

Optical Forces and Fresnel Drag in Atomic Vapor “Slow-Light” Media

**Akbar Safari,¹ Israel De Leon,¹ Mohammad Mirhosseini,²
Omar S. Magana-Loaiza,² and Robert W. Boyd^{1,2,3}**

1. Department of Physics, University of Ottawa

2. Institute of Optics, University of Rochester

3. School of Physics and Astronomy, University of Glasgow

Presented at the Institute for Advanced Studies in Basic Sciences, Zanjan,
Iran, February 15, 2015.





Why Care about Optical Forces?

- Optical levitation
- Optical tweezers
- Optomechanical systems
- But can we **control** optical forces.
 - Yes! Photon momentum and optical forces depend on both refractive index and group index of optical materials.
 - The “slow light” community knows how to manipulate the group velocity of light.

Controlling the Velocity of Light

“Slow,” “Fast” and “Backwards” Light

– Light can be made to go:

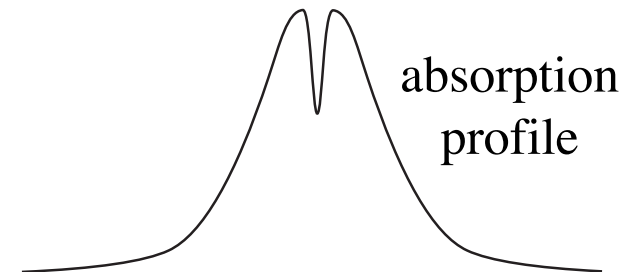
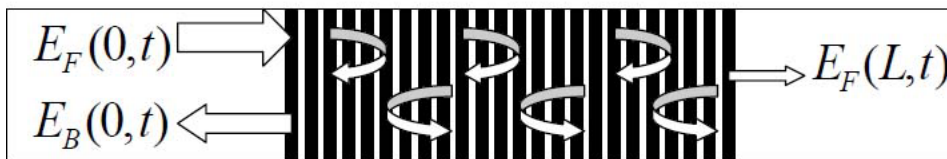
slow: $v_g \ll c$ (as much as 10^6 times slower!)

fast: $v_g > c$

backwards: v_g negative

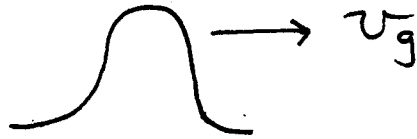
Here v_g is the **group velocity**: $v_g = c/n_g$ $n_g = n + \omega (dn/d\omega)$

– Velocity controlled by structural or material resonances



Group Velocity

Pulse
(wave packet)



Group velocity given by $v_g = \frac{d\omega}{dk}$

$$\text{For } k = \frac{n\omega}{c} \quad \frac{dk}{d\omega} = \frac{1}{c} \left(n + \omega \frac{dn}{d\omega} \right)$$

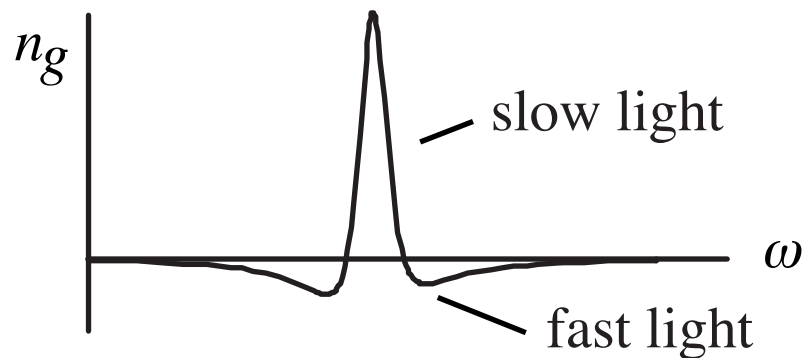
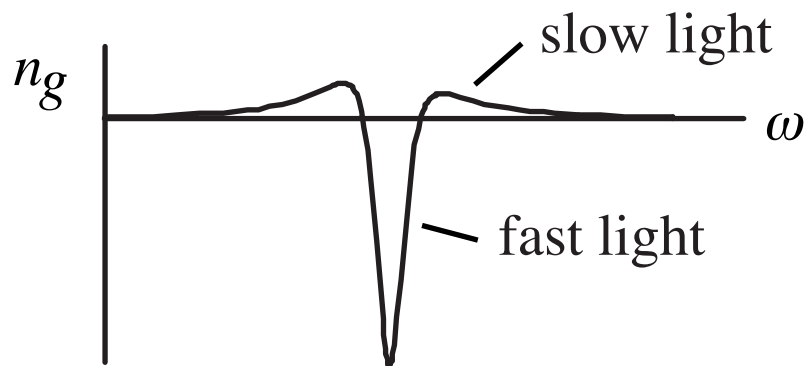
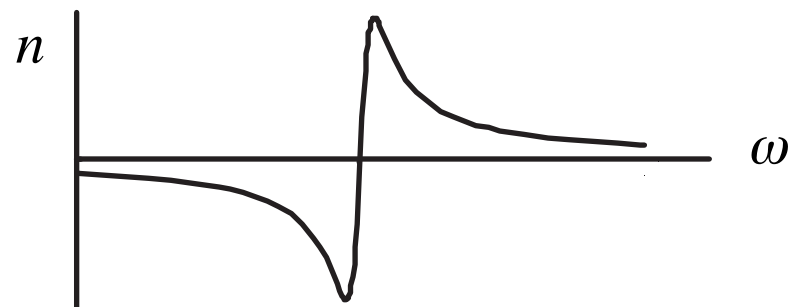
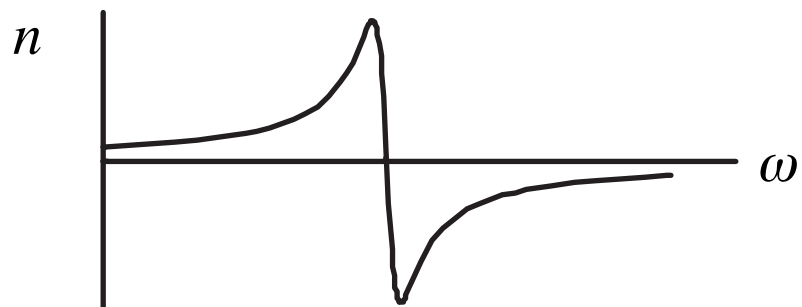
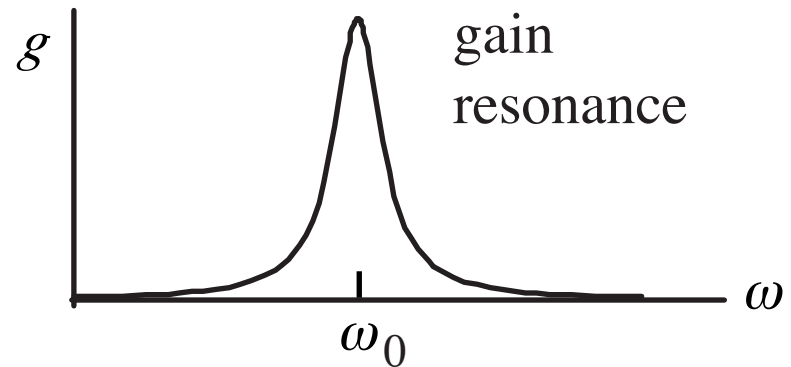
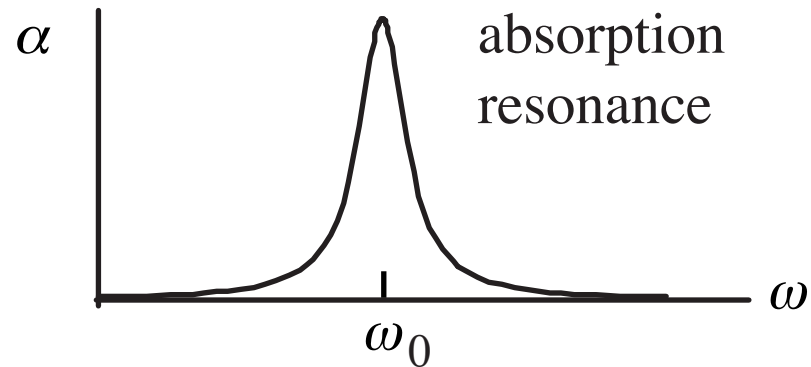
Thus

$$v_g = \frac{c}{n + \omega \frac{dn}{d\omega}} \equiv \frac{c}{n_g}$$

Thus $n_g \neq n$ in a dispersive medium!

Slow Light Fundamentals: How to Create Slow and Fast Light I

Use Isolated Gain or Absorption Resonance



$$n_g = n + \omega (dn/d\omega)$$

Kinematic Properties of Slow and Fast Light

Poynting's Theorem when derived for a dispersive medium leads to the conclusion that

$$S = \frac{1}{2} n \epsilon_0 c E^2 \quad (\text{intensity})$$

$$u = \frac{1}{2} n n_g \epsilon_0 E^2 \quad (\text{energy density})$$

where

$$v_g = c/n_g \quad (\text{group velocity}).$$

It thus follows that

$$S = u v_g.$$

Note:

Large enhancement of stored energy

But no enhancement of E!

See, e.g., Haus, Landau and Lifshitz, Milonni, or Harris and Hau

What is the Momentum of a Photon?

In vacuum: $p = (\hbar\omega/c)$

Abraham form (for matter)

$P = E \times H / c^2$ (EM momentum density)

$p = (\hbar\omega/c)(1/n_g)$ (photon momentum)

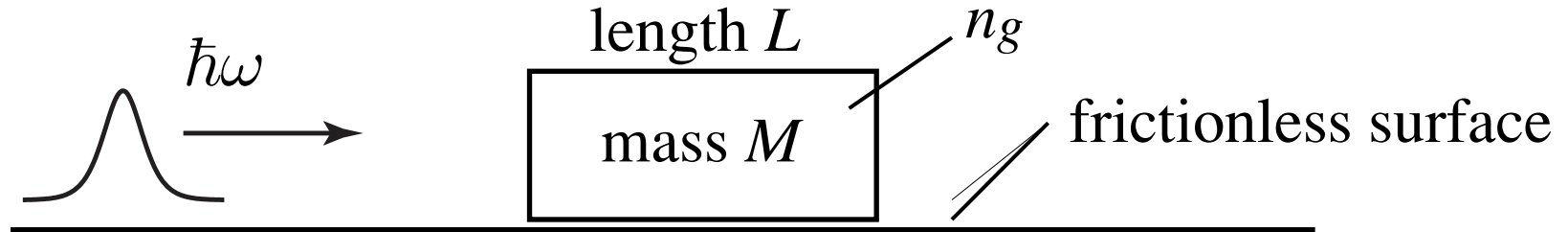
Minkowski form (for matter)

$P = D \times B$ (EM momentum density)

$p = (\hbar\omega/c)(n^2/n_g)$ or $p = (\hbar\omega/c) n$ photon momentum

One way or other, photon momentum very small in slow-light medium

Einstein-Balazs Argument Supports the Abraham Form



- Argue that center of mass-energy must move with a constant velocity
- When photon wavepack enters block, it slows down. Block thus receives a kick into the forward direction.
- When photon leaves block, block receives backward kick and returns to rest.

- Block undergoes longitudinal displacement of

$$\Delta z = (n_g - 1)L \hbar\omega / (Mc^2)$$

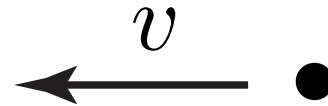
- Simple kinematic argument shows that momentum of photon in block is

$$p = \hbar\omega / (n_g c) \quad \text{Abraham form!}$$

Fermi's Argument Supports the Minkowski Form



photon in medium of
refractive index $n(\omega)$



atom with mass m and
resonance frequency ω_0

- Fermi describes Doppler effect in terms of atomic recoil (RMP, 1932)
- Atom can absorb only if $\omega \approx \omega_0(1 - nv/c)$
- Conservation of energy and momentum

$$\text{Initial energy} = \hbar\omega + \frac{1}{2}mv^2 \qquad \text{Final energy} = \hbar\omega_0 + \frac{1}{2}mv'^2$$

$$\text{Initial momentum} = p + mv \qquad \text{Final momentum} = mv'$$

- Solve: find photon momentum p in medium given by

$$p = n \hbar\omega / c \qquad \text{Minkowski form!}$$

Which is Correct, Abraham or Minkowski?

PRL **104**, 070401 (2010)

PHYSICAL REVIEW LETTERS

week ending
19 FEBRUARY 2010



Resolution of the Abraham-Minkowski Dilemma

Stephen M. Barnett

Department of Physics, SUPA, University of Strathclyde, Glasgow G4 0NG, United Kingdom

(Received 7 October 2009; published 17 February 2010)

The dilemma of identifying the correct form for the momentum of light in a medium has run for a century and has been informed by many distinguished contributions, both theoretical and experimental. We show that *both* the Abraham and Minkowski forms of the momentum density are correct, with the former being the kinetic momentum and the latter the canonical momentum. This identification allows us to explain why the experiments supporting each of the rival momenta gave the results that they did. The inclusion of dispersion and absorption provides an interesting subtlety, but does not change our conclusion.

DOI: [10.1103/PhysRevLett.104.070401](https://doi.org/10.1103/PhysRevLett.104.070401)

PACS numbers: 03.50.De, 42.50.Nn, 42.50.Wk

$$\mathbf{p}_{\text{kin}}^{\text{med}} + \mathbf{p}_{\text{Abr}} = \mathbf{p}_{\text{can}}^{\text{med}} + \mathbf{p}_{\text{Min}}.$$

Total momentum (field plus material) the same in either treatment!

What is the Momentum of a Photon?

In vacuum: $p = (\hbar\omega/c)$

Abraham form (for matter)

$P = E \times H / c^2$ (EM momentum density)

$p = (\hbar\omega/c)(1/n_g)$ (photon momentum)

It is the kinetic (as in mv) momentum

It is the momentum of the field (alone)

It is what comes out of Balazs's moving block analysis

Minkowski form (for matter)

$P = D \times B$ (EM momentum density)

$p = (\hbar\omega/c)(n^2/n_g)$ or $p = (\hbar\omega/c) n$ photon momentum

It is the canonical momentum (as in $h/\lambda_{\text{deBroglie}}$)

It is the momentum of field and (at least part of that of the) matter

It is what comes out of a Doppler shift analysis

One way or other, photon momentum very small in slow-light medium

See, e.g., Barnett, PRL (2010), Milonni and Boyd, AOP (2010).

Experiment of She, Yu, and Feng (PRL, 2008)

PRL 101, 243601 (2008)

PHYSICAL REVIEW LETTERS

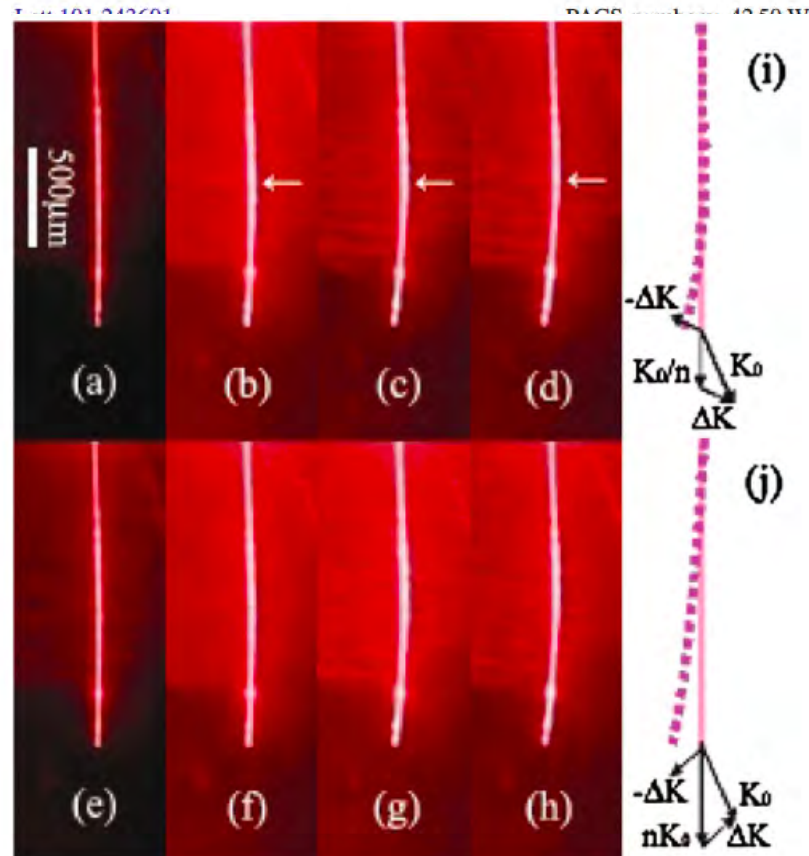
week ending
12 DECEMBER 2008

Observation of a Push Force on the End Face of a Nanometer Silica Filament Exerted by Outgoing Light

Weilong She,^{*} Jianhui Yu,[†] and Raohui Feng

State Key Laboratory of Optoelectronic Materials and Technologies, Sun Yat-Sen University, Guangzhou 510275, China
(Received 12 February 2008; revised manuscript received 15 September 2008; published 8 December 2008)

There are two different proposals for the momentum of light in a transparent dielectric of refractive index n : Minkowski's version nE/c and Abraham's version $E/(nc)$, where E and c are the energy and vacuum speed of light, respectively. Despite many tests and debates over nearly a century, momentum of light in a transparent dielectric remains controversial. In this Letter, we report a direct observation of the inward push force on the free end face of a nanometer silica filament exerted by the outgoing light. Our results suggest that Abraham's momentum is correct.



Photon Recoil Momentum in Dispersive Media

Gretchen K. Campbell, Aaron E. Leanhardt,^{*} Jongchul Mun, Micah Boyd, Erik W. Streed,
Wolfgang Ketterle, and David E. Pritchard[†]

*MIT-Harvard Center for Ultracold Atoms, Research Laboratory of Electronics and Department of Physics,
Massachusetts Institute of Technology, Cambridge, Massachusetts 02139, USA*

(Received 31 January 2005; published 4 May 2005)

A systematic shift of the photon recoil momentum due to the index of refraction of a dilute gas of atoms has been observed. The recoil frequency was determined with a two-pulse light grating interferometer using near-resonant laser light. The results show that the recoil momentum of atoms caused by the absorption of a photon is $n\hbar k$, where n is the index of refraction of the gas and k is the vacuum wave vector of the photon. This systematic effect must be accounted for in high-precision atom interferometry with light gratings.

DOI: 10.1103/PhysRevLett.94.170403

PACS numbers: 03.75.Dg, 39.20.+q, 42.50.Ct

Note that momentum is $n\hbar k$ = Minkowski form = “canonical momentum” as derived by Fermi.

Photon Drag Effects with Slow Light

We would like to use the dependence of the photon momentum on the group index as a means to control optical forces.

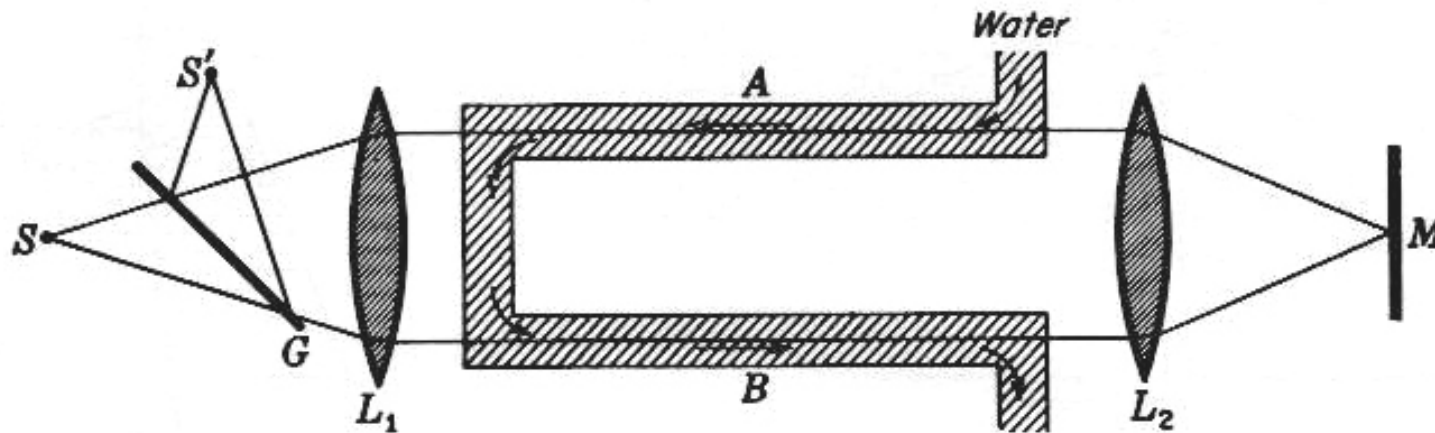
As a first step down this pathway, we are studying how to control photon drag effects using slow light.

Velocity of (Slow) Light in Moving Matter: Photon Drag (or Ether Drag) Effects

Fizeau (1859): Longitudinal photon drag:

Velocity of light in flowing water.

$V = 700 \text{ cm/sec}$; $L = 150 \text{ cm}$; displacement of 0.5 fringe.



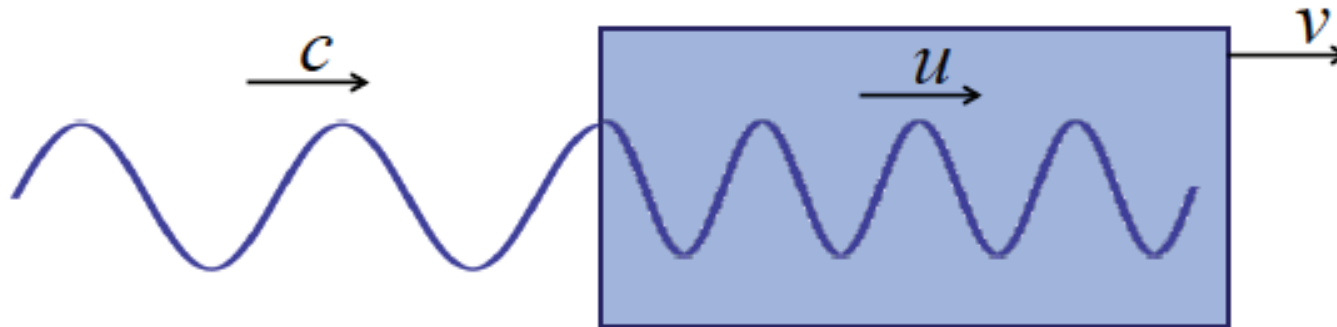
Modern theory: relativistic addition of velocities

$$v = \frac{c/n + V}{1 + (V/c)(1/n)} \approx \frac{c}{n} + V \left(1 - \frac{1}{n^2} \right)$$

Fresnel “drag” coefficient

Fresnel Drag Effect in Nondispersive and Dispersive Media

- Nondispersive medium



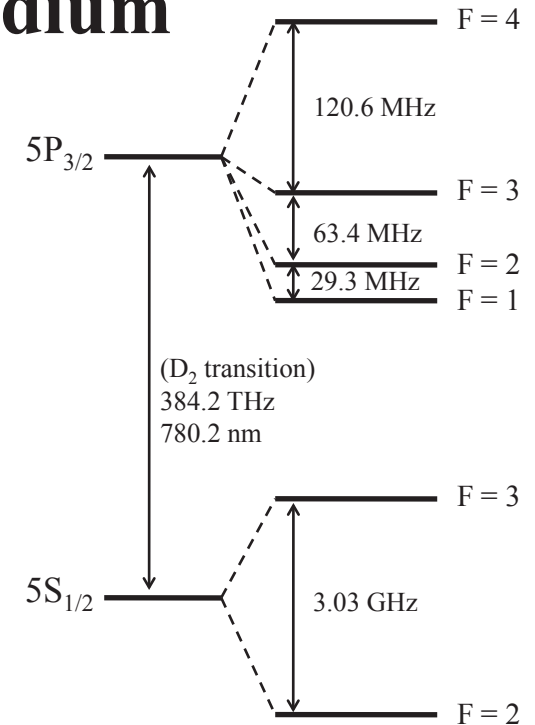
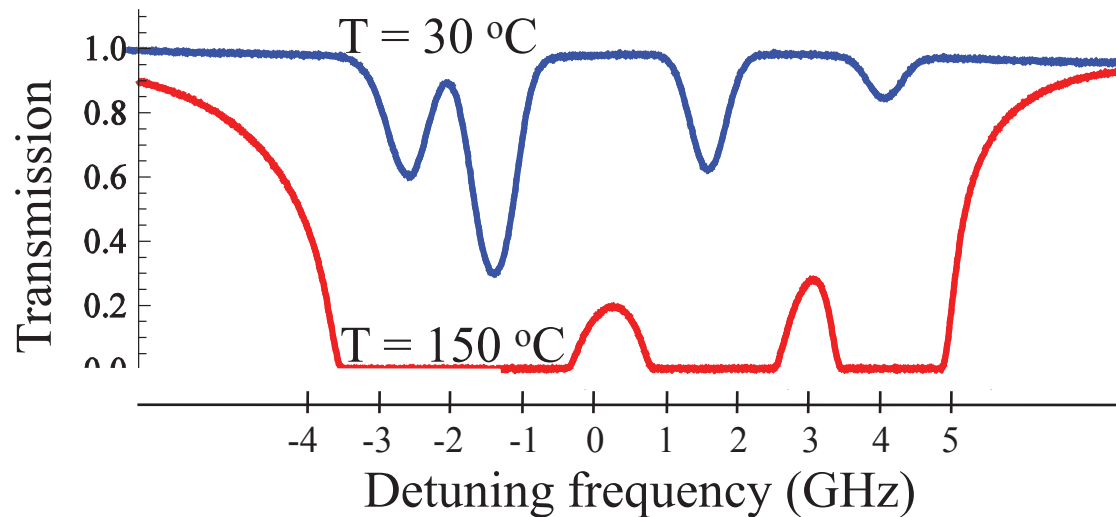
$$u = \frac{c/n \pm v}{1 + v/nc} \simeq \frac{c}{n} \pm v \left(1 - \frac{1}{n^2} \right)$$

- Dispersive medium

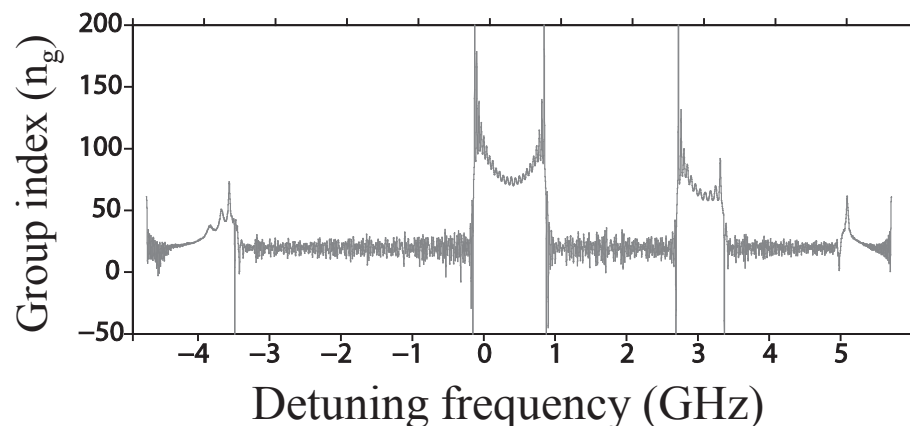
$$u \simeq \frac{c}{n} \pm v \left(1 - \frac{1}{n^2} + \frac{n_g - n}{n^2} \right) \quad \text{where} \quad n_g \equiv n + \omega \frac{dn}{d\omega}$$

We Use Rubidium as Our Slow Light Medium

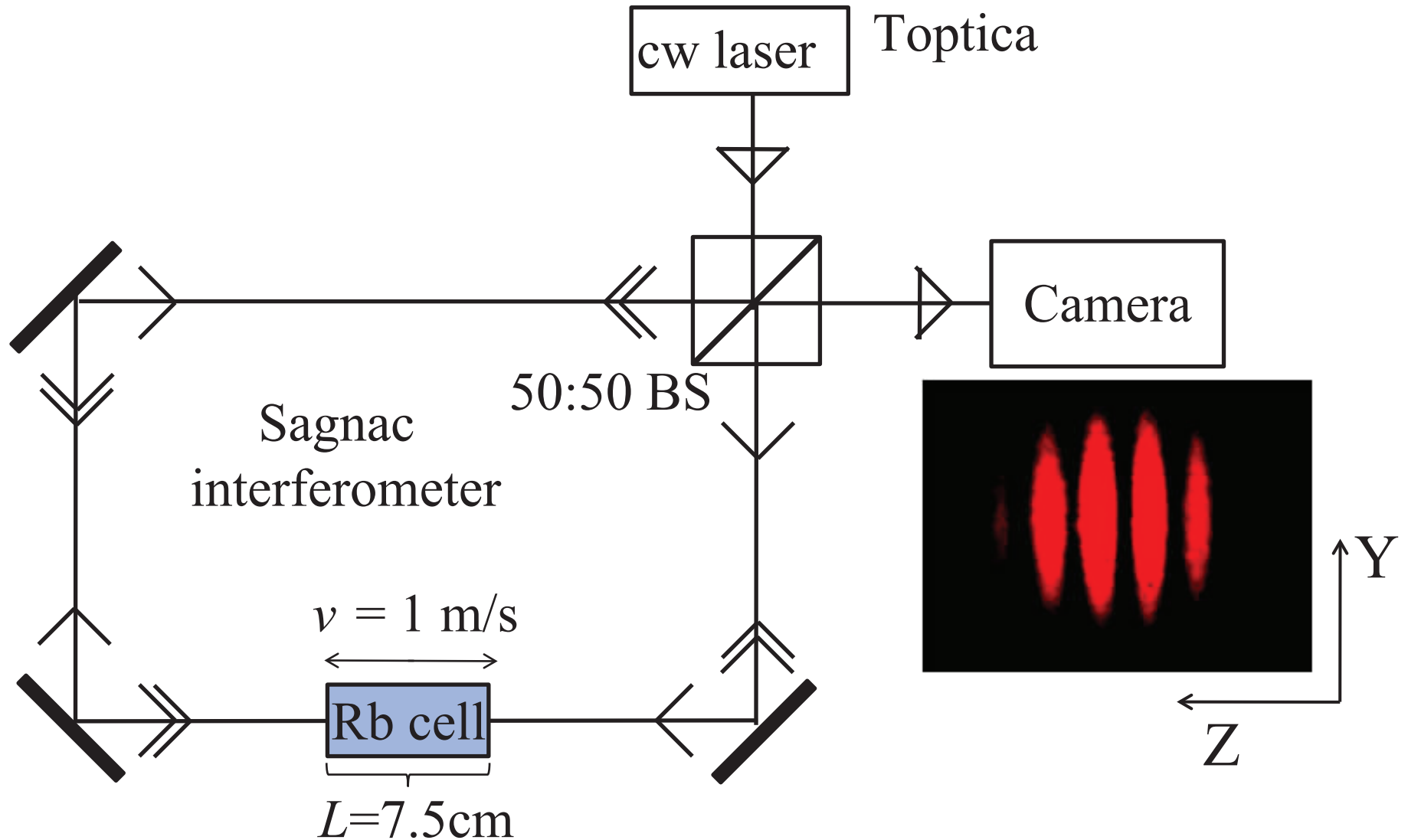
- Transmission spectrum of Rb around D_2 transition:



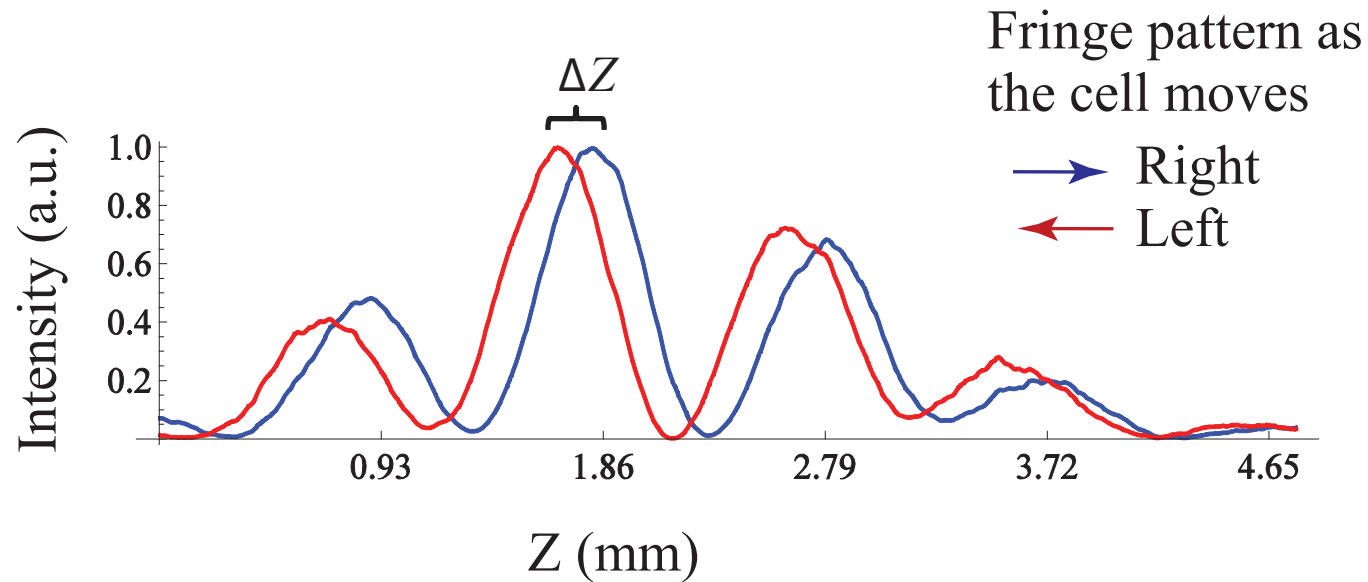
- There is large dispersion where rapid changes in transmission are observed
- Group index of Rb around D_2 transition line at $T=130\text{ }^{\circ}\text{C}$:



Our Experimental Setup

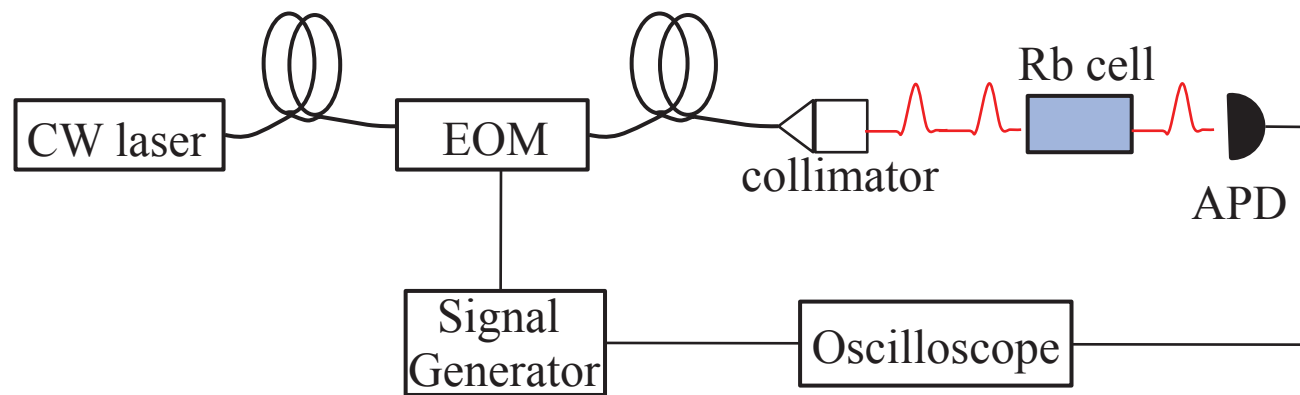


The Fringe Pattern Shifts According to Velocity of the Rubidium Cell



Velocity of cell is (+/-) 1 m/s.

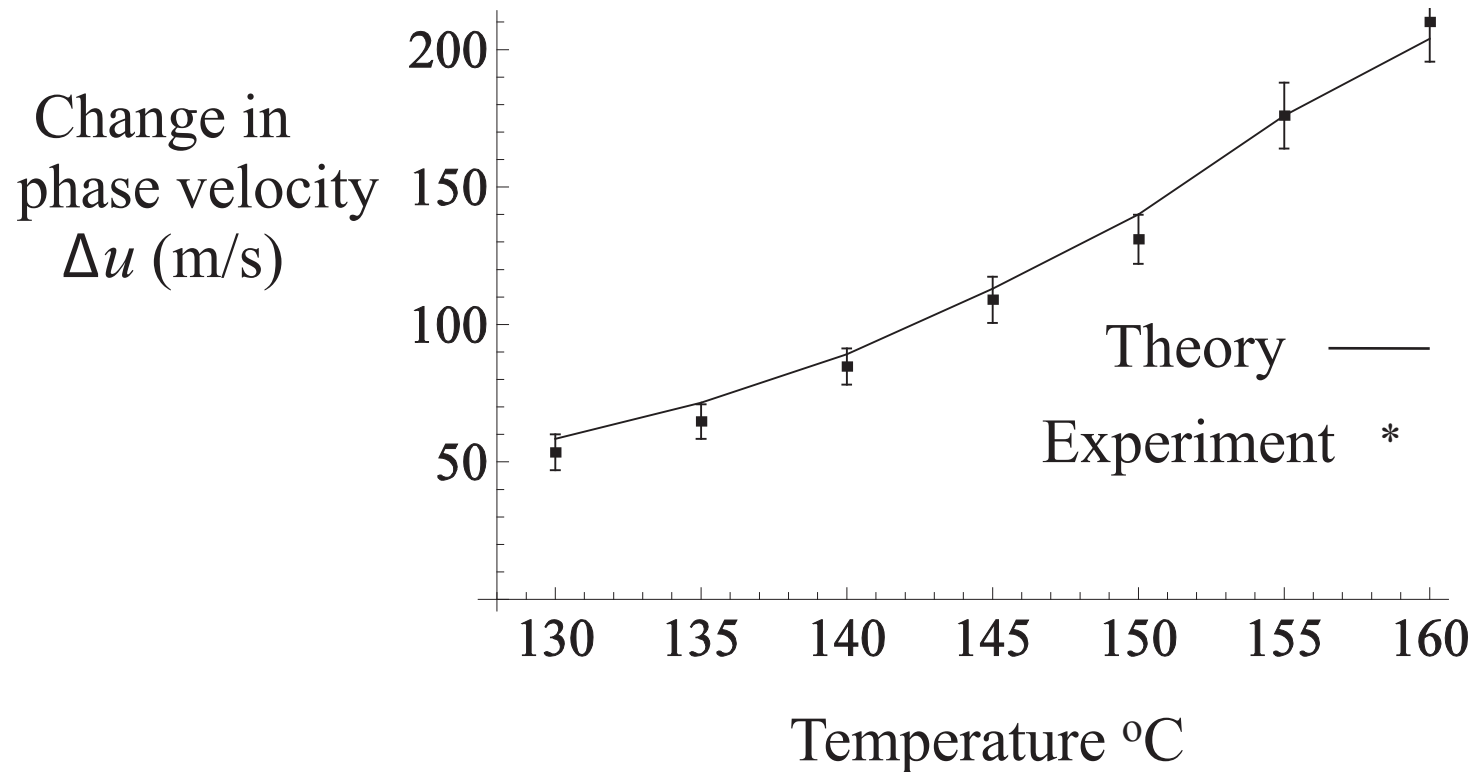
Direct Measurement of the Group Index of Rubidium



Variation of n_g with temperature of the Rb cell:

T	130	135	140	145	150	155	160
n_g	59.4	72.6	90.2	114	141	177	205

Light Dragging Experimental Results



Recall that the rubidium cell was moving at only 1 m/s.

Conclusions

- A maximum drag speed of 205 m/s was measured in a highly dispersive medium (hot Rb vapor).
 - This effect is at least two orders of magnitude larger than that observed to date.
 - Much larger dispersion can be achieved in Rb atoms using electromagnetically induced transparency (n_g as high as 10^7).
-

This project is the PhD thesis topic of one of my graduate students, Akbar Safari. Please note the correct spelling of Akbar.

Note Carefully: Akbar, not Ackbar



Akbar Safari



Admiral Ackbar

the end



Early history: (Fringe shift)

- ❑ 1851: Fizeau
 - Water
 - %16 accuracy
- ❑ 1886: Michelson-Morely
 - Water
 - %5 accuracy
- ❑ 1895: Lorentz
 - Theory
 - Effect of dispersion

$$u = \frac{c}{n} + v \left(1 - \frac{1}{n^2} - \frac{\lambda}{n} \frac{dn}{d\lambda} \right)$$

(Fixed boundaries)

-
-
- (Many experiments to see the dispersion effect)



History of light dragging



- ❑ 1911: Harress
 - Dispersion in glass
 - %2 accuracy (after subtracting the Sagnac effect)

- ❑ 1912-1922: Zeeman
 - Dispersion in glass
 - %1.7 accuracy

$$u = \frac{c}{n} + v \left(1 - \frac{1}{n^2} - \frac{\lambda}{n^2} \frac{dn}{d\lambda} \right)$$

(moving boundaries)
(Laub drag coefficient)

Modern history: (Frequency spilling in a ring resonator)

- ❑ 1964: Macek et al
- ❑ 1972: Bilger and Zavodny
- ❑ 1977: Bilger and Stowell
- ⋮
- ❑ 1988: Sanders and Ezekiel (%0.01 to %0.1 accuracy)

Interpretation of these Results

- How do we understand these results in terms of the physical mechanism that leads to the slow-light effect?
- Our experiment made use of “self-pumped” slow light based on coherent population oscillations (CPO).
- (Need to explain how this works)

Does Orbital Angular Momentum Depend on the Group Index?

JOURNAL OF MODERN OPTICS, 2003, VOL. 50, NO. 10, 1555–1562



Taylor & Francis
Taylor & Francis Group

The angular momentum of light inside a dielectric

MILES PADGETT[†], STEPHEN M. BARNETT[‡] and
RODNEY LOUDON[§]

[†] Department of Physics and Astronomy, University of Glasgow,
Glasgow G12 8QQ, Scotland, UK

[‡] Department of Physics, University of Strathclyde, Glasgow G4 0NG,
Scotland, UK

[§] Department of Electronic Systems Engineering, University of Essex,
Colchester CO4 3SQ, England, UK

E-mail: m.padgett@physics.gla.ac.uk

(Received 30 August 2002)

Abstract. We consider whether or not a short pulse of light carrying angular momentum will exert a torque when propagating through a transparent disc. The approach is based on the ‘Einstein-box’ argument which we apply to discuss linear optical momentum in a medium. Two competing theories due to Minkowski and Abraham, at least superficially, suggest that the disc will not or will rotate. Our analysis suggests that the disc will rotate and that an experiment using optical tweezers should be able to detect the rotation.