Filamentation: Historical Perspectives and Recent Results

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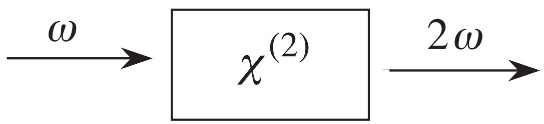
Simple Formulation of the Theory of Nonlinear Optics

$$P = \chi^{(1)} E + \chi^{(2)} E^2 + \chi^{(3)} E^3 + \ldots$$

Here P is the induced dipole moment per unit volume and E is the field amplitude

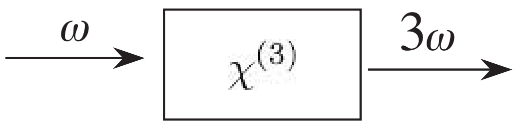
$\chi^{(1)}$ describes linear optics, e.g., how lenses work: 

$\chi^{(2)}$ describes second-order effects, e.g., second-harmonic generation (SHG)

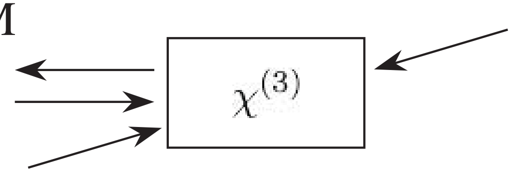


$\chi^{(3)}$ describes third-order effects such as third-harmonic generation, four-wave mixing, and the intensity dependence of the index of refraction.

THG



FWM



NL index

$$n = n_0 + n_2 I \quad \text{where} \quad n_2 = \frac{3}{4 n_0^2 \epsilon_0 c} \chi^{(3)}$$

Intense Field and Attosecond Physics

E 70, NUMBER 11 PHYSICAL REVIEW LETTERS 15 MARCH

Above Threshold Ionization Beyond the High Harmonic Cutoff

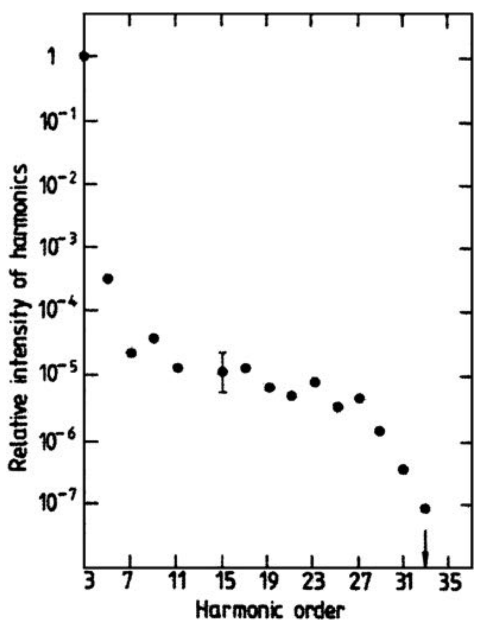
K. J. Schafer,[1] Baorui Yang,[2] L. F. DiMauro,[2] and K. C. Kulander[1]
[1] *Lawrence Livermore National Laboratory, Livermore, California 94550*
[2] *Chemistry Department, Brookhaven National Laboratory, Upton, New York 11973*
(Received 2 December 1992)

VOLUME 71, NUMBER 13 PHYSICAL REVIEW LETTERS 27 SEPTEMBER 1993

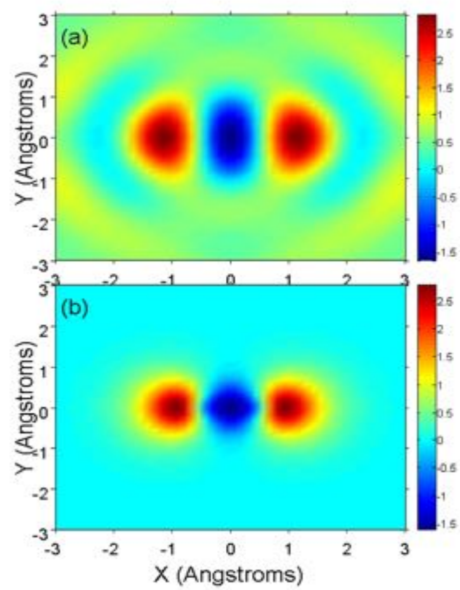
Plasma Perspective on Strong-Field Multiphoton Ionization

P. B. Corkum
National Research Council of Canada, Ottawa, Ontario, Canada K1A 0R6
(Received 9 February 1993)

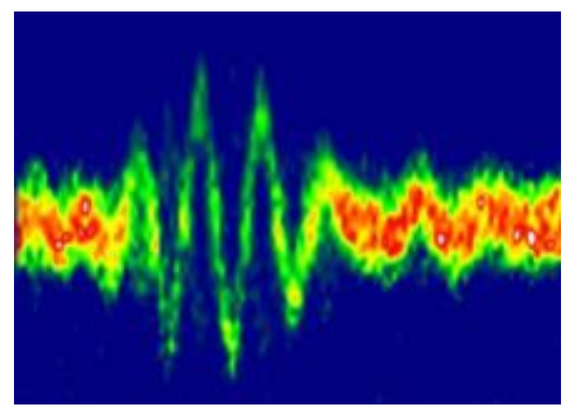
$I > 10^{15}$ W/cm^2



High-harmonic generation



Measuring the molecular nitrogen wavefunction

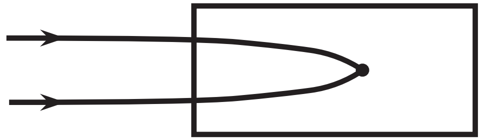


Attosecond pulses to sample a visible E-field

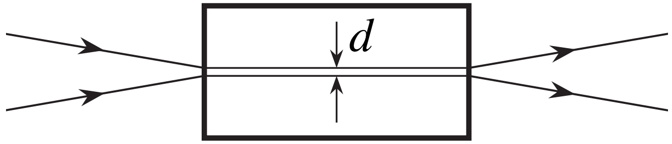
Self Action Effects in Nonlinear Optics

Self-action effects: light beam modifies its own propagation

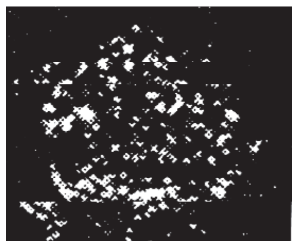
self focusing



self trapping



small-scale filamentation

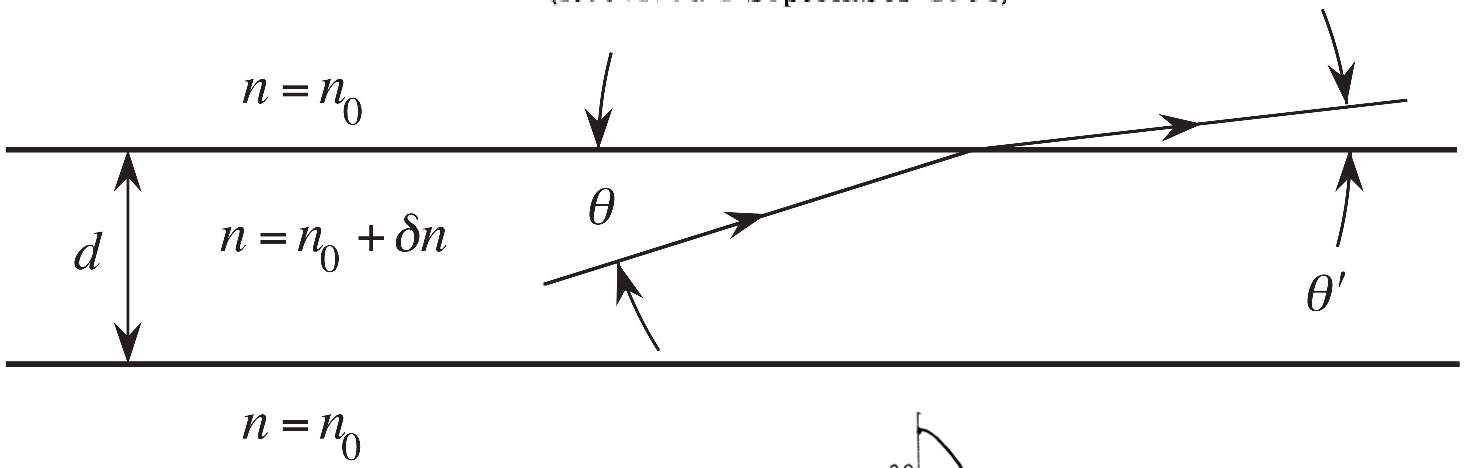


Prediction of Self Trapping

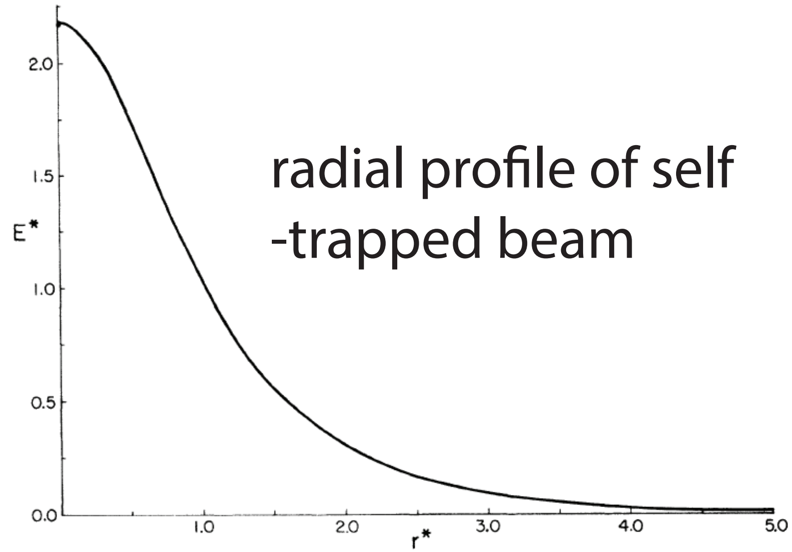
VOLUME 13, NUMBER 15 PHYSICAL REVIEW LETTERS 12 OCTOBER 1964

SELF-TRAPPING OF OPTICAL BEAMS*

R. Y. Chiao, E. Garmire, and C. H. Townes
Massachusetts Institute of Technology, Cambridge, Massachusetts
(Received 1 September 1964)



$$P_{\mathrm{cr}} = \frac{\pi(0.61)^2\lambda_0^2}{8n_0n_2}$$



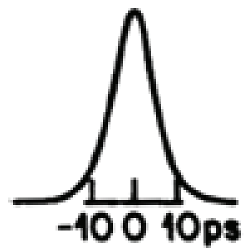
radial profile of self -trapped beam

Optical Solitons

Field distributions that propagate without change of form

Temporal solitons (nonlinearity balances gvd)

$$\frac{\partial \tilde{A}_s}{\partial z} + \tfrac{1}{2} i k_2 \frac{\partial^2 \tilde{A}_s}{\partial \tau^2} = i\gamma |\tilde{A}_s|^2 \tilde{A}_s.$$

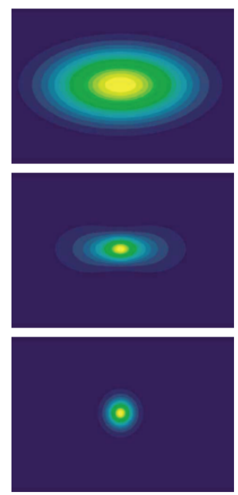


1973: Hasegawa & Tappert

1980: Mollenauer, Stolen, Gordon

Spatial solitons (nonlinearity balances diffraction)

$$2ik_0 \frac{\partial A}{\partial z} + \frac{\partial^2 A}{\partial x^2} = -3\chi^{(3)} \frac{\omega^2}{c^2} |A|^2 A$$



1964: Garmire, Chiao, Townes

1974: Ashkin and Bjorkholm (Na)

1985: Barthelemy, Froehly (CS2)

1991: Aitchison et al. (planar glass waveguide

1992: Segev, (photorefractive)

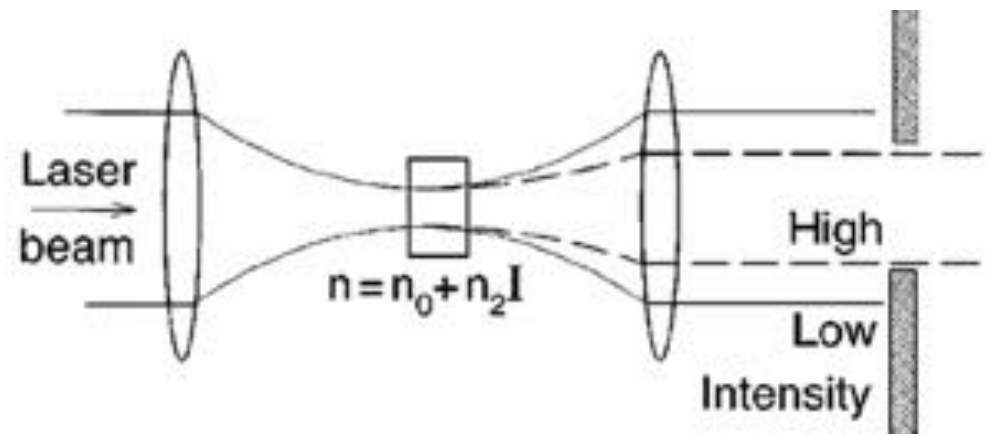
Solitons and self-focussing in Ti:Sapphire

42 OPTICS LETTERS / Vol. 16, No. 1 / January 1, 1991

60-fsec pulse generation from a self-mode-locked Ti:sapphire laser

D. E. Spence, P. N. Kean, and W. Sibbett

J. F. Allen Physics Research Laboratories, Department of Physics and Astronomy, University of St. Andrews, North Haugh, St. Andrews, Fife, KY16 9SS, Scotland



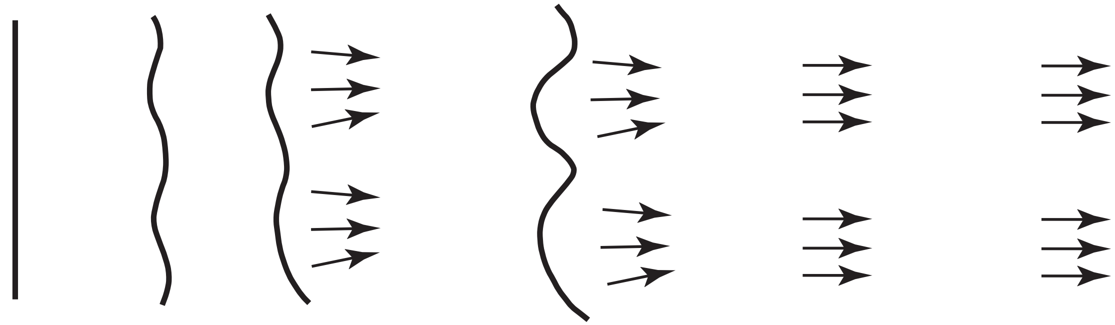
Diffraction-management controls the spatial self-focussing

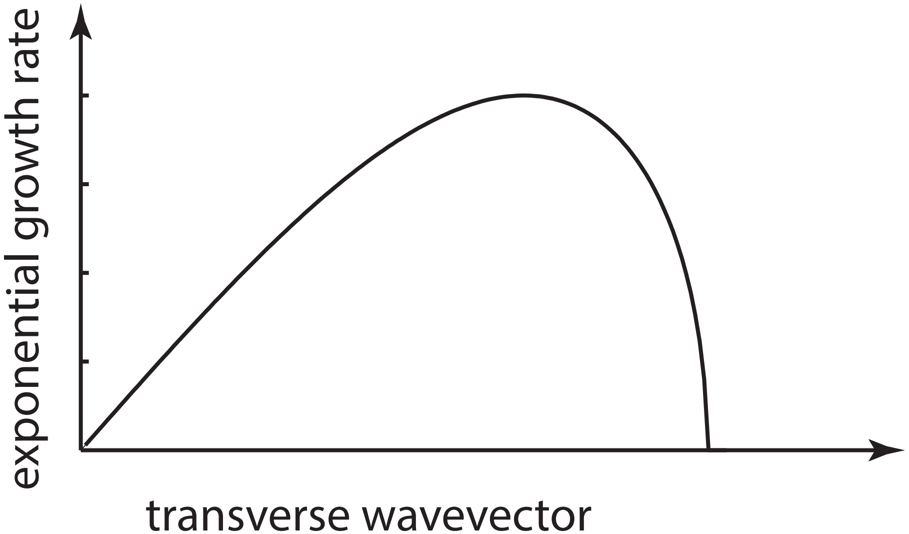
Dispersion-management controls the temporal self-focussing

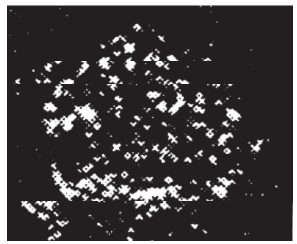
Beam Breakup by Small-Scale Filamentation

Predicted by Bespalov and Talanov (1966)

Exponential growth of wavefront imperfections by four-wave mixing processes

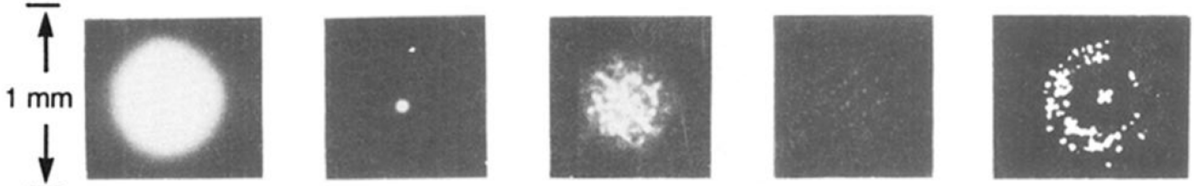






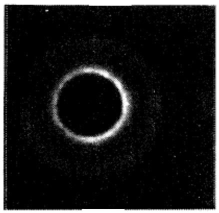
Rabi Sideband Generation in Sodium from Four-Wave Mixing in Filaments

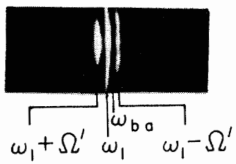
Beam at exit of cell:



Incident beam | Single filament | Multiple filaments | Patterns for structured input beams

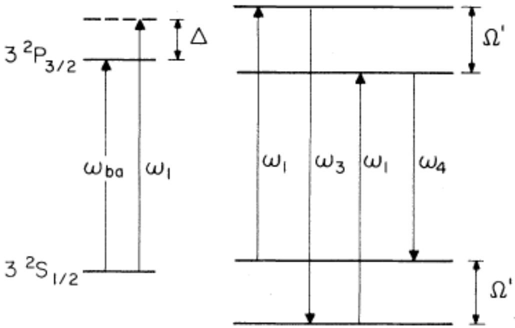
Beam in far field (conical emission)



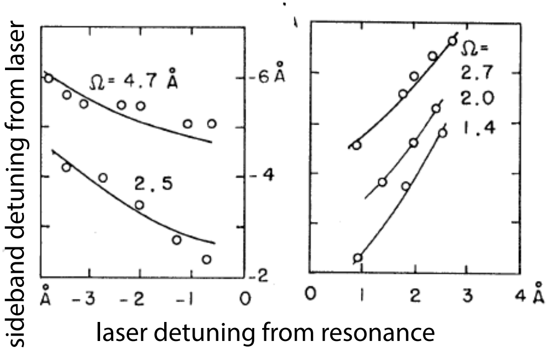


Spectrum of beam shows Rabi side-bands

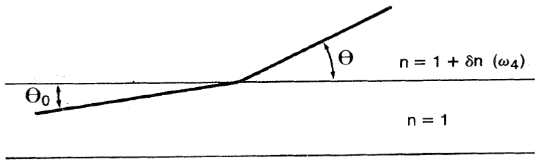
Rabi-sideband model



$$\Omega' = (\Delta^2 + \Omega^2)^{1/2}$$



Four-wave mixing in filament (wave-guide) explains spectrum and cone emission angle



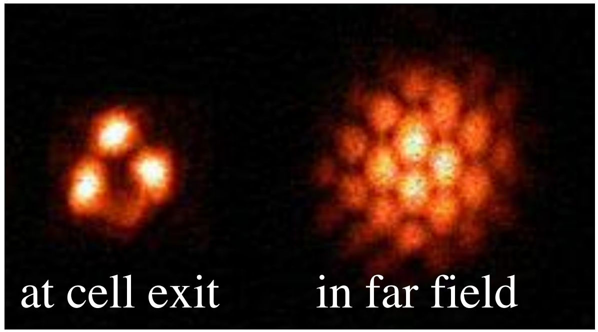
Harter et al., PRL 46, 1192 (1981); PRA 29, 739 (1984); Boyd et al., PRA 24, 411 (1981).

Honeycomb Pattern Formation

Output from cell with a single gaussian input beam

At medium input power

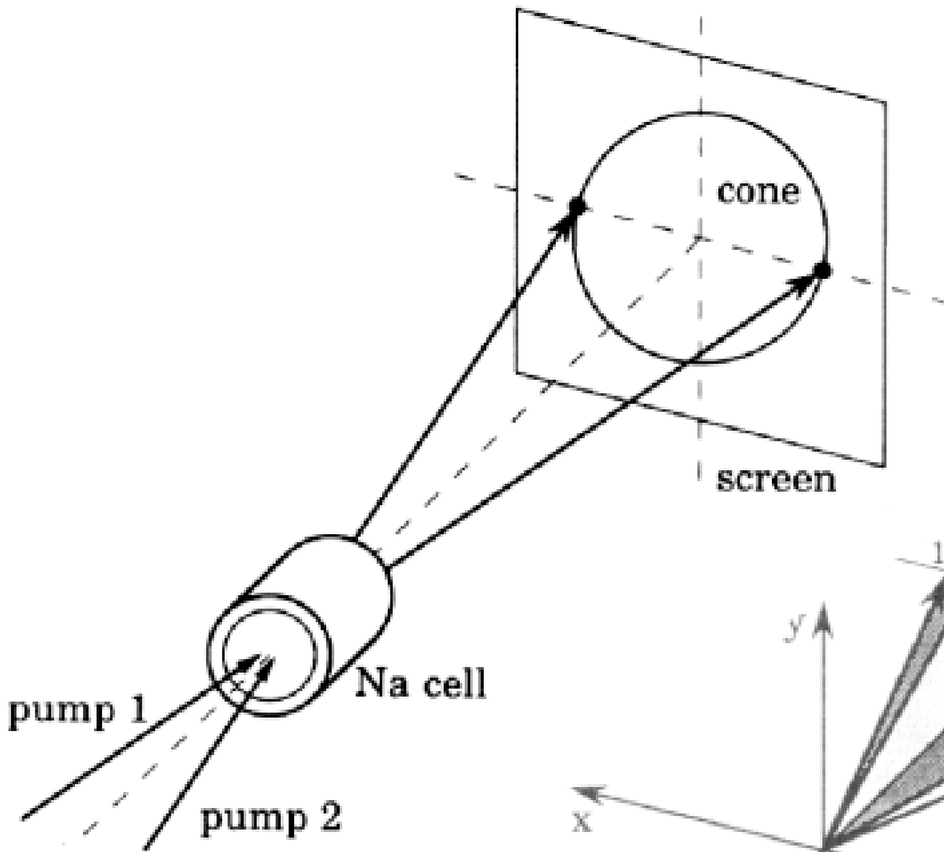
At high input power

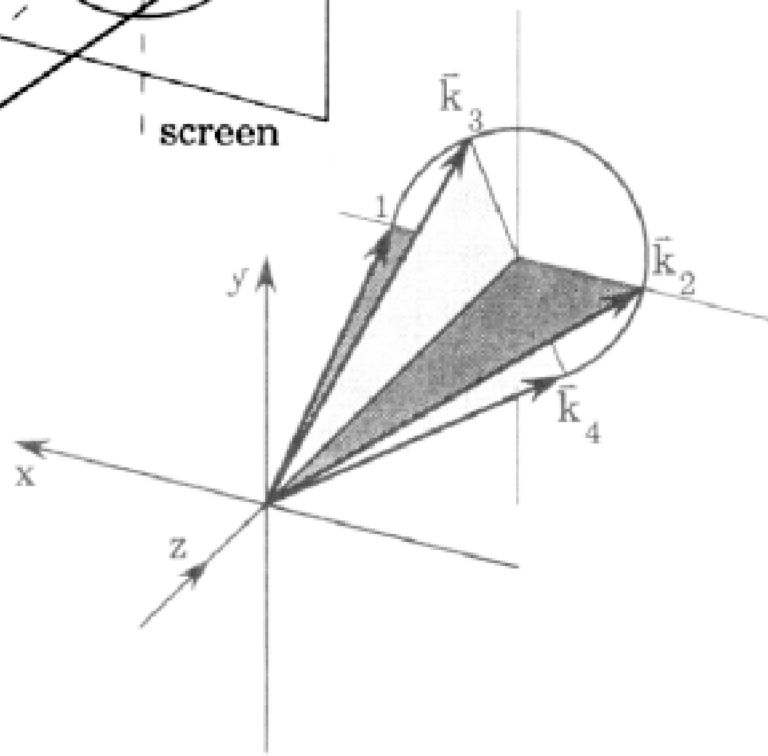


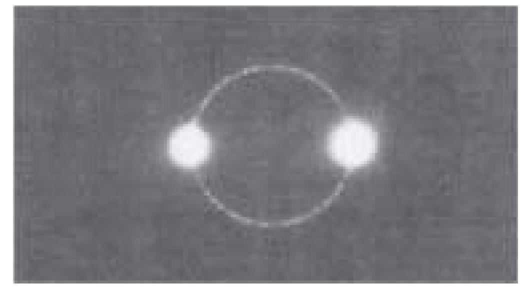
Input power 100 to150 mW
Input beam diameter 0.22 mm

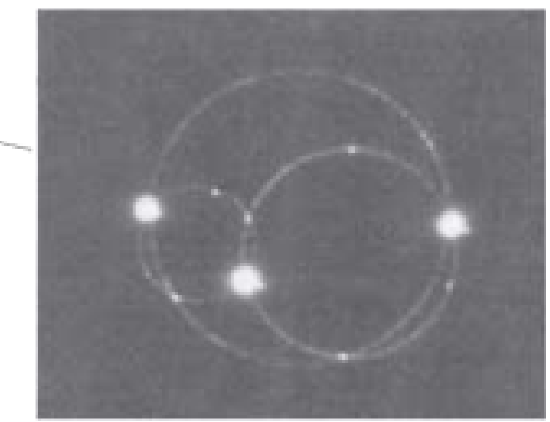
Sodium vapor cell T = 220^{o} C
Wavelength = 588 nm
Bennink et al., PRL 88, 113901 2002.

Generation of Quantum States of Light by Two-Beam Excited Conical Emission







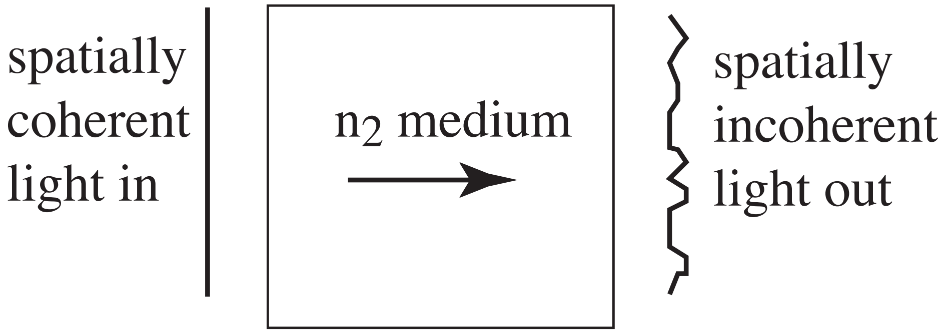


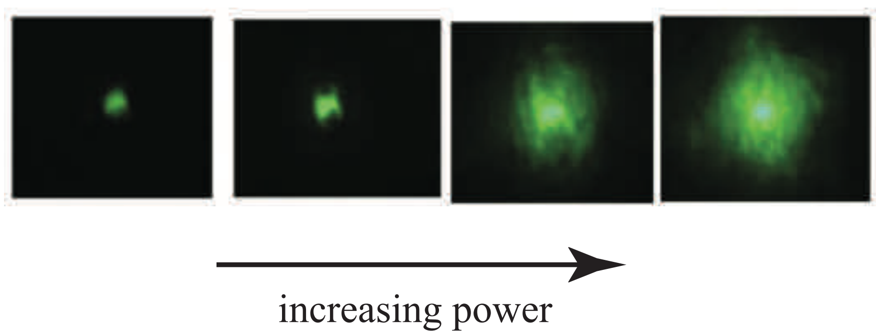
Kauranen et al, Opt. Lett. 16, 943, 1991; Kauranen and Boyd, Phys. Rev. A, 47, 4297, 1993.

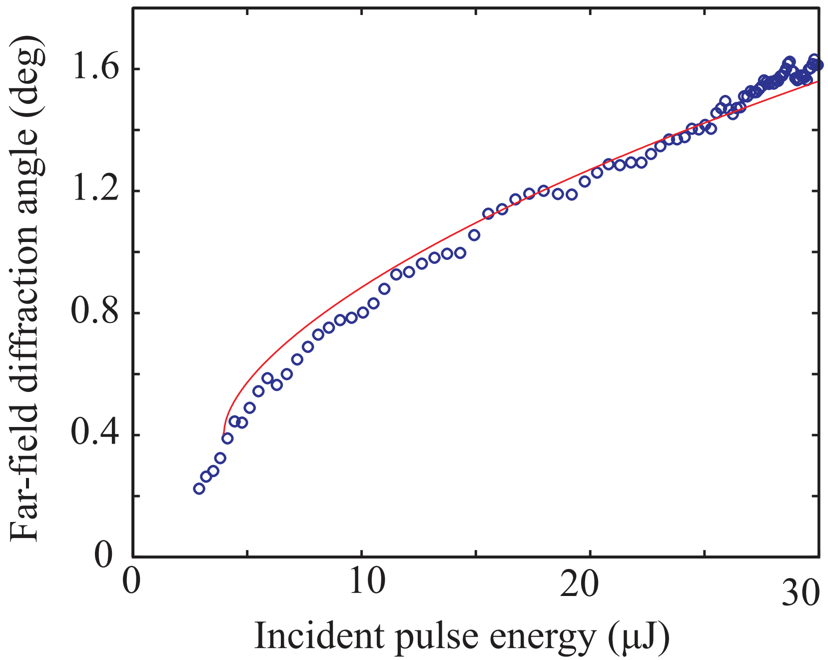
Optical Radiance Limiter Based on Spatial Coherence Control

Controlled small-scale filamentation used to modify spatial degree of coherence

Alternative to standard appropches to optical power limiting



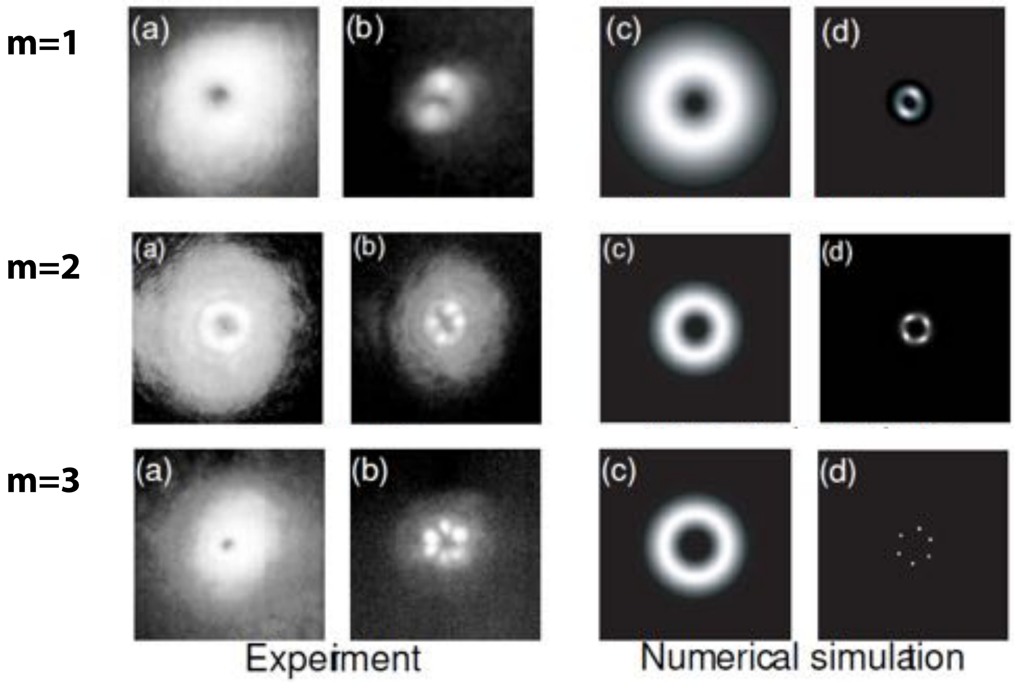




Schweinsberg et al., Phys. Rev. A 84, 053837 (2011).

Breakup of Ring Beams Carrying Orbital Angular Momentum (OAM) in Sodium Vapor

- Firth and Skryabin predicted that ring shaped beams in a saturable Kerr medium are unstable to azimuthal instabilities.
- Beams with OAM of $m\hbar$ tend to break into $2m$ filaments.
  (But aberrated OAM beams tend to break into $2m+1$ filaments.)



M.S. Bigelow, P. Zerom, and R.W. Boyd, Phys. Rev. Lett 92, 083902 (2004)

Nonlinear evolution of space-varying polarized light beams

Hugo Larocque, Frédéric Bouchard, Alison M. Yao, Christopher Travis, Israel De Leon, Lorenzo Marrucci, Ebrahim Karimi, Gian-Luca Oppo, and Robert W. Boyd

Max Planck - University of Ottawa Centre
for Extreme and Quantum Photonics

SQO Group

Department of Physics

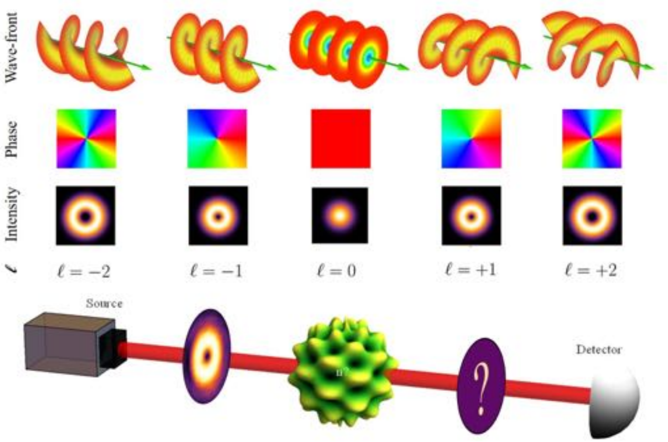
Max Planck Centre for Extreme
and Quantum Photonics

University of Ottawa

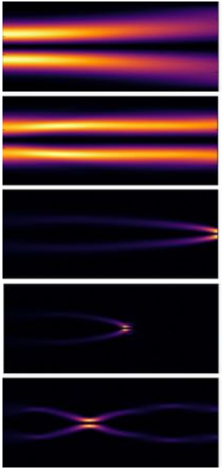
May 24, 2016

Orbital Angular Momentum (OAM)

- Helical wavefronts
- Laguerre-Gauss modes ($\mathrm{LG}_{p,\ell}$)
  - ℓ appears in $e^{i\ell\varphi}$ term
  - $\mathrm{LG}_{p,\ell}$: $\ell\hbar$ OAM per photon
  - $\mathrm{LG}_{p,\ell}$: wavefront consists of ℓ intertwined helices
  - Set of orthonormal solutions to the paraxial wave equation



Experimental Setup – Nonlinear Medium



Linear Diffraction*

Kerr Nonlinearities: Self-Trapping

Kerr Nonlinearities: Self-Focusing

Kerr Nonlinearities: Filamentation

Saturable Kerr Nonlinearities**

- Paraxial wave equation (*)

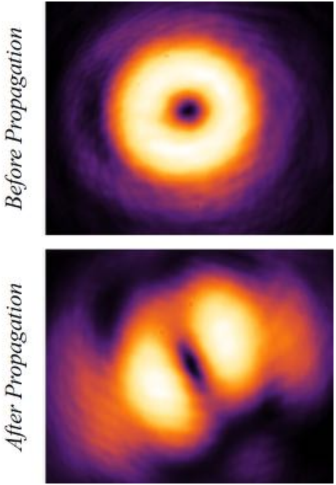
$$\frac{\partial E}{\partial \zeta} - \frac{i}{2}\nabla_\perp^2 E = 0$$

- Nonlinear Schrödinger equation (**):
  - γ: nonlinear parameter
  - σ: saturation parameter

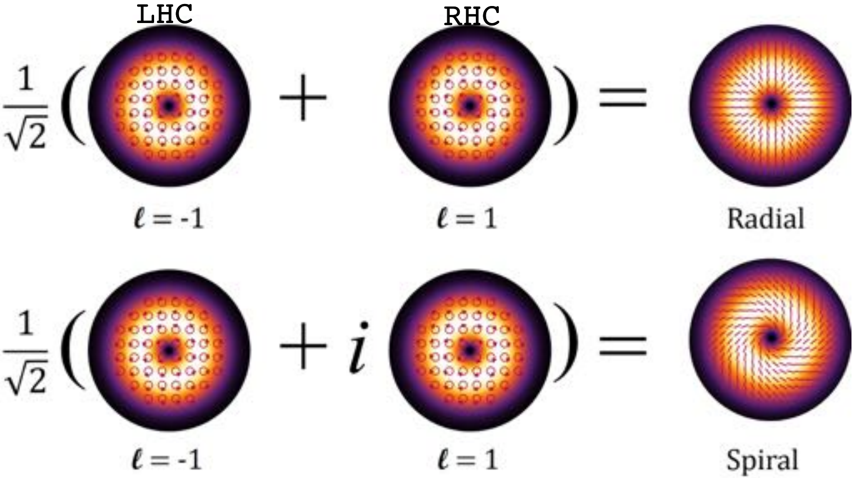
$$\frac{\partial E}{\partial \zeta} - \frac{i}{2}\nabla_\perp^2 E = i\gamma \frac{|E|^2}{1+\sigma|E|^2} E$$

OAM Carrying Beams in Nonlinear Media

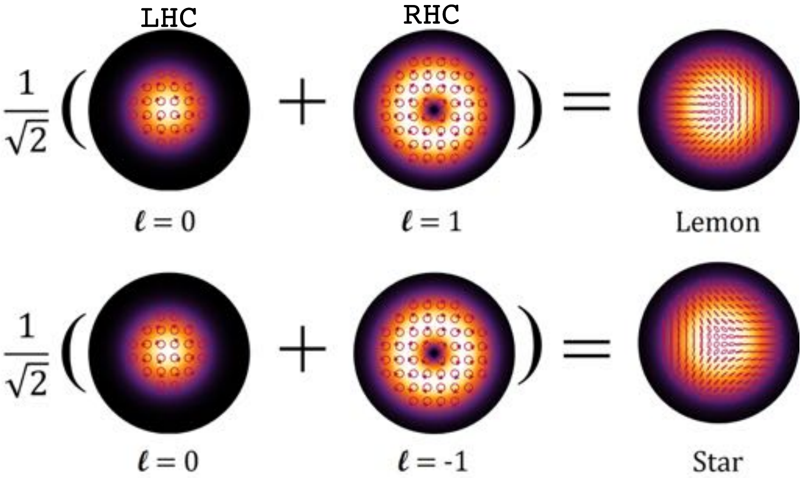
- Modulational instabilities in OAM carrying beams $\rightarrow$ Alterations to their intensity profile
  - Beam breakup
  - Filamentation
  - Soliton formation (specific ICs)
- Alternatives: *Structured*/space-varying polarized light beams in a nonlinear medium.



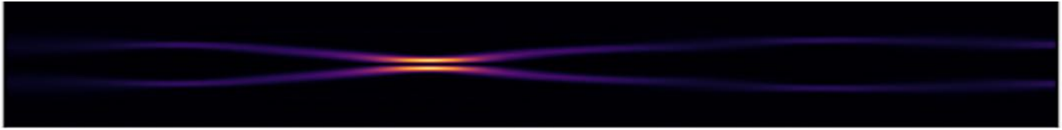
Space-varying Polarized light beams – Vector Vortex Beams



Space-varying Polarized light beams – Poincaré Beams



Space-varying Polarized light beams – Nonlinear Propagation

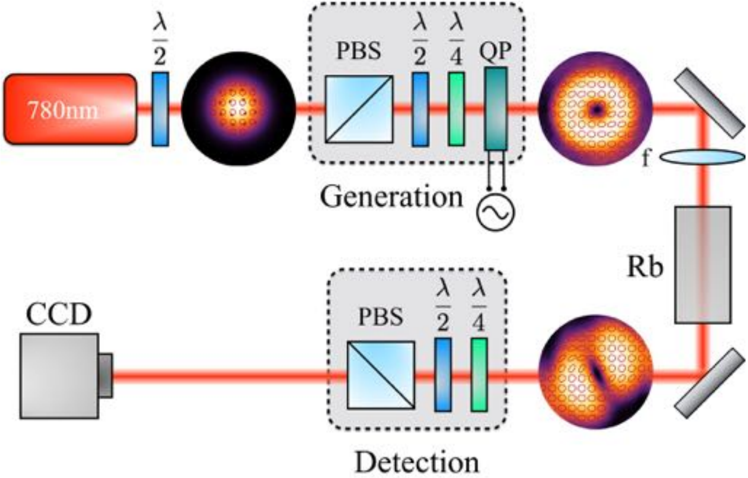


$$\frac{\partial E_L}{\partial \zeta} - \frac{i}{2}\nabla_\perp^2 E_L = i\gamma \frac{|E_L|^2 + \nu |E_R|^2}{1 + \sigma \left(|E_L|^2 + \nu |E_R|^2\right)} E_L$$

$$\frac{\partial E_R}{\partial \zeta} - \frac{i}{2}\nabla_\perp^2 E_R = i\gamma \frac{|E_R|^2 + \nu |E_L|^2}{1 + \sigma \left(|E_R|^2 + \nu |E_L|^2\right)} E_R$$

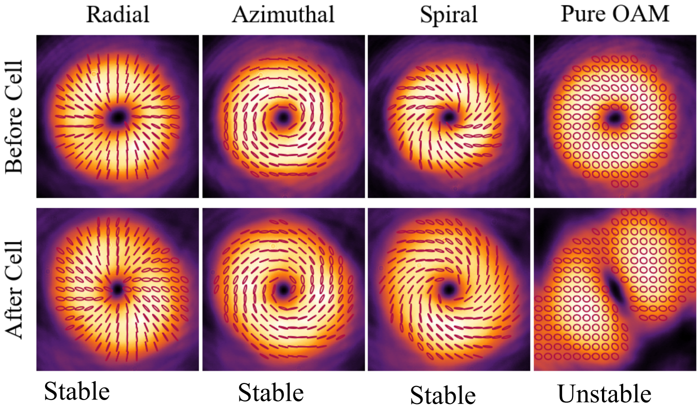
- Coupled nonlinear Schrödinger equations
- ν: coupling parameter
- L: left-handed circular polarization
- R: right-handed circular polarization

Experimental Setup



Results – Vector Beams

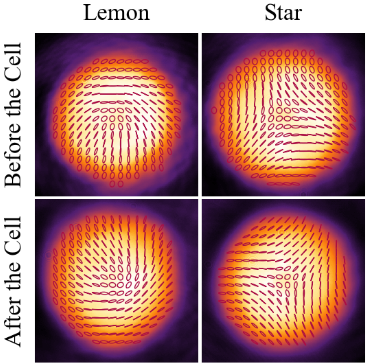
(Experimental Results)



Intensity and polarization distributions of vector and LG beams before and after propagating through the Rb atomic vapour.

Results – Poincaré Beams

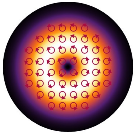
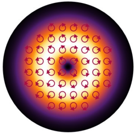
Note that both the lemon and star beams are extremely stable and only undergo an overall rotation.



Intensity and polarization distributions of fundamental topology Poincaré beams before and after propagating through the Rb atomic vapour.

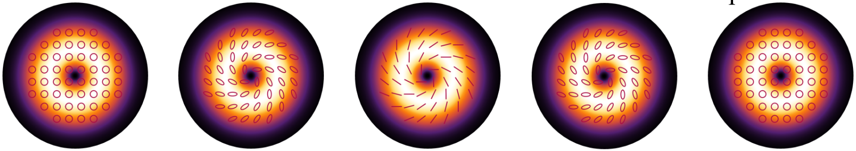
Biased Superpositions

- Generated beams :

$\cos\gamma$  $+ \ \sin\gamma\, e^{i\beta}$ 

ℓ = -1 $\qquad$ ℓ = 1

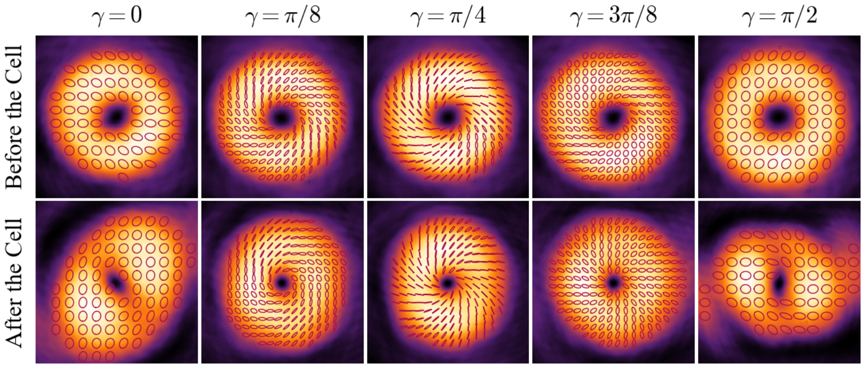
- e.g. : $\exp(i\beta) = -i$



"scalar" beam (uniform circular polarization) | elliptical | vector beams linear | ellipical | "scalar" beam (uniform circular polarization)

$\gamma = 0$ $\quad$ $\gamma = \pi/8$ $\quad$ $\gamma = \pi/4$ $\quad$ $\gamma = 3\pi/8$ $\quad$ $\gamma = \pi/2$

Biased Superpositions – Results

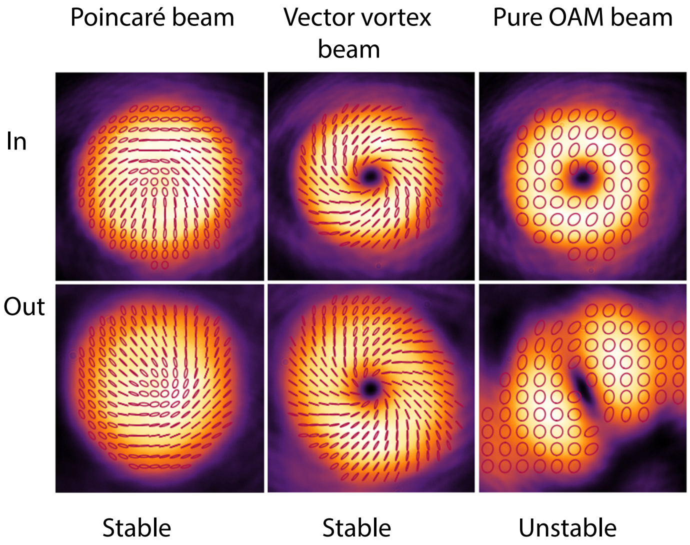


Unstable | Stable | Stable | Stable | Unstable

Scalar beam | Vector beams | Scalar beam

Conclusions: propagation through a nonlinear medium

- Pure OAM beam: beam breakup
- Vector vortex beams: stable propagation
- Poincaré beams: stable propagation



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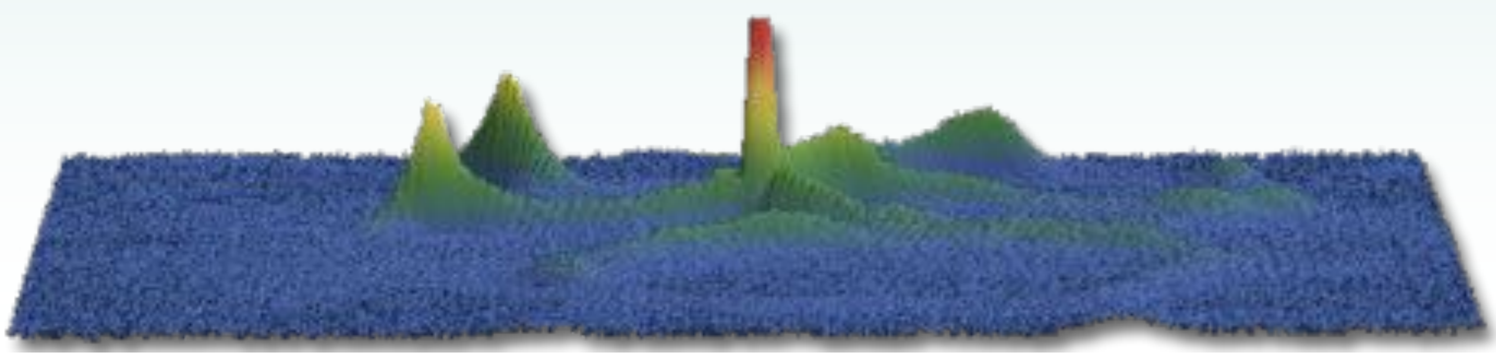
Effect of Nonlinearity on Optical Rogue Waves

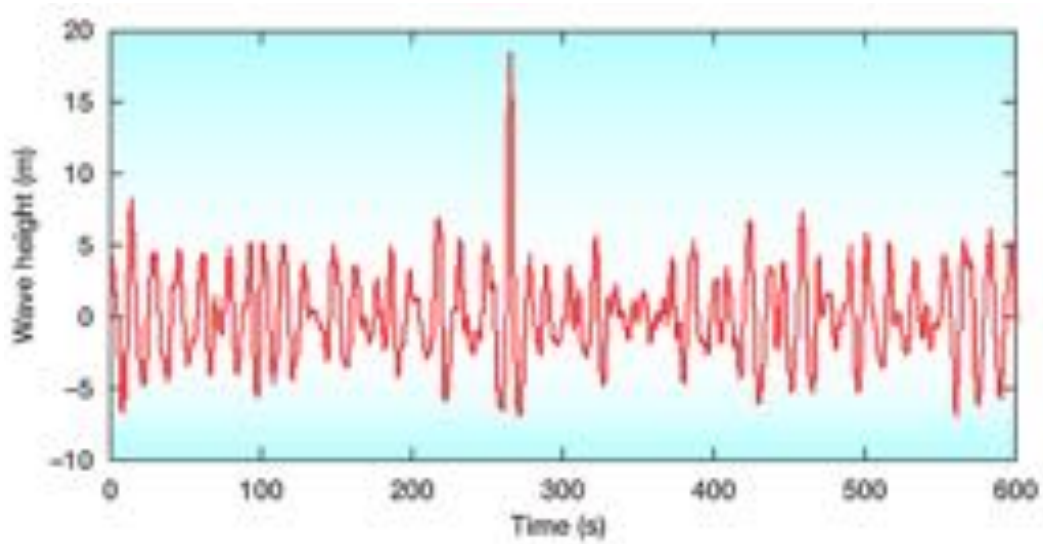
Akbar Safari[1], Robert Fickler[1], Miles J Padgett[2] and Robert W. Boyd[1,2,3]

[1]Department of Physics and Max Planck Centre for Extreme and Quantum Photonics, University of Ottawa, Ottawa, ON, Canada
[2]School of Physics and Astronomy, University of Glasgow, Glasgow G12 8QQ, UK
[3]The Institute of Optics, University of Rochester, Rochester, NY, USA



Rogue wave - Introduction

- Rogue wave: Amplitude significantly larger than the other waves.
- First scientific observation: In an oil platform (Draupner) north of Norway in 1995.

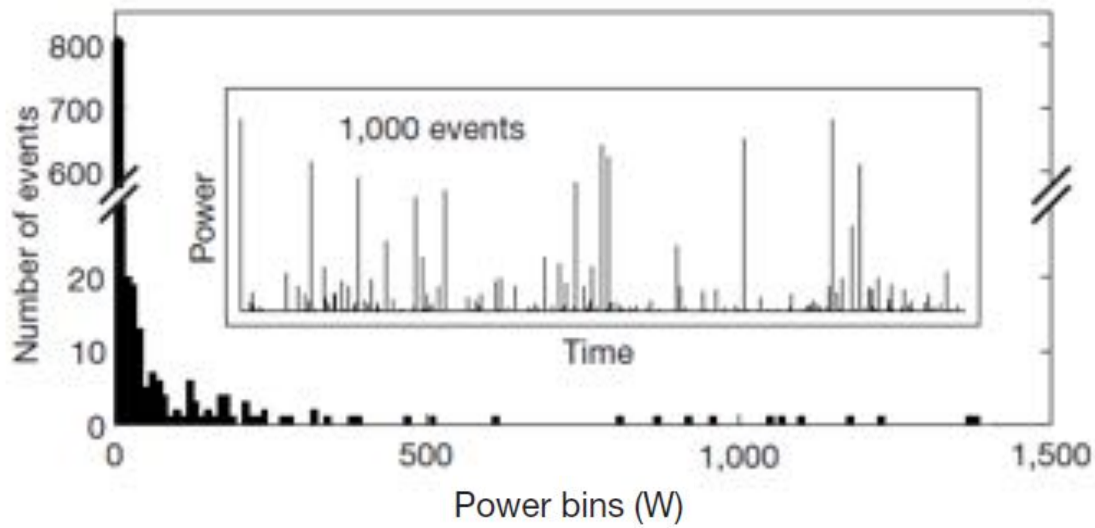


- Rogue waves appear from nowhere and disappear without a trace.
- Rogue waves ≠ accidental constructive interference
- They occur much more frequently than expected in ordinary wave statistics.
- Not limited to ocean: Many other wave systems including optics.

APS Viewpoint: Exciting rogue waves (2009)

Optical rogue wave

First optical rogue wave: supercontinuum fiber[1].



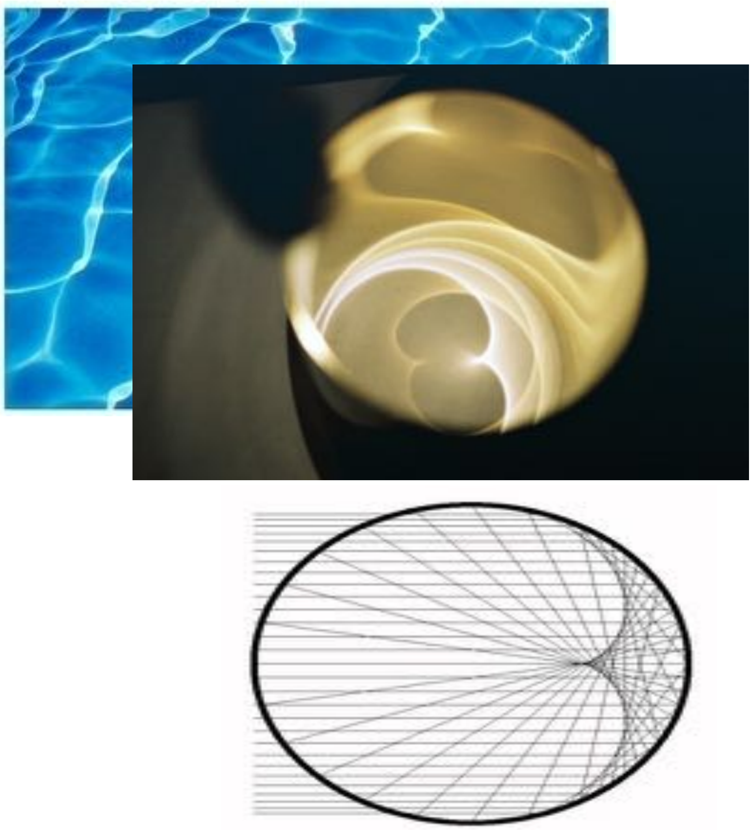
Water waves in oceans
Optical waves in nonlinear fiber } nonlinear Schrödinger equation

- Therefore, nonlinearity could be a key ingredient of rogue waves.
- Rogue waves are observed in linear systems as well!

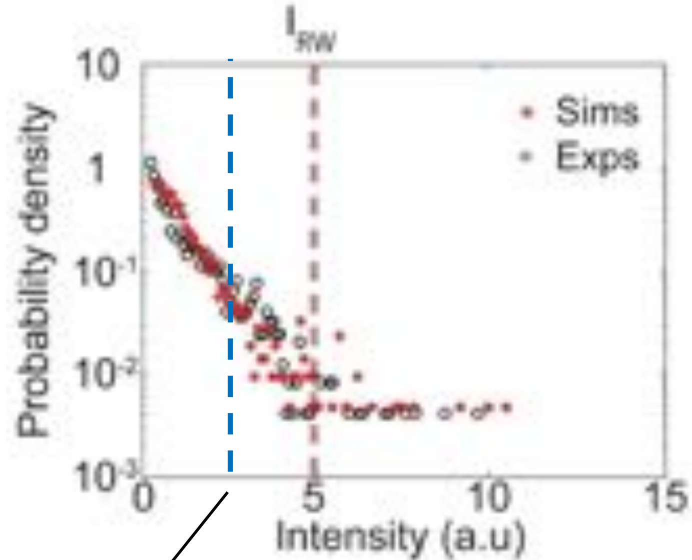
How important is nonlinearity?

1- D. R. Solli, C. Ropers, P. Koonath & B. Jalali, Nature **450**, 1054 (2007)

Spatial rogue wave – Caustic pattern



Caustics: singularities in ray optics.
They have long-tailed statistics.



Significant wave height

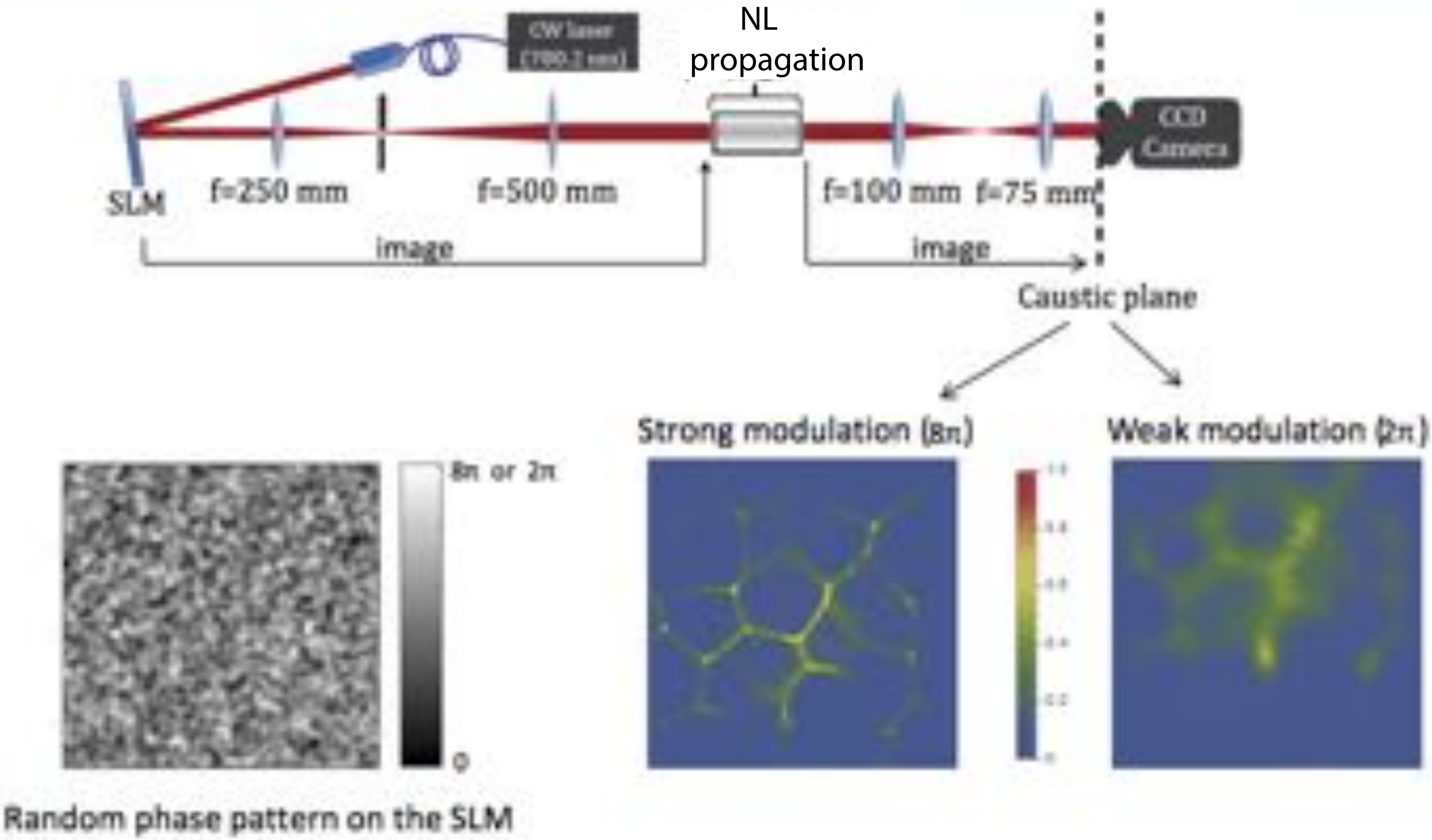
Rogue wave (quantitative definition):

Waves with amplitude higher than two times the significant wave height.
Significant wave height = average of upper third of events

R. Höhmann, U. Kuhl, H. Stöckmann, L. Kaplan, & E. Heller. *Phys. Rev. Lett.* **104**, 093901 (2010).

A. Mathis, L. Froehly, S. Toenger, F. Dias, G. Genty & J. Dudley. *Scientific Reports* **5**, 1 (2015).

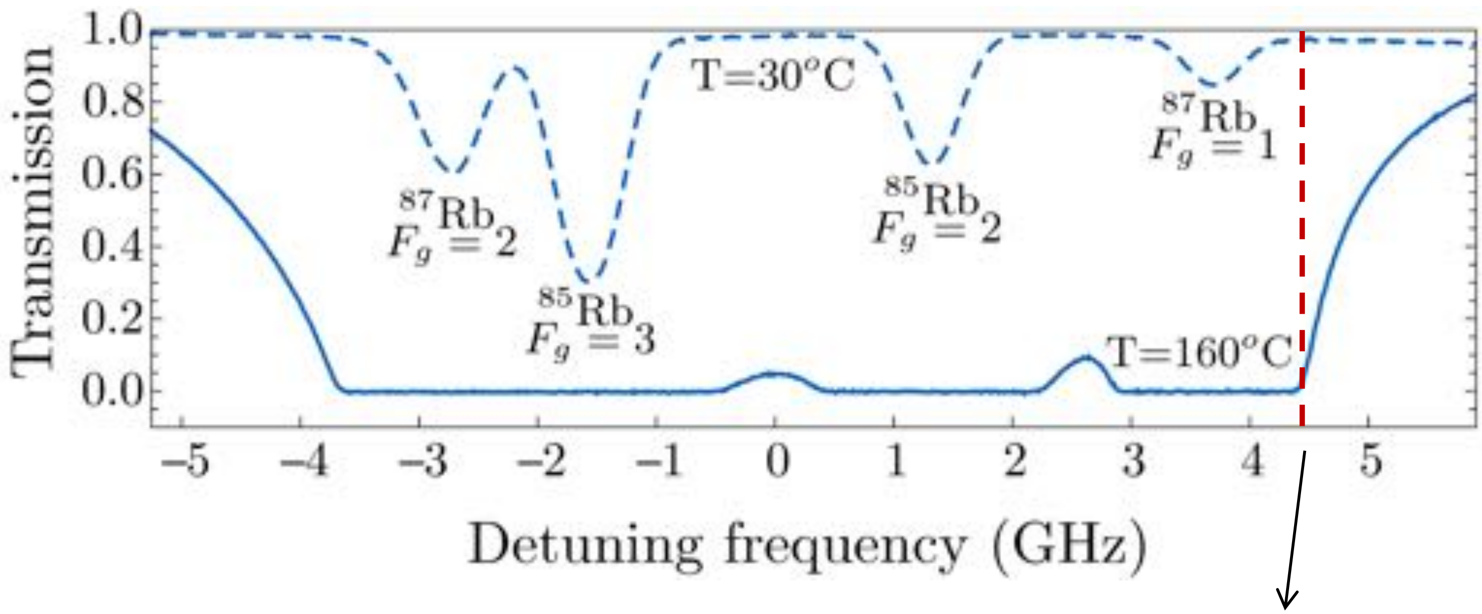
Experimental setup



We study linear and nonlinear propagation in two transverse di

$\chi^{(3)}$ in rubidium

Transmission spectrum of natural rubidium of length L=75mm

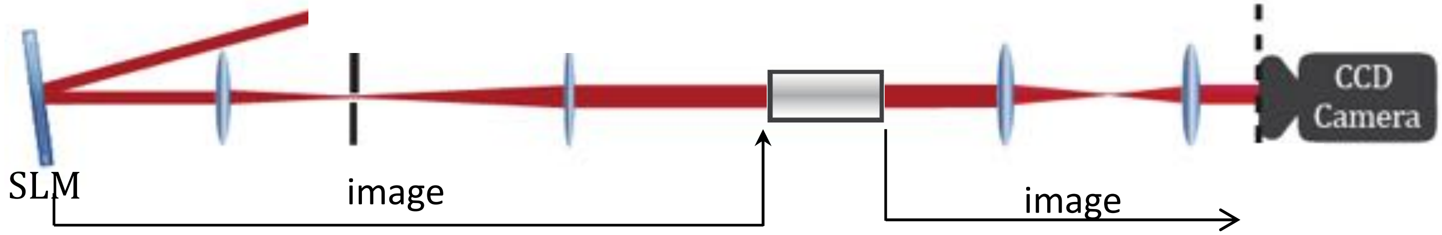


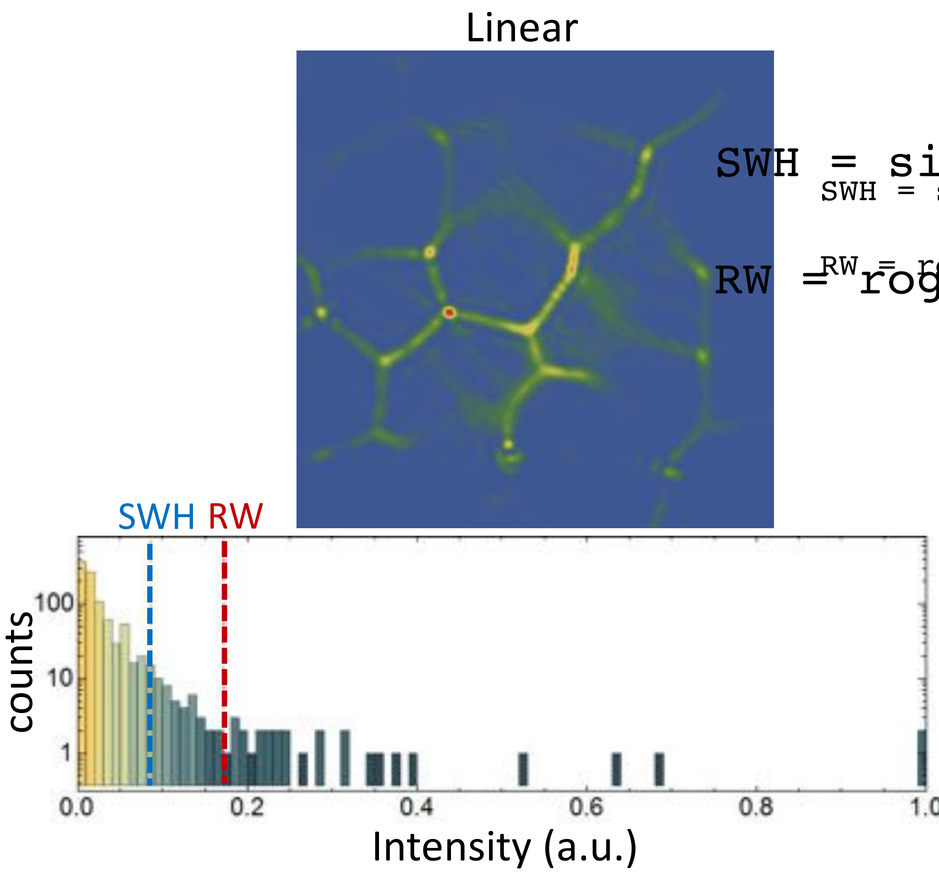
$\mathrm{Re}(\chi^{(3)}) \approx 8\times10^{-12}\ \mathrm{m^2/V^2}$

$\mathrm{Im}(\chi^{(3)}) \approx -5\times10^{-14}\ \mathrm{m^2/V^2}$

(From theoretical model)

Experimental results
(Strong phase modulation)



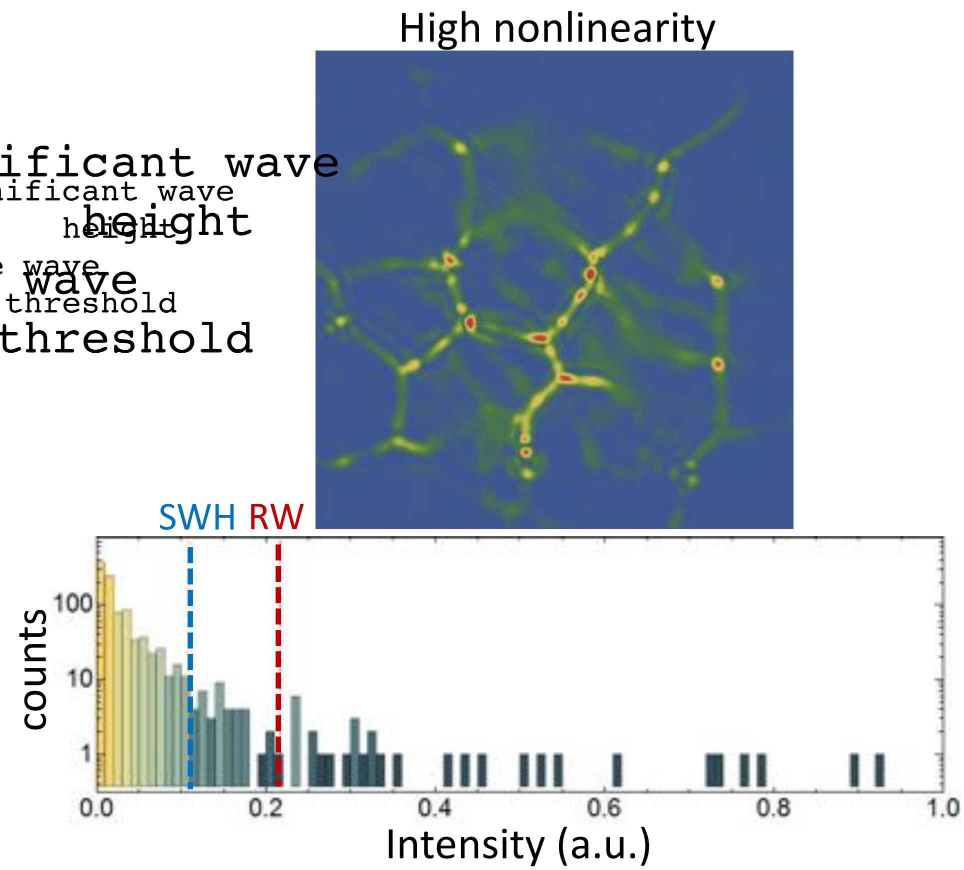


SWH = significant wave height

RW = rogue wave threshold

SWH = significant wave height

RW = rogue wave threshold

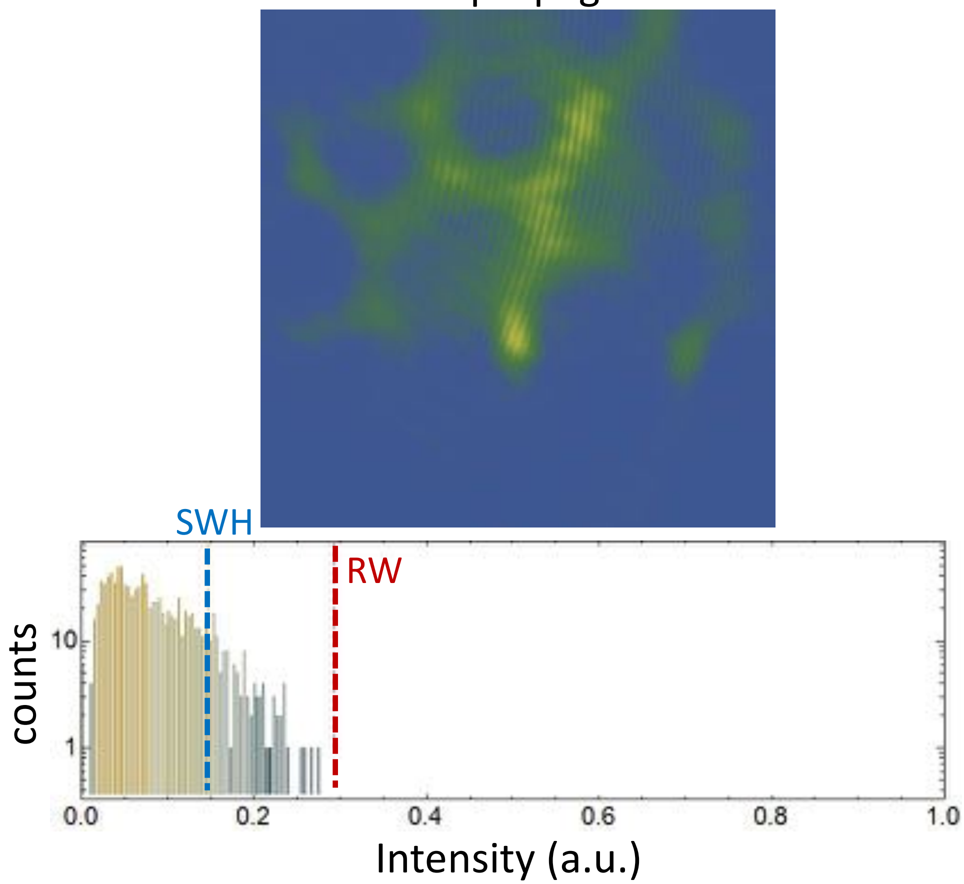


All histograms are taken from 1000 measurements and picking the intensity at the center of each image

Experimental results
(Weak phase modulation)

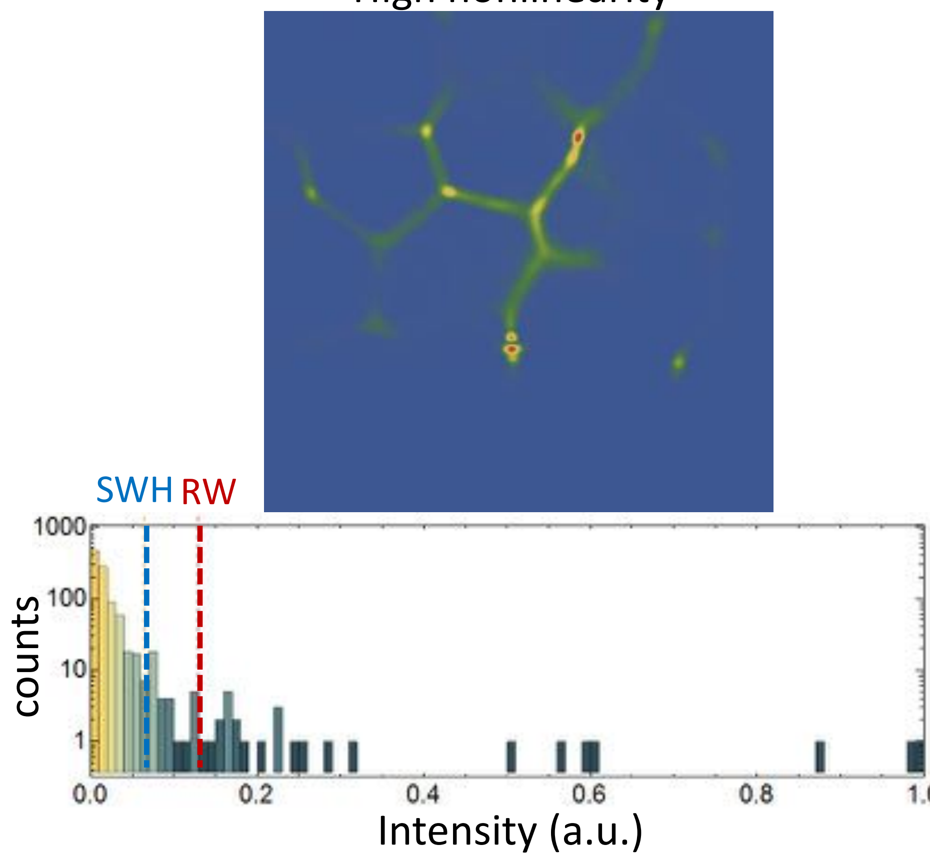
Phase variation = 2π

Linear propagation



No rogue wave

High nonlinearity



Rogue wave (Long tail statistics)

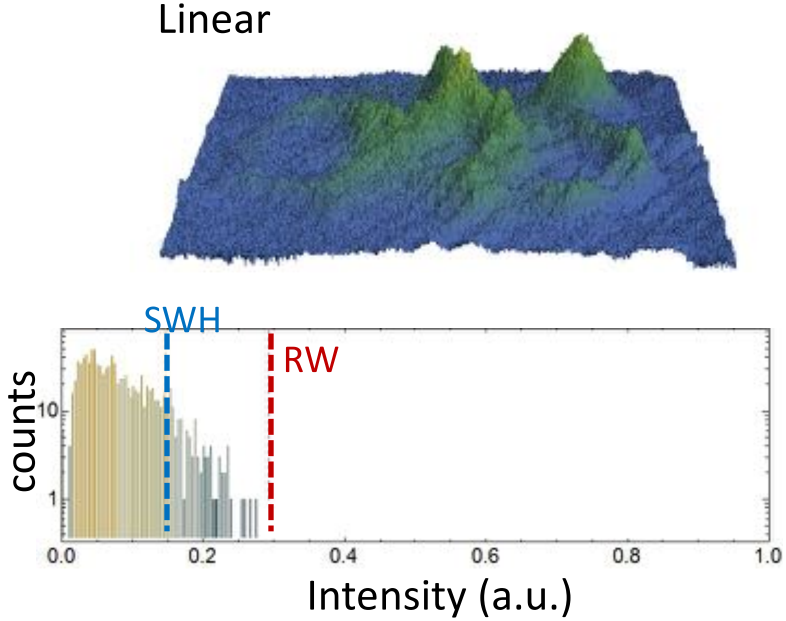
SWH = significant wave height
RW = rogue wave threshold

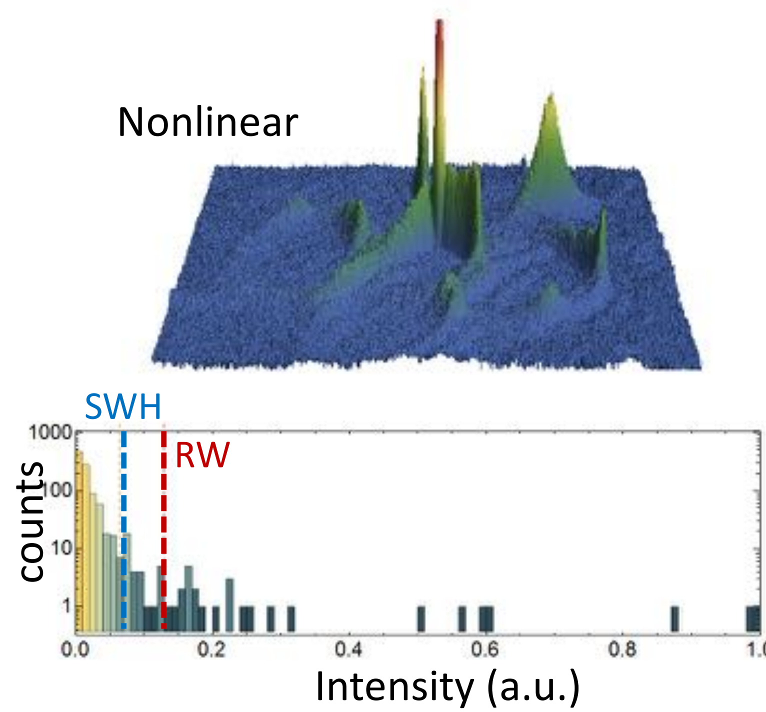
SWH = significant wave height
RW = rogue wave threshold

Conclusion

- Rogue wave in linear propagation requires strong modulation.
- With nonlinear propagation, even a small modulation generates rogue wave.
- Results confirmed by numerical simulation (FFT beam propagation with split-steps)

Nonlinearity enhances the rogue behavior significantly.





Thank you for your attention!