
v = \frac{c}{n} + V \approx \frac{c}{n} + V \left( 1 - \frac{1}{n^2} \right)
\]

Fresnel “drag” coefficient
Fresnel Drag Effect in Nondispersive and Dispersive Media

- Nondispersive medium

\[ u = \frac{c}{n} \pm \frac{v}{1 + \frac{v}{nc}} \approx \frac{c}{n} \pm v \left(1 - \frac{1}{n^2}\right) \]

- Dispersive medium

\[ u \approx \frac{c}{n} \pm v \left(1 - \frac{1}{n^2} + \frac{n_g - n}{n^2}\right) \quad \text{where} \quad n_g \equiv n + \omega \frac{dn}{d\omega} \]
We Use Rubidium as Our Slow Light Medium

- Transmission spectrum of Rb around D$_2$ transition:

![Graph showing transmission spectrum at T=30 °C and T=150 °C](image)

- There is large dispersion where rapid changes in transmission are observed

- Group index of Rb around D$_2$ transition line at T=130 °C:

![Graph showing group index](image)
The Fringe Pattern Shifts According to Velocity of the Rubidium Cell

Fringe pattern as the cell moves

Velocity of cell is (±) 1 m/s.
Direct Measurement of the Group Index of Rubidium

Variation of $n_g$ with temperature of the Rb cell:

<table>
<thead>
<tr>
<th>T</th>
<th>130</th>
<th>135</th>
<th>140</th>
<th>145</th>
<th>150</th>
<th>155</th>
<th>160</th>
</tr>
</thead>
<tbody>
<tr>
<td>$n_g$</td>
<td>59.4</td>
<td>72.6</td>
<td>90.2</td>
<td>114</td>
<td>141</td>
<td>177</td>
<td>205</td>
</tr>
</tbody>
</table>
Recall that the rubidium cell was moving at only 1 m/s.
For details of the light dragging experiment, please see the poster of Akbar Safari this evening.

**Conclusions**

- A maximum drag speed of 205 m/s was measured in a highly dispersive medium (hot Rb vapor).

- This effect is at least two orders of magnitude larger than that observed to date.

- Much larger dispersion can be achieved in Rb atoms using electromagnetically induced transparency ($n_g$ as high as $10^7$).
Note Carefully: Akbar, not Ackbar

Akbar Safari

Admiral Ackbar
the end
History of light dragging

Early history: (Fringe shift)

- 1851: Fizeau
  - Water
  - %16 accuracy

- 1886: Michelson-Morely
  - Water
  - %5 accuracy

- 1895: Lorentz
  - Theory
  - Effect of dispersion
    - \[ u = \frac{c}{n} + v \left( 1 - \frac{1}{n^2} - \frac{\lambda}{n d} \frac{dn}{d\lambda} \right) \]
    (Fixed boundaries)
  - (Many experiments to see the dispersion effect)
History of light dragging

- 1911: Harress
  - Dispersion in glass
  - %2 accuracy (after subtracting the Sagnac effect)

- 1912-1922: Zeeman
  - Dispersion in glass
  - %1.7 accuracy

- \[ u = \frac{c}{n} + v \left( 1 - \frac{1}{n^2} - \frac{\lambda}{n^2} \frac{d n}{d \lambda} \right) \] (moving boundaries)
  (Laub drag coefficient)

Modern history: (Frequency spilling in a ring resonator)

- 1964: Macek et al
- 1972: Bilger and Zavodny
- 1977: Bilger and Stowell
- ...
- 1988: Sanders and Ezekiel (%0.01 to %0.1 accuracy)
Observation of a Push Force on the End Face of a Nanometer Silica Filament Exerted by Outgoing Light

Weilong She, Jianhui Yu, and Raohui Feng

State Key Laboratory of Optoelectronic Materials and Technologies, Sun Yat-Sen University, Guangzhou 510275, China
(Received 12 February 2008; revised manuscript received 15 September 2008; published 8 December 2008)

There are two different proposals for the momentum of light in a transparent dielectric of refractive index $n$: Minkowski’s version $nE/c$ and Abraham’s version $E/(nc)$, where $E$ and $c$ are the energy and vacuum speed of light, respectively. Despite many tests and debates over nearly a century, momentum of light in a transparent dielectric remains controversial. In this Letter, we report a direct observation of the inward push force on the free end face of a nanometer silica filament exerted by the outgoing light. Our results suggest that Abraham’s momentum is correct.
Interpretation of these Results

- How do we understand these results in terms of the physical mechanism that leads to the slow-light effect?

- Our experiment made use of “self-pumped” slow light based on coherent population oscillations (CPO).

- (Need to explain how this works)
The angular momentum of light inside a dielectric

MILES PADGETT†, STEPHEN M. BARNETT‡ and RODNEY LOUDON§
† Department of Physics and Astronomy, University of Glasgow, Glasgow G12 8QQ, Scotland, UK
‡ Department of Physics, University of Strathclyde, Glasgow G4 0NG, Scotland, UK
§ Department of Electronic Systems Engineering, University of Essex, Colchester CO4 3SQ, England, UK
E-mail: m.padgett@physics.gla.ac.uk

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Abstract. We consider whether or not a short pulse of light carrying angular momentum will exert a torque when propagating through a transparent disc. The approach is based on the ‘Einstein-box’ argument which we apply to discuss linear optical momentum in a medium. Two competing theories due to Minkowski and Abraham, at least superficially, suggest that the disc will not or will rotate. Our analysis suggests that the disc will rotate and that an experiment using optical tweezers should be able to detect the rotation.