Weak Values and Direct Measurement of the Quantum Wavefunction

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Presented at the Institute for Advanced Studies in Basic Sciences, Zanjan, Iran, February 17, 2015.
How the Result of a Measurement of a Component of the Spin of a Spin-$\frac{1}{2}$ Particle Can Turn Out to be 100

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(Received 30 June 1987)

We have found that the usual measuring procedure for preselected and postselected ensembles of quantum systems gives unusual results. Under some natural conditions of weakness of the measurement, its result consistently defines a new kind of value for a quantum variable, which we call the weak value. A description of the measurement of the weak value of a component of a spin for an ensemble of preselected and postselected spin-$\frac{1}{2}$ particles is presented.

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standard expectation value: \[< A > = \langle \Psi | \hat{A} | \Psi \rangle \]

weak value: \[A_w \equiv \langle \psi_f | \hat{A} | \psi_{\text{in}} \rangle / \langle \psi_f | \psi_{\text{in}} \rangle.\]

Why are weak values important?
can lead to amplification of small signals
can lead to direct measurement of the quantum wavefunction
Birefringence separates polarized beams by 0.64 μm, but gaussian in (b) is displaced by 12 μm.

PRL 1991
Ultrasensitive Beam Deflection Measurement via Interferometric Weak Value Amplification

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(Received 12 January 2009; published 27 April 2009)

We report on the use of an interferometric weak value technique to amplify very small transverse deflections of an optical beam. By entangling the beam’s transverse degrees of freedom with the which-path states of a Sagnac interferometer, it is possible to realize an optical amplifier for polarization independent deflections. The theory for the interferometric weak value amplification method is presented along with the experimental results, which are in good agreement. Of particular interest, we measured the angular deflection of a mirror down to $400 \pm 200$ frad and the linear travel of a piezo actuator down to $14 \pm 7$ fm.
Direct Measurement of the Quantum Wavefunction

The wavefunction of quantum mechanics is notoriously difficult to measure. Measurement at any one position causes the entire wavefunction to “collapse.”

(The problem is related to the Heisenberg Uncertainty Relation; if you measure position you cannot also know momentum.)

Historically, the wavefunction has been measured only indirectly and inefficiently, using “quantum state tomography.”

Recent work has demonstrated how to measure the wavefunction directly.

The idea is to perform a “weak measurement” on one variable (which thus only minimally disturbs the system), followed by a “strong measurement.”

J. Lundeen et al., Nature 474, 188 (2011)
J. Z. Salvail et al., Nature Photonics, 10.1038 (2013)
Direct measurement of the quantum wavefunction

Jeff S. Lundeen¹, Brandon Sutherland¹, Aabid Patel¹, Corey Stewart¹ & Charles Bamber¹

\[ \langle A \rangle_W = \frac{\langle c | A | \Psi \rangle}{\langle c | \Psi \rangle} \]

Returning to our example of a single particle, consider the weak measurement of position \( A = \pi_x \equiv |x \rangle \langle x| \) followed by a strong measurement of momentum giving \( P = p \). In this case, the weak value is:

\[ \langle \pi_x \rangle_W = \frac{\langle p | x \rangle \langle x | \Psi \rangle}{\langle p | \Psi \rangle} \]

\[ = \frac{e^{ipx/\hbar} \Psi(x)}{\Phi(p)} \]

In the case \( p = 0 \), this simplifies to

\[ \langle \pi_x \rangle_W = k \Psi(x) \]

where \( k = 1/\Phi(0) \) is a constant (which can be eliminated later by normalizing the wavefunction). The average result of the weak measurement gives the expectation value of the particle.
Direct Measurement of the Photon “Wavefunction”

Measurement setup

Typical results

J. Lundeen et al., Nature 474, 188 (2011)
Wavefunction of the Photon?

Many people feel it is inaccurate to speak of the “wavefunction” of the photon.

I personally try to avoid the term wavefunction of the photon. A photon is an excitation of a mode of the field. I prefer to distinguish the photon from the mode in which it lives.

Lundeen et al. use the term “wavefunction of the photon,” but comment that it is sometimes called the “spatial mode of the photon.”

1. We have made a direct measurement of the state of polarization of the photon. This is thus the first direct measurement of a qubit.


2. We have measured the statevector of a state imbedded in a 27-dimensional OAM Hilbert space. One expects direct measurement to be increasingly useful with increasing size of the Hilbert space.


3. We have performed amplification of angular rotations using weak measurements.

Full characterization of polarization states of light via direct measurement

Jeff Z. Salvail\textsuperscript{1*}, Megan Agnew\textsuperscript{1}, Allan S. Johnson\textsuperscript{1}, Eliot Bolduc\textsuperscript{1}, Jonathan Leach\textsuperscript{1}
and Robert W. Boyd\textsuperscript{1,2}

\[ |\psi\rangle = \alpha |H\rangle + \beta |V\rangle. \]
Experiment 1

Experiment 2

Wavefunction

Dirac distribution

LP [D] or [A]

Calcite NF FF [D] [H/V]

Quartz crystal NPBS H/V [ψ] /2 /4

State preparation Weak measurement

Readout

CCD D/A

strong measurement weak measurement
Direct Measurement of the Full Density Matrix


Salvail et al, 2013
First weak measurements made on optical polarization states

Mar 11, 2013  4 comments

Physicists in Canada and the US claim to be the first to make a direct measurement of the polarization quantum state of light—a feat that at first glance appears to defy Heisenberg's uncertainty principle. The technique, which relies on a process known as weak measurement, could help in fundamental studies on quantum mechanics or in the development of quantum computing.

Jeff Salvail

Canadian researchers take a sneak peek at Schrödinger’s Cat and a step toward a quantum computer

In an Ottawa lab, scientists have succeeded in side-stepping an obstacle of Heisenberg's Uncertainty Principle, a strange law of the quantum world.

Lawrence Krauss: The Large Hadron Collider helps science do a quantum leap.

Neill Turok: Furthering our understanding of the universe.

Fusing antimatter could allow scientists to study the strangest stuff in existence.

Canadian researcher.

Scientists discover a way around Heisenberg’s Uncertainty Principle

A new technique for quantum mechanics.

Science Recorder | Mark Petillo | Monday, March 04, 2013

It could be big news of the world of quantum physics. According to a pair of scientists from the University of Rochester and the University of Ottawa, there may be a way around Heisenberg’s famous Uncertainty Principle. According to a report published this week in Nature Photonics, a recently developed technique that allows scientists to directly measure the polarization states of light could be the key... To continue reading, subscribe to Science Recorder today.
1. We have made a direct measurement of the state of polarization of the photon. This is thus the first direct measurement of a qubit.


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3. We have performed amplification of angular rotations using weak measurements.

We are constructing a QKD system in which each photon carries many bits of information

We encode in states that carry OAM such as the Laguerre-Gauss states

As a diagnostic, we need to be able to measure the statevector of OAM states

**Single Photon States**

*Laguerre-Gaussian Basis* \( \ell = -13, \ldots, 13 \)

```
0 1 2 .... 12 13 14 .... 25 26 27
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*“Angular” Basis (mutually unbiased with respect to LG)*

```
0 1 2 .... 12 13 14 .... 25 26 27
```

\[
\Psi_{AB}^N = \frac{1}{\sqrt{27}} \sum_{l=-13}^{13} \text{LG}_{l,0} \exp \left( i2\pi N l / 27 \right)
\]
Direct Measurement of a High-Dimensional OAM State


Readout

strong measurement of angular position

read out two orthogonal polarizations

weak measurement of OAM

strong measurement of OAM

weak measurement of OAM

Results: Direct Measurement of a High-Dimensional OAM State
Direct Measurement Procedure Properly Measures Phase

(a) Proper Amplitude $|\psi(\ell)|$ vs. OAM mode number $\ell$

(b) Proper Density $|\psi(\ell)|^2$ vs. OAM mode number $\ell$

(c) Phase difference $\Delta \phi(\ell)$ vs. OAM mode number $\ell$
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Amplification of Angular Rotations using Weak Measurements

First demonstration of weak--value amplification in the azimuthal variables of angular position and orbital angular momentum.

Amplification of angular rotations using WM

State preparation or pre-selection

\[ f(\phi) = e^{\frac{\phi^2}{2 \sigma_\phi^2}} \]
\[ |\Psi_{pr}\rangle = \frac{1}{\sqrt{2}} (|H\rangle + |V\rangle) \]

\[ |\Psi_i\rangle = |f(\phi)\rangle |\Psi_{pr}\rangle \]

Amplification of angular rotations using WM

\[ \Psi_i = f(\phi) \Psi_{pr} \]

\[ |\Psi_i\rangle = |f(\phi)\rangle |\Psi_{pr}\rangle \]

\[ \hat{H}_{\text{int}} = \hat{H}_{SO} + \hat{H}_{g} \]

\[ |\Psi_f\rangle = e^{-i\theta} |f(\phi - \Delta\phi / 2)\rangle |H\rangle + e^{i\theta} |f(\phi + \Delta\phi / 2)\rangle |V\rangle \]

This is the rotation that we want to amplify!

Polarization sensitive Sagnac interferometer

Amplification of angular rotations using WM

Real Weak Values

Experimental data for the real part of the weak value

Amp: Amplification of rotation
PA: Post-selection angle

The expected value of the angular position $\langle \hat{\phi} \rangle \propto \frac{\Re(\sigma_w) \Delta \phi}{2}$
Conclusions

• We have made the first step towards the study of WVA in the azimuthal degree of freedom.

• The shift in angular position is related to the real part of the weak value of a polarization operator.

• The OAM spectrum is shifted as a consequence of the breakup in the polarization symmetry realized by a differential geometric phase, which produces imaginary weak values.

• We believe that our protocol opens the possibility for new schemes in optical metrology.