Quantum Nonlinear Optics: Nonlinear Optics Meets the Quantum World

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Outlook: NLO is a superb platform from which to explore new physical processes and to develop photonics applications.

Prospectus

1. Introduction to Nonlinear Optics and Quantum NLO
2. New Applications of “Slow Light”
3. Möbius Strips of Polarization
4. Quantum Communication with Multiple Bits per Photon
Simple Formulation of the Theory of Nonlinear Optics

\[ P = \chi^{(1)} E + \chi^{(2)} E^2 + \chi^{(3)} E^3 + \ldots \]

Here \( P \) is the induced dipole moment per unit volume and \( E \) is the field amplitude.

\( \chi^{(1)} \) describes linear optics, e.g., how lenses work.

\( \chi^{(2)} \) describes second-order effects, e.g., second-harmonic generation (SHG).

\( \chi^{(3)} \) describes third-order effects such as third-harmonic generation, four-wave mixing, and the intensity dependence of the index of refraction.

\[ n = n_0 + n_2 I \quad \text{where} \quad n_2 = \frac{3}{4n_0^2 \epsilon_0 c} \chi^{(3)} \]
Intense Field and Attosecond Physics

**Above Threshold Ionization Beyond the High Harmonic Cutoff**

K. J. Schafer, Baorui Yang, L. F. DiMauro, and K. C. Kulander

(1) Lawrence Livermore National Laboratory, Livermore, California 94550
(2) Chemistry Department, Brookhaven National Laboratory, Upton, New York 11973
(Received 2 December 1992)

**Plasma Perspective on Strong-Field Multiphoton Ionization**

P. B. Corkum

National Research Council of Canada, Ottawa, Ontario, Canada K1A 0R6
(Received 9 February 1993)

I > $10^{15}$ W/cm$^2$

High-harmonic generation

Measuring the molecular nitrogen wavefunction

Attosecond pulses to sample a visible E-field
Why Interest in Quantum Nonlinear Optics?

Explore the relation between traditional nonlinear optics (NLO) and phenomena in quantum information science (QIS).

QIS holds great promise for secure communication, quantum logic, quantum computing, etc.

Many processes in QIS rely on nonlinear optical interactions.
Parametric Downconversion: A Source of Entangled Photons

The signal and idler photons are entangled in:
(a) polarization
(b) time and energy
(c) position and transverse momentum
(d) angular position and orbital angular momentum

Entanglement is important for:
(a) Fundamental tests of QM (e.g., nonlocality, Bell tests)
(a) Quantum technologies (e.g., secure communications, Q teleportation)
• We discriminate among four orthogonal images using single-photon interrogation in a coincidence imaging configuration.

• Note that a single photon can carry more than one bit of information.

Quantum Lithography: Concept of Jonathan Dowling

- Entangled photons can be used to form an interference pattern with detail finer than the Rayleigh limit.
- Resolution \( \approx \frac{\lambda}{2N} \), where \( N = \) number of entangled photons.


- No practical implementation to date, but some laboratory results.


Squeezed Light Generation

Entanglement and squeezing share a common origin:

Entangled light
beam splitter
squeezed light
Precision Measurement beyond the Shot-Noise Limit

Enhanced sensitivity of the LIGO gravitational wave detector by using squeezed states of light


Affiliations | Contributions | Corresponding author

Received 23 April 2013 | Accepted 04 June 2013 | Published online 21 July 2013
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Controlling the Velocity of Light

“Slow,” “Fast” and “Backwards” Light

– Light can be made to go:
  slow: \( v_g \ll c \) (as much as \( 10^6 \) times slower!)
  fast: \( v_g > c \)
  backwards: \( v_g \) negative

Here \( v_g \) is the group velocity: \( v_g = c/n_g \) \( n_g = n + \omega (dn/d\omega) \)

– Velocity controlled by structural or material resonances

Slow and Fast Light Using Isolated Gain or Absorption Resonances

\[ n_g = n + \omega \left( \frac{dn}{d\omega} \right) \]
Observation of Superluminal and “Backwards” Pulse Propagation

- A strongly counterintuitive phenomenon
- But entirely consistent with established physics
- Observed by Gehring, Schweinsberg, Barsi, Kostinski, and Boyd Science 312, 985 2006.
Can We Beat the $1/L$ Resolution Limit of Standard Spectrometers?

- The limiting resolution of a broad class of spectrometers is given (in wave-numbers) by the inverse of a characteristic dimension $L$ of the spectrometer.

$$\Delta \nu \text{(res)} \approx 1/L$$

- We use slow-light methods to design spectrometers with resolution that exceeds this conventional limit by a factor as large as the group index.

- This ability allows us to miniaturize spectrometers with no loss of resolution, for “lab-on-a-chip” applications.
Our Goal

Replace this:

with this:
Our Approach: Chip-Scale Slow-Light Spectrometer

- The spectral sensitivity of an interferometer is increased by a factor as large as the group index of a material placed within the interferometer.

- We want to exploit this effect to build chip-scale spectrometers with the same resolution as large laboratory spectrometers.

- Here is why it works:

  \[
  \frac{d \Delta \phi}{d \omega} = \frac{d}{d \omega} \frac{\omega n L}{c} = \frac{L}{c} (n + \omega \frac{dn}{d\omega}) = \frac{L n_g}{c}
  \]

- We use line-defect waveguides in photonic crystals as our slow light mechanism.

  Slow-down factors of greater than 100 have been observed in such structures.

Laboratory Characterization of the Slow-Light Mach-Zehnder Interferometer

- Interference fringes

• Resolution (quarter wave) is 17 pm or 2.1 GHz or 0.071 cm\(^{-1}\)

• (Slow-light waveguide is only 1 mm long!)

Magaña-Loaiza, Gao, Schulz, Awan, Upham, Dolgaleva, and Boyd, in review.
Challenge: Fabricate a chip-scale spectrometer that can discriminate acetylene (H$_2$C$_2$) from hydrogen cyanide (HCN)?

(data from our own lab)
On-chip spectrometer based on high-Q photonic crystal cavities

- The concept

- Cavity design

- Spectroscopy results

The Velocity of Light in Moving Matter: Fresnel Drag (or Ether Drag) Effects

• Fizeau (1859): Longitudinal photon drag:

Velocity of light in flowing water.

\[ V = 700 \text{ cm/sec}; \quad L = 150 \text{ cm}; \quad \text{displacement of 0.5 fringe}. \]

\[ V = \frac{c}{n} + V \left(1 - \frac{1}{n^2}\right) \quad \text{Fresnel “drag” coefficient} \]

• But what about slow-light media?
Fresnel Drag in a Highly Dispersive Medium

Light Drag in a Slow Light Medium (Lorentz)

\[ u \approx \frac{c}{n} \pm v \left(1 - \frac{1}{n^2} + \frac{n_g - n}{n^2}\right) \]

We Use Rubidium as Our Slow Light Medium

- Transmission spectrum of Rb around D₂ transition:

  ![Transmission Spectrum](image1)

- Group index of Rb around D₂ line at T=130

  ![Group Index](image2)

- Change in phase velocity is much larger than velocity of rubidium cell. Implications for new velocimeters?
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Observation of Optical Polarization Möbius Strips

- Möbius strips are familiar geometrical structures, but their occurrence in nature is extremely rare.

- We generate such structures in the nanoscale in tightly focused vector light beams and confirm experimentally their Möbius topology.

Prediction of Optical Möbius Strips

- Isaac Freund discovered, described, and investigated these unusual structures.
- To observe these structures, one needs to create a very special field distribution (e.g., a Poincaré beam).
- One also needs to observe the field distribution in a very special way (measure polarization as a function of position around a very tightly focused light beam).

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1. Wikipedia
2. Isaac Freund, Bar-Ilan Univ., Talk: *Optical Möbius Strips and Twisted Ribbons*, Conf. on Singular Optics, ICTP Trieste, Part II, 30 May 2011

Full vectorial beam measurement on the nanoscale

Nanoparticle-based probing technique for vector beam reconstruction
1. A dipole-like spherical nanoparticle (90 nm diameter) is scanned through the beam
2. The forward- and backward-scattered light for each position of the nanoparticle relative to the beam in the focal plane is measured

Full amplitude and phase reconstruction scheme:
Lab Setup to Observe a Polarization Möbius Strip

- q-plate: waveplate with a spatially varying orientation (q is the topological charge)
- output beam has a spatially varying state of polarization (vector beam, Poincaré beam, etc.)

Tight focusing enhances the Möbius effect, which depends on the z component of the field.
Observation of Polarization Möbius Strips

q = -1/2
3/2 twists

q = -3/2
5/2 twists

Remarks
• First observation of a polarization Möbius strip
• Light fields can possess rich spatial structure on subwavelength scales
• Current technology is capable of controllably creating beams with such structures and measuring it at subwavelength distances.

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Use of Quantum States for Secure Optical Communication

- The celebrated BB84 protocol for quantum key distribution (QKD) transmits one bit of information per received photon.

- We have built a QKD system that can carry more than one bit per photon.
  – Note that in traditional telecom, one uses many photons per bit!

- Our procedure is to encode using beams that carry orbital angular momentum (OAM), such as the Laguerre-Gauss states, which reside in an infinite dimensional Hilbert space.
We are constructing a QKD system in which each photon carries many bits of information.

We encode in states that carry OAM such as the Laguerre-Gauss states.

We also need a second basis composed of linear combinations of these states.

### Single Photon States

**Laguerre-Gaussian Basis** \( \ell = -13, \ldots, 13 \)

<table>
<thead>
<tr>
<th>(-13)</th>
<th>(-12)</th>
<th>(-11)</th>
<th>(-1)</th>
<th>(0)</th>
<th>(1)</th>
<th>(11)</th>
<th>(12)</th>
<th>(13)</th>
</tr>
</thead>
</table>

**“Angular” Basis (mutually unbiased with respect to LG)**

<table>
<thead>
<tr>
<th>(0)</th>
<th>(1)</th>
<th>(2)</th>
<th>(12)</th>
<th>(13)</th>
<th>(14)</th>
<th>(25)</th>
<th>(26)</th>
<th>(27)</th>
</tr>
</thead>
</table>

\[
\Psi_{AB}^N = \frac{1}{\sqrt{27}} \sum_{\ell=-13}^{13} \text{LG}_{\ell,0} \exp\left(i2\pi N\ell/27\right)
\]
Our Laboratory Setup

We use a seven-dimensional state space.


We use a seven-dimensional state space.
We use a 7-letter alphabet, and achieve a channel capacity of 2.1 bits per sifted photon.

We do not reach the full 2.8 bits per photon for a variety of reasons, including dark counts in our detectors and cross-talk among channels resulting from imperfections in our sorter.

Nonetheless, our error rate is adequately low to provide full security,
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Thank you for your attention!
Why We Shouldn’t Always Trust Google

Robert W. Boyd

Robert William Boyd is an American physicist noted for his work in optical physics and especially in nonlinear optics. Wikipedia

Born: 1948, Buffalo, NY
Education: University of California, Berkeley
Doctoral advisor: Charles H. Townes
Residence: United States of America, Canada

Books

Nonlinear Optics, Second Edition
1962
Radiometry and the detection of light
1983
Not by Genes Alone
2005
Mathematical models of social evolution
2007
Indium Tin Oxide (ITO) displays enormously strong NLO properties:

- $n_2$ is $2.5 \times 10^5$ times that of fused silica
- nonlinear change in refractive index as large as 0.8
- response time of 270 fs

$$n_2 = \frac{3\chi^{(3)}}{4\epsilon_0 c n_0 \text{Re}(n_0)}$$

Some possible new effects

- Waveguiding outside the “weakly-guiding” regime
- Efficient all-optical switching
- No need for phase-matching